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Options: Pricing, Usage, and Greeks

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Options play an important role within most undergraduate and graduate finance programs. Teaching about options and their pricing and usage as well as the theory of implied volatility can be challenging. This study provides content to access actual options data and calculates intrinsic value, time value, implied volatility, and Greeks. Recognizing the importance of visual learning, this study also provides content to draw charts for profit and loss, time value, and implied volatility. All calculations and charts are done across different option types (call and put), various strike prices, different maturities, and varying underlying securities for comparative teaching. Study methods can easily be employed by instructors and students using Stata. Specific study results can also be replicated.

Keywords: Derivatives, financial options, implied volatility, Greeks, financial data

Introduction

Options are derivative securities that have gained significant usage and recognition with the integration of technology into financial markets. The Chicago Board Options Exchange (CBOE) was the "first marketplace for trading listed options" and opened in 1973 (http://www.cboe.com/aboutcboe/ history). The same year, Black and Scholes (1973) introduced the famous Black and Scholes option-pricing model, along with the extensions provided by Merton (1973) and Thorp (1973). Merton (1976) and Cox, Ross and Rubinstein (1979) provided further extensions to option-pricing models after the immediate popularity of the Black and Scholes model. Merton and Scholes were awarded the Nobel Memorial Prize in Economic Sciences in 1997 "for a new method to determine the value of derivatives" (https://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/1997/press.html).

Options have grown greatly in importance since 1973. Market statistics provided by the CBOE (available via http://www.cboe.com/data/historical-options-data/annual-market-statistics) show that there were over 1 billion options contracts traded during 2015. The dollar value for the total options trading volume was about $604 billion.

Recognizing the importance of options, many financial textbooks in corporate finance (e.g., Ross, Westerfield and Jordan, 2008), investments (e.g., Bodie, Kane and Marcus, 2009), international finance (e.g., Madura, 2011), and derivatives (e.g., Hull, 2006) have dedicated chapters for options. Accordingly, many pedagogical researchers have studied options with respect to financial education (e.g., Scallan, 2016; Wann, 2015; S. Johnson and Stretcher, 2013; Chatterjea and Jarrow, 2012; Smolira and Travis, 2011; Silveira Barbedo and Lemgruber, 2009; R. S. Johnson, Zuber and Gandar, 2008; Arnold, Crack and Schwartz, 2006; Arnold and Henry, 2005; Winsen, 2005; Pavlik and Nienhaus, 2004; Raju, 2004; Alexander and Sher, 2003; Shirvani and Wilbratte, 2003; Johnstone, 2002; Sterk, 2001).

Teaching options, however, has its challenges. Since historic data for options are not readily available, most of the content focuses on pricing theory. Even the profit and loss charts for options need to show intrinsic values instead of time value. The studies cited above can therefore be extended to include teaching methods with actual options data.

The ability to access market data in the classroom upgrades hypothetical prices into real prices. It also enables instructors to present pricing schemes, time value charts, and implied volatility charts across various maturities and across the full spectrum of stocks and ETFs. This study is intended to provide these important tools to modify option teaching from a hypothetical context to a real one.

Microsoft Excel is one of the most common software packages in financial teaching. Many studies that addressed options in the literature also implemented methods using Excel (e.g., Scallan, 2016; Wann, 2015; S. Johnson and Stretcher, 2013; Silveira Barbedo and Lemgruber, 2009; Arnold, Crack and Schwartz, 2006; Arnold and Henry, 2005; Raju, 2004). While Excel is almost a default software for students of finance to learn, it does have its shortcomings. Macros provided for earlier versions of Excel usually do not work with later versions of Excel. Excel spreadsheets, especially those with macros, can contain hostile code, and therefore many institutions will not even allow their use within their IT networks. Excel is system dependent, and what works on a PC might not work on a Mac, with UNIX, or on Linux systems. Repetitious tasks become singular manual jobs using Excel.

Econometrics packages, such as Stata (https://www.stata.com), have gained popularity within finance in recent years. Stata is not a substitute for Excel, but its power in data management, econometrics, and matrix language (MATA) and, more importantly for our purposes, its ability to conduct repetitious tasks with ease make it our choice of software for the classroom and for research.

The purpose of this study is to provide the necessary methods to access actual options data as well as to calculate intrinsic values, time value, and implied volatilities. We recognize the importance of teaching with visual tools and provide methods to draw charts of time value and implied volatilities across strike prices and maturities as well as across stocks and ETFs.

Downloading options data

We begin our discussion by providing a way to download option prices. Note that there are no historical option prices available for public use. Thus, the maturity dates for options included in the following Stata commands must be updated to the current date. *fetchyahoooptions* is a user-written command for Stata to download the financial options data from Yahoo! Finance (Dicle, 2013). The author provides the most current version via his web site. Options data are specific to each publicly traded company, to each day, and to each maturity date for the options. The following Stata code will install the *fetchyahoooptions* package onto Stata:

net install http://researchata.com/stata/010/fetchyahoooptions.pkg, force

The following Stata code will download all the options data for AAPL and SPY for three different maturities:

fetchyahoooptions AAPL SPY, m(2017-10-20 2017-11-17 2017-12-15)

Please note that there are no historic data available for options. This study's replication data can be downloaded from our servers using the following Stata code:

use "http://researchata.com/data_public/20170927_options_sample_1.dta", clear

The following variables are downl oaded into memory: Underlying (for underlying security), Maturity, Type (call or put), Symbol (of the specific option), Price, IRX (for risk-free interest rate), Strike, Last (last traded price), Bid, Ask, Change (in US Dollars), Change_per (in percent), Volume (number of contracts), Open_Interest (number of contracts), and IV_Yahoo (implied volatility calculated by Yahoo! Finance).

Each observation for this dataset is an option contract with a specific strike price, a specific maturity date, and a specific option type. The downloaded dataset includes all call and put options for all available strike prices for AAPL (Apple Inc.) and for S&P-500 ETF SPY for October 20, 2017; November 17, 2017; and December 15, 2017. A sample of the downloaded options data for AAPL and SPY is provided in Figure 1.

Figure 1
Downloaded Options Data for AAPL and SYP

Data cleanup

Downloaded options data comes with some outdated observations. Some of the option strikes do not trade every day, so they need to be cleaned out. With the intention of keeping only the actual options data, we filter each option contact based on the last traded date, volume, and open interest. The data in this study were downloaded on September 27, 2017, and it is therefore expected that all options data would have a "last traded date" as September 27, 2017. In order to be included within our sample, each options contract is expected to have some volume and some open interest. The following Stata code implements these data filters.

keep if Last_traded_date == date("27sep2017","DMY")

```
drop if Volume==. | Volume==0
drop if Open_Interest==. | Open_Interest==0
```

Intrinsic values

Intrinsic value refers to the exercise value for the option. Intrinsic values for call and for put options are calculated as follows:

$$\boldsymbol{Intrinsic_{Call} = max(Price - Strike, 0)}$$

Equation 1

$$\boldsymbol{Intrinsic_{Put} = max(Strike - Price, 0)}$$

Equation 2

For Equations 1 and 2, $\boldsymbol{Price}$ refers to the spot price of the underlying security. $\boldsymbol{Price}$ is downloaded for each of the securities, along with the options data. The following Stata code calculates these intrinsic values:

```
gen intrinsic = max(Price - Strike, 0) if Type=="Call"
replace intrinsic = max(Strike - Price, 0) if Type=="Put"
```

Time value of the option

The time value of the option refers to the difference between the option price and the intrinsic value of the option. A simple filter is employed to clean out abnormal prices. The following code calculates the time value of the option:

```
generate time_value = Ask - intrinsic
drop if time_value < 0
```

The following code draws the time value charts for different maturities for AAPL and for SPY.

```
twoway (line time_value Strike if Type=="Call" &
Maturity==date("20oct2017","DMY") & Underlying =="AAPL") (line time_value Strike
if Type=="Call" & Maturity==date("17nov2017","DMY") & Underlying =="AAPL")
(line time_value Strike if Type=="Call" & Maturity==date("15dec2017","DMY") &
Underlying =="AAPL"), legend(on cols(2) order(1 "20oct2017" 2 "17nov2017" 3
"15dec2017")) scale(0.5)

twoway (line time_value Strike if Type=="Call" &
Maturity==date("20oct2017","DMY") & Underlying =="SPY") (line time_value Strike if
Type=="Call" & Maturity==date("17nov2017","DMY") & Underlying =="SPY") (line
time_value Strike if Type=="Call" & Maturity==date("15dec2017","DMY") &
Underlying =="SPY"), legend(on cols(2) order(1 "20oct2017" 2 "17nov2017" 3
"15dec2017")) scale(0.5)
```

Figure 2
Time value for AAPL call options for three different maturities

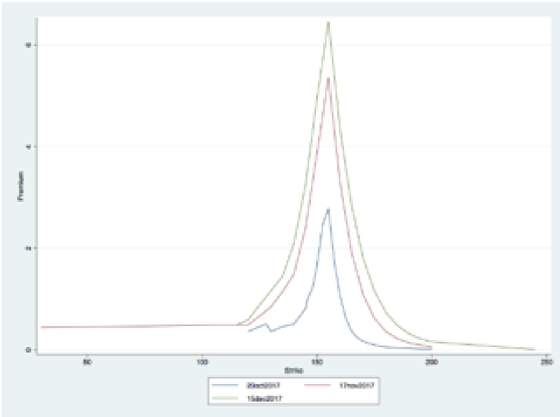
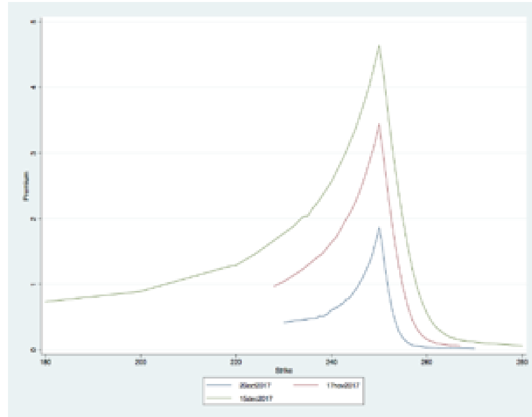


Figure 3
Time value for SPY call options for three different maturities



Based on Figures 2 and 3, note that the time values shift up for later maturities for AAPL as well as for SPY.

Before continuing on to the next section, we will save the downloaded and processed data using the following Stata code:

save options_data.dta, replace

Implied volatility

As part of the options data downloaded from Yahoo! Finance, implied volatility is available. This variable, "IV_Yahoo," is calculated by Yahoo! Finance. The following code calculates the average implied volatilities, as provided by Yahoo! Finance, for all options downloaded.

use options_data.dta, clear

collapse (mean) IV_Yahoo, by(Underlying Maturity Type)

format %8.4fc IV_Yahoo

Figure 4
Average implied volatilities

Underlying	Maturity	Type	IV_Yahoo
AAPL	20oct2017	Call	27.0764
AAPL	20oct2017	Put	24.8317
AAPL	17nov2017	Call	36.2006
AAPL	17nov2017	Put	31.6120
AAPL	15dec2017	Call	25.8729
AAPL	15dec2017	Put	27.0917
SPY	20oct2017	Call	9.0227
SPY	20oct2017	Put	18.1886
SPY	17nov2017	Call	10.4605
SPY	17nov2017	Put	19.5590
SPY	15dec2017	Call	11.8996
SPY	15dec2017	Put	18.3644

Based on the results provided in Figure 4, note that the average implied volatility is much higher for AAPL than the average implied volatility for SPY.

To calculate our own implied volatilities for downloaded options, an iteration procedure is used based on the Black and Scholes (1973) option-pricing model, as follows:

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left[\left(r_f + \frac{\sigma^2}{2}\right)T\right]}{\sigma\sqrt{T}}$$

Equation 3

$$d_2 = d_1 - \sigma\sqrt{T}$$

Equation 4

$$c = S_0 N(d_1) - Ke^{-r_f T} N(d_2)$$

Equation 5

$$p = Ke^{-r_f T} N(-d_2) - S_0 N(-d_1)$$

Equation 6

where S_0 refers to the spot price of the underlying security, K is the strike price, r_f is the risk free rate, σ is the standard deviation, and T is the years to maturity. The following Stata code calculates the implied volatilities following Equations 3, 4, 5, and 6:

```
use options_data.dta, clear
fetchyahoooptions AAPL SPY, calconly iv
```

Note that if the downloaded data are not current (e.g., the data made available for this study) the system date is adjusted to 180 days prior to the maximum maturity date. The following code draws the implied volatility charts for AAPL and SPY. In order to make the chart more familiar to popular volatility smile charts, we will keep all strike prices that are within 20% of the spot price of the underlying security.

```
keep if (Strike < (Price*1.20)) & (Strike > (Price*0.80))

twoway (line IV Strike if Maturity==date("20oct2017","DMY") & Type=="Call" & Underlying=="AAPL") (line IV Strike if Maturity==date("17nov2017","DMY") & Type=="Call" & Underlying=="AAPL") (line IV Strike if Maturity==date("15dec2017","DMY") & Type=="Call" & Underlying=="AAPL"), legend(on cols(3) order(1 "20oct2017" 2 "17nov2017" 3 "15dec2017")) scale(0.75)

twoway (line IV Strike if Maturity==date("20oct2017","DMY") & Type=="Call" & Underlying=="SPY") (line IV Strike if Maturity==date("17nov2017","DMY") & Type=="Call" & Underlying=="SPY") (line IV Strike if Maturity==date("15dec2017","DMY") & Type=="Call" & Underlying=="SPY"), legend(on cols(3) order(1 "20oct2017" 2 "17nov2017" 3 "15dec2017")) scale(0.75)
```

Figure 5
Implied volatilities for AAPL

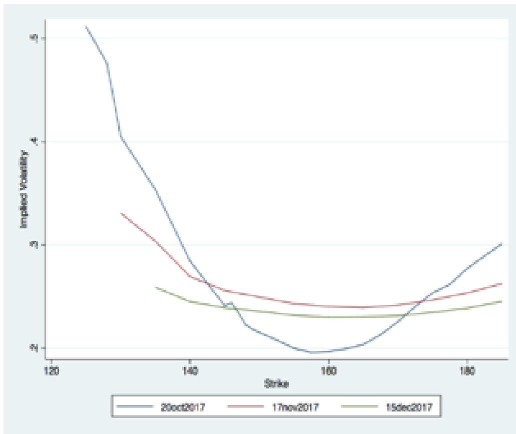
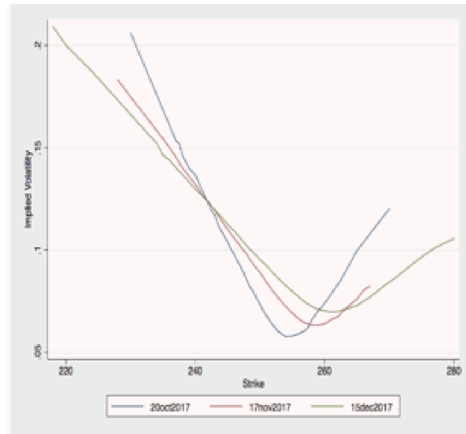


Figure 6
Implies volatilities for SPY



Based on Figures 5 and 6, note that the smile curvature is steeper for earlier maturities as compared to later maturities. Also, the smile curve is steeper for AAPL (0.2-0.5) as compared to the smile for SPY (0.05-0.2).

Greeks

The following Stata code calculates the variables needed for the calculations of Greeks:

```
use options_data.dta, clear
* The commented line is for the actual data
* gen double T=(Maturity-date(c(current_date),"DMY"))/365
* The current code is for the data made available for this study
gen double T=(Maturity- 20988)/365
label variable T "Years to maturity"
sort Underlying Maturity T Type Strike
gen double d1=.
gen double d2=.
gen double Call=.
gen double Put=.
gen double IV=.
label variable IV "Implied Volatility"
local option_price="Ask"
forval bb=1/50000 {
    local aa: di %5.4f (`bb'/10000)
```

```
replace d1=(ln(Price/Strike) + ((IRX + ((`aa'^2)/2))*T)) / (`aa'*(T^.5)) if IV==.
replace d2=d1-(`aa'*(T^.5)) if IV==.
replace Call=round((Price*normal(d1))-(Strike*exp(-IRX*T)*normal(d2)),0.01) if IV==.
replace Put=round((Strike*exp(-IRX*T)*normal(-d2))-(Price*normal(-d1)),0.01) if IV==.
replace IV=`aa' if ((Type=="Call") & (Call==round(`option_price',0.01)) & (IV==.)) | ((Type=="Put") & (Put==round(`option_price',0.01)) & (IV==.))
}
```

We refer to Hull (2006) for the equations used to calculate the Greeks for the options. We also use the same Greek letters as Hull (2006).

Delta

Delta (Δ) measures the (expected) change in the option's value ($\boldsymbol{V}$) with respect to the change in the underlying security's spot price ($\boldsymbol{S_0}$).

$$\Delta = \frac{\partial \boldsymbol{V}}{\partial \boldsymbol{S_0}}$$

Equation 7

The calculation of Delta is different for call and put options. For call options, the calculation is as follows:

$$\Delta_{Call} = \boldsymbol{N(d_1)}$$

Equation 8

For put options, the calculation is below:

$$\Delta_{Put} = \boldsymbol{N(d_1) - 1}$$

Equation 9

The following Stata code shows how to calculate the Delta for call and put options:

```
gen double Delta = .
replace Delta = normal(d1) if Type == "Call"
replace Delta = normal(d1) - 1 if Type == "Put"
```

Gamma

Gamma ($\boldsymbol{\Gamma}$) measures the (expected) change in Delta with respect to the change in the underlying security's spot price.

$$\boldsymbol{\Gamma = \frac{\partial \Delta}{\partial S_0}}$$

Equation 10

The calculation of Gamma is as follows:

$$\boldsymbol{\Gamma = \frac{e^{\frac{-d_1^2}{2}}}{S_0 \sigma \sqrt{T}\sqrt{2\pi}}}$$

Equation 11

The following Stata code shows how to calculate the Gamma:

*gen double Gamma = (1 / (Price * IV * sqrt(T) * sqrt(2*c(pi)))) * exp(-(d1^2)/2)*

Theta

Theta ($\boldsymbol{\theta}$) measures the (expected) change in the option's value ($\boldsymbol{V}$) with respect to the change in time ($\boldsymbol{\tau}$). In other words, theta is the time decay in an option's value.

$$\boldsymbol{\theta = \frac{\partial V}{\partial \tau}}$$

Equation 12

The calculation of Theta is different for call and put options. For call options, the calculation is below:

$$\boldsymbol{\theta_{Call} = \frac{1}{365}\left(-\frac{S_0 \sigma e^{\frac{-d_1^2}{2}}}{2\sqrt{T}\sqrt{2\pi}} + r_f K e^{-r_f T} N\left(d_1 - \sigma\sqrt{T}\right)\right)}$$

Equation 13

For put options, the calculation is as follows:

$$\boldsymbol{\theta_{Put} = \frac{1}{365}\left(-\frac{S_0 \sigma e^{\frac{-d_1^2}{2}}}{2\sqrt{T}\sqrt{2\pi}} - r_f K e^{-r_f T} N\left(\sigma\sqrt{T} - d_1\right)\right)}$$

Equation 14

The following Stata code shows how to calculate the Theta for call and put options:

gen double Theta = .

replace Theta=(1/365)(-(Price*IV*exp(-(d1^2)/2)/(2*sqrt(T)*sqrt(2*c(pi)))) +(IRX*Strike*exp(-IRX*T)*normal(d1-(IV*sqrt(T))))) if Type == "Call"*

replace Theta=(1/365)(-(Price*IV*exp(-(d1^2)/2)/(2*sqrt(T)*sqrt(2*c(pi)))) - (IRX*Strike*exp(-IRX*T)*normal((IV*sqrt(T))-d1))) if Type == "Put"*

Vega

Vega ($\boldsymbol{\nu}$) measures the (expected) change in the option's value ($\boldsymbol{V}$) with respect to the change in volatility of the underlying security ($\boldsymbol{\sigma}$).

$$\boldsymbol{\nu = \frac{\partial V}{\partial \sigma}}$$

Equation 15

The calculation of Vega is as follows:

$$\boldsymbol{\nu = \frac{1}{100}\left(\frac{S_0\sqrt{T}e^{\frac{-d_1^2}{2}}}{\sqrt{2\pi}}\right)}$$

Equation 16

The following Stata code shows how to calculate the Vega for call and put options:

*gen double Vega = (1/100) * Price * sqrt(T) * (1 / sqrt(2*c(pi))) * exp(-(d1^2)/2)*

Rho

Rho ($\boldsymbol{\rho}$) measures the (expected) change in the option's value ($\boldsymbol{V}$) with respect to the change in the risk-free interest rate ($\boldsymbol{r_f}$).

$$\boldsymbol{\rho = \frac{\partial V}{\partial r_f}}$$

Equation 17

The calculation for Rho is different for call and put options. For call options, it is as follows:

$$\boldsymbol{\rho_{Call} = \frac{1}{100}\left(KTe^{-r_fT}N(d_2)\right)}$$

Equation 18

For put options, the calculation is below:

$$\rho_{Put} = \frac{1}{100}\left(KTe^{-r_f T}N(-d_2)\right)$$

Equation 19

The following Stata code shows how to calculate the Rho for call and put options:

```
gen double Rho = .
replace Rho = (1/100) * Strike * T * exp(-IRX*T) * normal(d2) if Type == "Call"
replace Rho = (1/100) * Strike * T * exp(-IRX*T) * normal(-d2) if Type == "Put"
```

Concluding remarks

Options are a significant part of financial markets and have become important within financial teaching as well. One of the main challenges of teaching options is the reliance on hypothetical data and scenario analysis. This study provides the methods, using Stata, to access actual options data and to calculate intrinsic values, time values, implied volatilities, and Greeks. These methods enable classroom teaching to compare the effects of each associated option variable on option outcomes. Comparisons are easily made across various strike prices, maturities, and stocks. The included code is versatile enough to extend the analysis to portfolios, index components, and market components.

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An Evaluation of Three Approaches to Pricing Interest-rate Derivatives

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In the pedagogy of fixed income, there exist three alternative pricing approaches to interest-rate derivatives: 1) the approach that calibrates forward rates; 2) the approach that derives risk-neutral probabilities; and 3) the approach that constructs payoff-replicating portfolios. With the different procedures and challenges involved in each approach, we may ask whether one is superior to another for effective teaching and learning. In this paper, we summarize each method and show through examples that they all produce the same pricing result. Further, we describe logics that link the three methods, thereby establishing their equivalence for pricing purpose. Specifically, the "calibrated forward rates" and the "risk-neutral probabilities" methods are algebraic transformations of one another, and both are rooted in the "replicating portfolios" method whose fundamental requirements are the tradability of bonds with varying maturities and the Law of One Price.

Keywords: Pricing of interest-rate derivatives, Calibrated forward rates, Risk-neutral probabilities, Replicating portfolios, Law of one price

Introduction

In the pedagogy of fixed income, instructors may choose from three alternative approaches to pricing interest-rate derivatives:

i. The approach that calibrates forward rates at each node in a binomial tree of interest rate evolution to match market prices of tradable bonds, see, e.g., in the widely popular text by Fabozzi (2016), pp.380-394. The price of any other bond – whether option-free or option-embedded, or any interest-rate derivative – is then calculated as simple expectation (under the real probabilities) of discounted value (using these *calibrated forward rates*). We refer to it as the *calibrated rates* (CR) method.

ii. The approach that transposes original probabilities into derived probabilities along the tree to match market prices of tradable bonds, see, e.g. in another widely popular text by Tuckman & Serrat (2012), pp.211-218. The price of any bond or interest-rate derivative is then calculated as simple expectation (under these *derived probabilities*) of discounted value (using the real forward rates). We refer to it as the *risk-neutral probabilities* (RN) method.

iii. The approach that constructs a portfolio of tradable bonds to replicate the stream of payoffs of any interest-rate derivative, see, e.g. in an advanced text by Veronesi (2010), pp.340-

343. The price of the derivative is then equal to the price of the replicating portfolio. We refer to it as the *replicating portfolio* (RP) method.

With three alternative methods available from different textbooks – each with its own procedure and challenge involved in pricing interest-rate derivatives – instructors are called upon to opt for one that would most effectively facilitate students' learning.

In this paper, we summarize each method, and show, through numerical examples and formal demonstrations, the differences and connections among these three methods in producing the same price result.

Literature Review

Asset pricing is a predominant theme in financial theory. In the literature of option pricing, three techniques are applied to bonds and their derivatives, as mentioned in our introduction.

With the distinctive procedure and varied complexity involved in each approach, it can seem confusing to students with regard to the "right" pricing method; is one superior to another? and why? Unfortunately, there has been no prior attempt to unify these three approaches in a single framework that demonstrates their coherence in producing the correct price results.

To substantiate our claim, we ran searches in ProQuest and Google Scholar using as keywords "calibrated forward rates," "risk-neutral probabilities," "replicating portfolio," "interest-rate derivatives," and "equivalence." We were unable to find any documented work addressing connections and equivalence among these three pricing techniques.

A formal analysis that clarifies the inherent logic for why the three aforementioned pricing methods necessarily produce the same result is certainly desirable, since such an attempt would establish the equivalence of these widely popular methods for pricing interest rate derivatives; this helps to dispel confusions in teaching the subject. As such, we will summarise all three pricing tools and show, through numerical examples and formal demonstrations, their connections and equivalence. Our goal is to share pedagogical experiences in employing each method and compare their relative ease and difficulty for students to grasp. For instructors new to teaching fixed income, this paper also serves as a concise document to facilitate their customized lecture notes on pricing interest-rate derivatives.

The Three Methods in Illustration

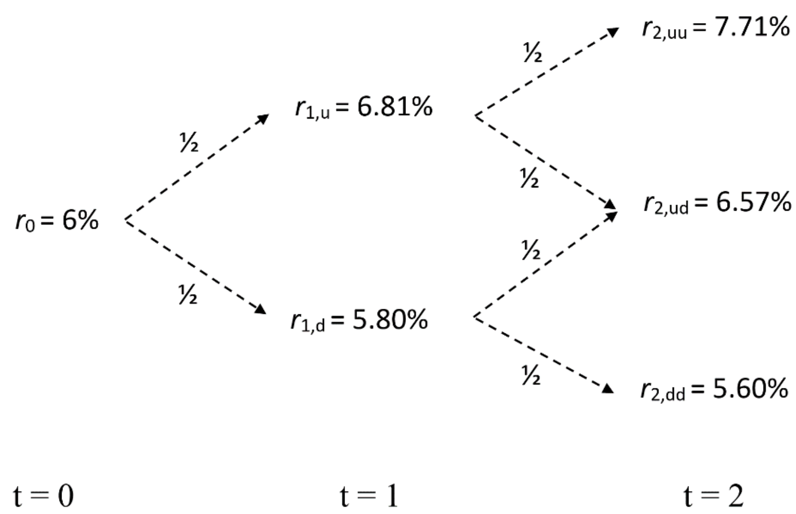
To start, suppose that the *market* prices of three elementary option-free bonds with the same face value and coupon rate but different maturities are as shown in Table 1 (we abstract away credit risk to focus on interest rate risk only).

Table 1
Market prices of bonds with FV = $^{\$}$1,000, CR = 4% (semi-annual coupon)

Bond	Maturity (year)	Market price
1	0.5	B_1 = $^{\$}$990.29
2	1	B_2 = $^{\$}$978.45
3	1.5	B_3 = $^{\$}$965.16

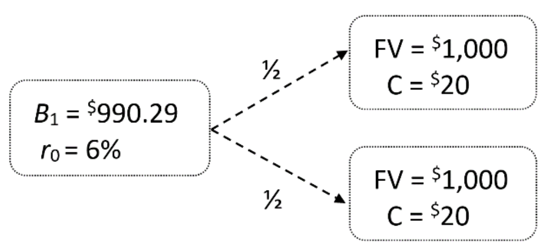
Also, suppose the market predicts that the 6-month real *market* interest rates will evolve in the following pattern – a lognormal binomial tree with original/real probability = ½, and volatility σ = 8% so that the upward rate and the downward rate at time t are related to each other in the way of $r_{t,u} = r_{t,d} \times e^{2\sigma}$:

Figure 1
Evolution of market interest rates with real probabilities



To see the relationship between the market prices of the bonds in Table 1 and the market interest rates in Figure 1, we proceed to the following calculations.

The 0.5-yr bond



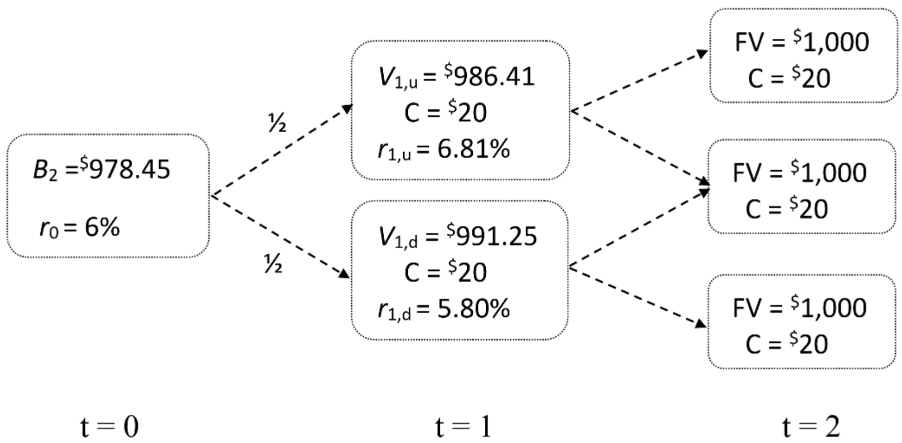
This bond matures in a half year (t = 1), paying the *sure* cash flow of $1,020 (face value plus coupon). Proceeding backward, the sure $1,020 at maturity, discounted at the real market interest rate, gives the market price:

$$B_1 = \frac{1{,}020}{1 + 6\%/2} = 990.29$$

Deriving *risk-neutral probabilities*

The 1-yr bond

Figure 2
Evolution of 1-yr bond prices under market rates and real probabilities



Proceeding backward, the *sure* payoff of $1,020 at maturity, discounted at the real market interest rates, gives the interim values:

$$V_{1,u} = \frac{1{,}020}{1+6.81\%/2} = 986.41 \qquad V_{1,d} = \frac{1{,}020}{1+5.8\%/2} = 991.25$$

Note 1: Under the real probability of ½, the expected discounted value is:

$$\frac{(½)(V_{1,u}+C)+(½)(V_{1,d}+C)}{1+r_0/2} = \frac{(½)(986.41+20)+(½)(991.25+20)}{1+6\%/2} = 979.45,$$

which is *higher* than the market price of B_2 = $978.45 in Table 1. Why the difference?

In fact, there is *price uncertainty* at t = 1 between $V_{1,u}$ and $V_{1,d}$ with real probability ½ each, and *risk-averse* investors dislike this uncertainty. Thus, market price reflects this dislike with a *reduced* price. The reduction indicates investors' subjective aversion to uncertainty.

We can modify the original/real probability ½ to new "*risk-neutral* probabilities" that account for the risk aversion, so that we can calculate price as a *simple expectation* of discounted value.

To find such risk-neutral probability of the 1st-period, p, we let

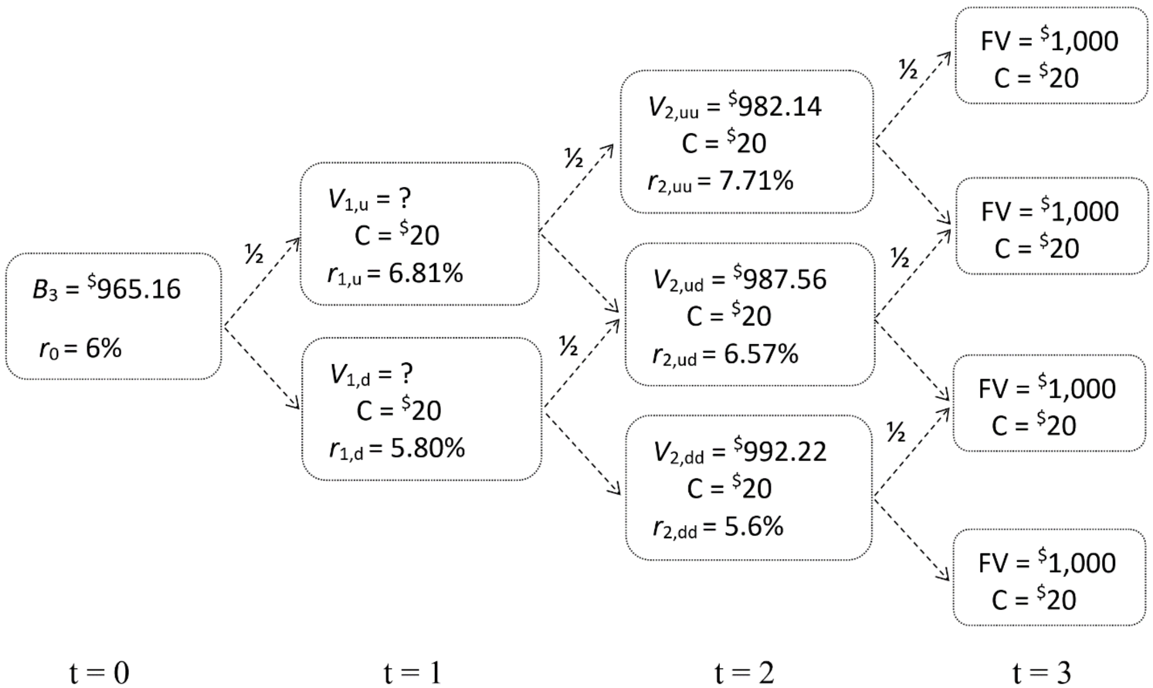
$$\frac{(p)(V_{1,u}+C)+(1-p)(V_{1,d}+C)}{1+r_0/2} = \text{market price } B_2$$

i.e., $$\frac{(p)(986.41+20)+(1-p)(991.25+20)}{1+6\%/2} = 978.45$$

Solving, $p = 0.7121$ (≠ the real probability of ½ in Figure 1).

The 1.5-yr bond

Figure 3
Evolution of 1.5-yr bond prices under market rates and real probabilities



Proceeding backward, the *sure* payoffs of $^{\$}$1,020 at maturity give interim values:

$$V_{2,uu} = \frac{1{,}020}{1+7.71\%/2} = 982.14 \qquad V_{2,ud} = \frac{1{,}020}{1+6.57\%/2} = 987.56 \qquad V_{2,dd} = \frac{1{,}020}{1+5.6\%/2} = 992.22$$

Now, what are the correct interim values for $V_{1,u}$ and $V_{1,d}$? Recalling from Note 1 on price uncertainty, we know they are not the expected (under real probability ½) discounted value. To calculate price as a simple expectation of discounted value, we need to modify again ½ to risk-neutral probability.

We search for such risk-neutral probability of the 2nd-period, q, from:

$$\frac{(p)(V_{1,u}+20)+(1\text{-}p)(V_{1,d}+20)}{1+6\%/2} = \text{market price } B_3 \text{ of } 965.16$$

$$\frac{(q)(982.14+20)+(1\text{-}q)(987.56+20)}{1+6.81\%/2} = V_{1,u}$$

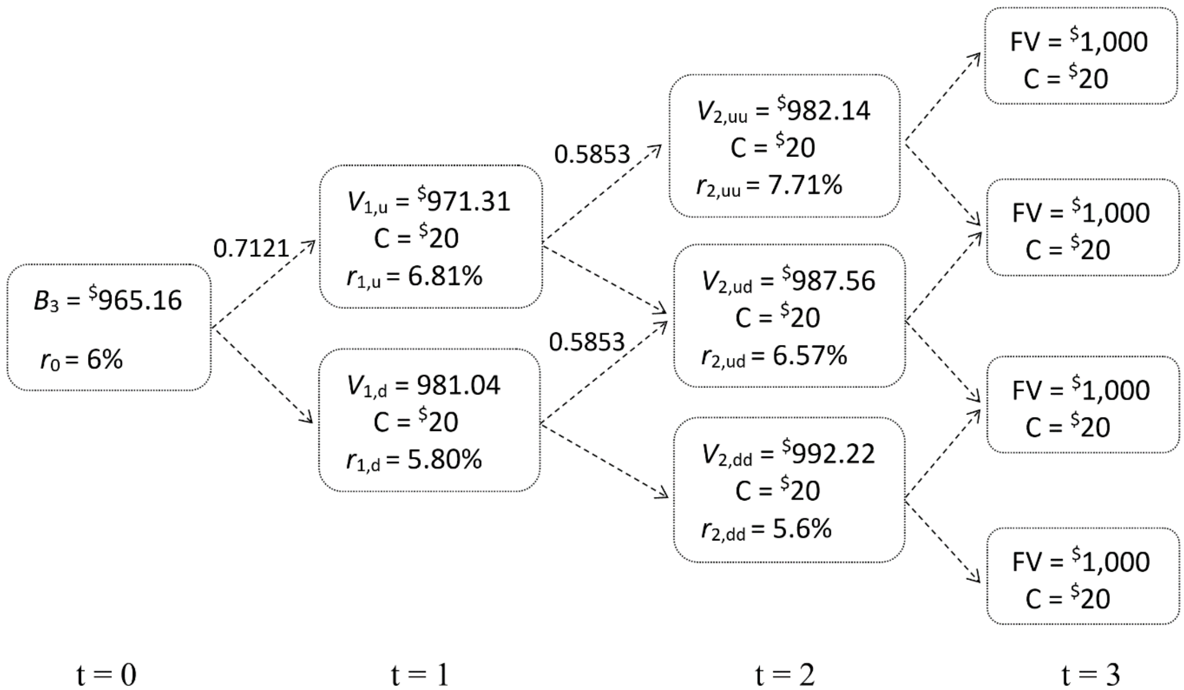
$$\frac{(q)(987.56+20)+(1\text{-}q)(992.22+20)}{1+5.8\%/2} = V_{1,d}$$

Solving this system of 3 equations, we obtain, $q = 0.5853$ ($\neq$ ½), and

$V_{1,u} = {}^{\$}971.31$ → interim value consistent with risk-neutral probabilities p and q,

$V_{1,d} = {}^{\$}981.04$ → interim value consistent with risk-neutral probabilities p and q.

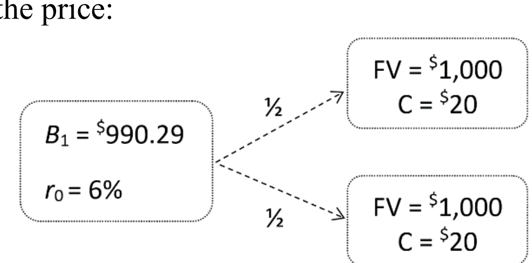
Figure 4
Evolution of 1.5-yr bond prices under market rates and *risk-neutral* probabilities



<u>Calibrating *forward rates*</u>

The 0.5-yr bond

As before, the *sure* cash flow of $1,020 at maturity, discounted at the current interest rate of 6%, gives the price:



$$\frac{1{,}020}{(1+0.06/2)} = 990.29$$

The 1-yr bond

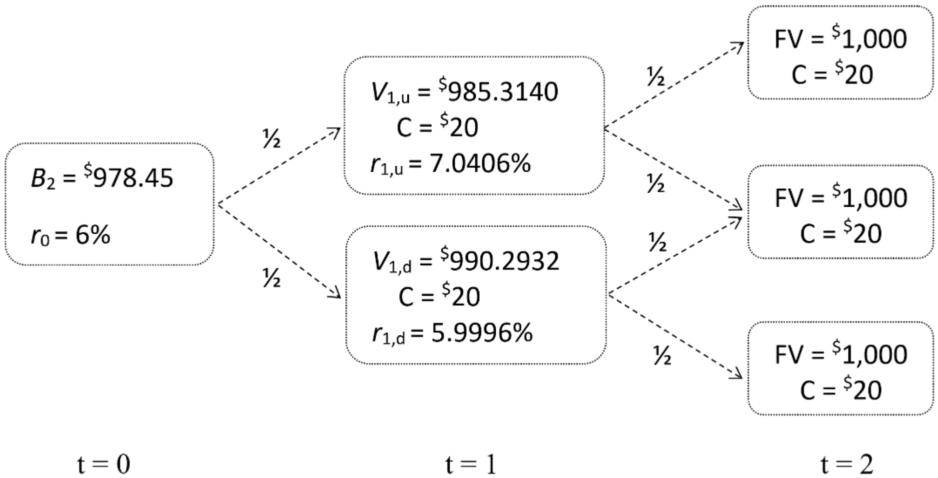
This bond's value in a half year (at t = 1) will depend on the then prevailing interest rate $r_{1,u}$ or $r_{1,d}$.

We want to find (i.e. calibrate) the value for the down rate $r_{1,d}$ and the up rate $r_{1,u} = r_{1,d} \times e^{2\sigma}$, such that the expectation (under the *real* prob. ½) of discounted value (at the *calibrated* rates) will *match* the market price of B_2 = $978.45. The calibrating procedure is a trial-and-error process.

Let us try/guess $r_{1,d}$ = 5.9996%, thus $r_{1,u} = 5.9996\% \times e^{2\times 0.08} = 7.0406\%$.

$$V_{1,d} = \frac{1{,}020}{(1+5.9996\%/2)} = 990.2932 \qquad V_{1,u} = \frac{1{,}020}{(1+7.0406\%/2)} = 985.3140$$

Figure 5
Evolution of 1-yr bond prices under *calibrated* rates and real probabilities



At t = 0, the expectation of discounted value of the interim values $V_{1,u}$ and $V_{1,d}$ plus coupons is:

$$V_0 = \frac{\frac{1}{2}(V_{1,u}+C)+\frac{1}{2}(V_{1,d}+C)}{(1+r_0/2)} = \frac{\frac{1}{2}(1005.3140)+\frac{1}{2}(1010.2932)}{(1+0.06/2)} = 978.45 \rightarrow \text{ matches the market price } B_2.$$

Thus, we keep these *calibrated* rates $r_{1,d}$ = 5.9996% (≠ the market rate of 5.80% in Figure 1) and $r_{1,u}$ = 7.0406% (≠ the market rate of 6.81% in Figure 1).

Note 2: We happen to guess $r_{1,d}$ = 5.9996% to arrive at a V_0 that matches the market price B_2 of $978.45. If the calculated V_0 is lower (higher) than the market price $978.45, then we restart the trial process with a larger (smaller) $r_{1,d}$, until the guessed $r_{1,d}$ produces a matched price.

The 1.5-yr bond

This bond's value in one year (at t = 2) will depend again on the then prevailing interest rate $r_{2,dd}$, $r_{2,du}$, or $r_{2,uu}$.

We want to find (i.e. calibrate) the values for the down rate $r_{2,dd}$ and the up rates $r_{2,du} = r_{2,dd} \times e^{2\sigma}$ and $r_{2,uu} = r_{2,dd} \times e^{4\sigma}$, such that expectation (under the real prob. ½) of discounted value (at the *calibrated* rates) will *match* the market price of B_3 = $965.16. The calibrating procedure is still a trial-and- error process.

Let us try/guess $r_{2,dd}$ = 5.8671%, thus $r_{2,du} = 5.8671\% \times e^{2\times 0.08} = 6.8851\%$, and $r_{2,uu} = 5.8671\% \times e^{4\times 0.08} = 8.0797\%$.

$$V_{2,uu} = \frac{1{,}020}{(1+r_{2,uu}/2)} = \frac{1{,}020}{(1+8.0797\%/2)} = 980.3934$$

$$V_{2,du} = \frac{1{,}020}{(1+r_{2,du}/2)} = \frac{1{,}020}{(1+6.8851\%/2)} = 986.0546$$

$$V_{2,\text{dd}} = \frac{1{,}020}{(1+r_{2,\text{dd}}/2)} = \frac{1{,}020}{(1+5.8671\%/2)} = 990.9306$$

$$V_{1,\text{u}} = \frac{\frac{1}{2}(V_{2,\text{uu}}+C)+\frac{1}{2}(V_{2,\text{du}}+C)}{(1+r_{1,u}/2)} = \frac{\frac{1}{2}(1000.3934)+\frac{1}{2}(1006.0546)}{(1+7.0406\%/2)} = 969.1084$$

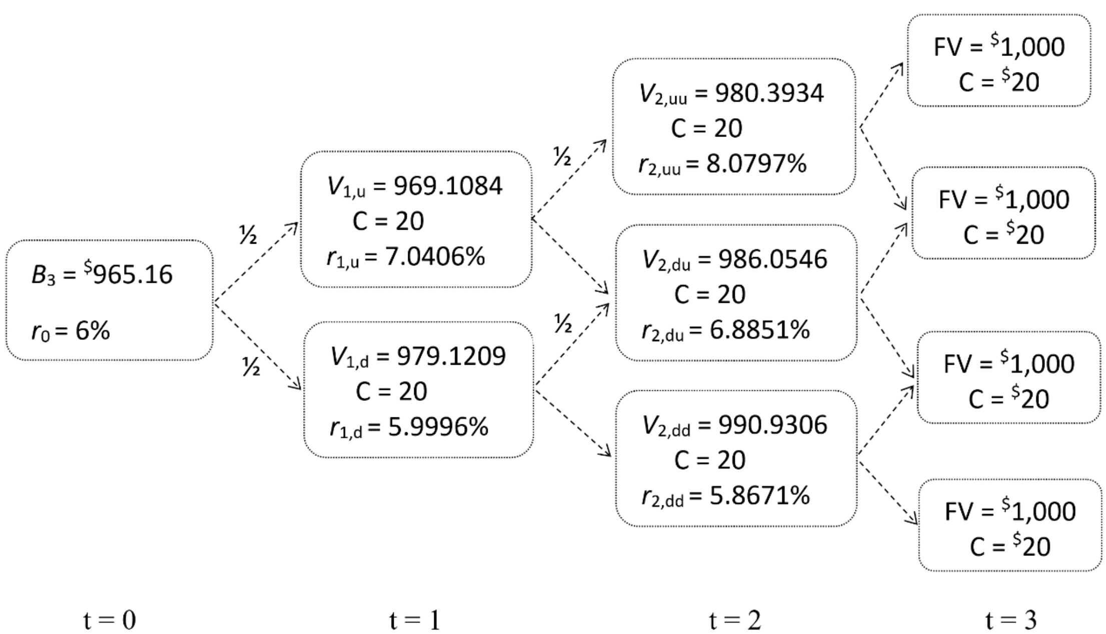
$$V_{1,\text{d}} = \frac{\frac{1}{2}(V_{2,\text{du}}+C)+\frac{1}{2}(V_{2,\text{dd}}+C)}{(1+r_{1,\text{d}}/2)} = \frac{\frac{1}{2}(1006.0546)+\frac{1}{2}(1010.9306)}{(1+5.9996\%/2)} = 979.1209$$

At t = 0, our expected present value is:

$$V_0 = \frac{\frac{1}{2}(V_{1,\text{u}}+C)+\frac{1}{2}(V_{1,\text{d}}+C)}{(1+r_0/2)} = \frac{\frac{1}{2}(989.1084)+\frac{1}{2}(999.1209)}{(1+0.06/2)} = {}^{\$}965.16 \rightarrow \text{ matches the market price } B_3.$$

Thus, we keep these *calibrated* rates $r_{2,\text{dd}}$ = 5.8671%, $r_{2,\text{du}}$ = 6.8851%, and $r_{2,\text{uu}}$ = 8.0797%. Again, if V_0 is lower (higher) than $965.16, then repeat the process with a larger (smaller) $r_{2,\text{dd}}$, until the calculated V_0 matches the market price B_3.

Figure 6
Evolution of 1.5-yr bond prices under *calibrated* rates and real probabilities



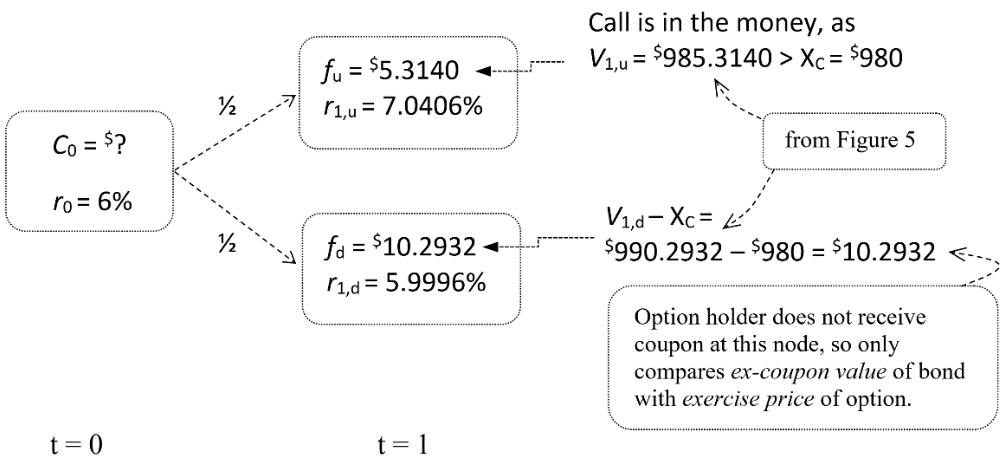
<u>Constructing *payoff-replicating portfolio*</u>

Now, equipped with these *calibrated* rates and *risk-neutral* probabilities we found above, along with *payoff-replicating* portfolio that we illustrate below, we can proceed to pricing any interest rate derivatives, such as option-free or option-embedded bonds, or options on bond, using one of the three pricing methods. As will be seen, all three methods produce the same results (due to space limitations, we illustrate only the pricing of vanilla call and put options on bond. Examples of pricing other interest rate derivatives over multiple periods are available from the authors upon request).

Example 1. Suppose a *call* option with exercise price X_C = $980 to buy the 1-year coupon bond in Table 1, expiring in 0.5-year (at t = 1). Determine the price of the option using the 3 methods.

The value of the bond in 0.5-year (at t = 1) is uncertain. Denote by f_u and f_d the payoff to the option in the up- and down-state of interest rate, respectively, at the option expiry time t = 1.

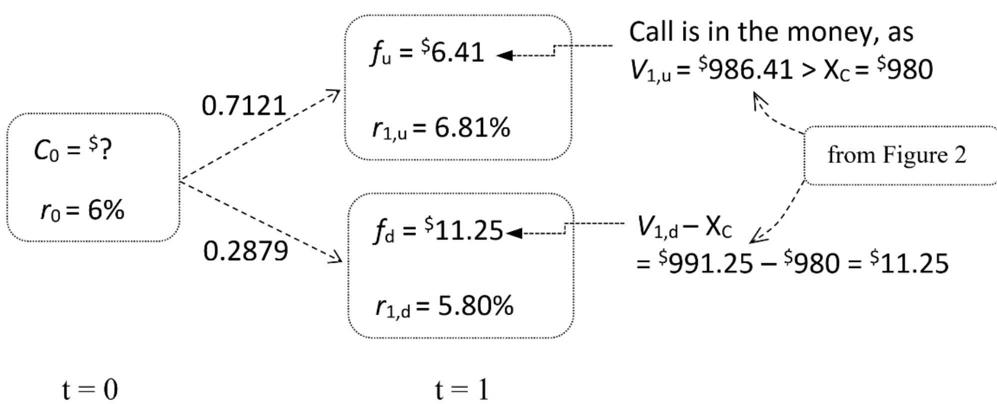
i. Using method of "calibrated forward rates":



The call option price is the simple expectation of payoffs using the real probabilities:

$$C_0 = \frac{\frac{1}{2}\times 5.3140 + \frac{1}{2}\times 10.2932}{1+6\%/2} = \$7.58$$

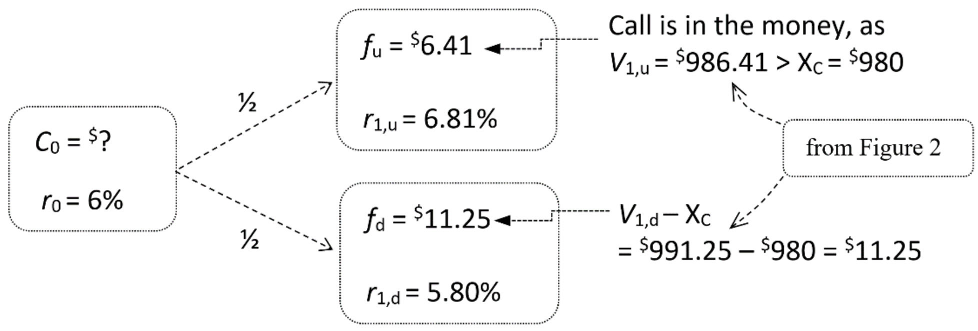
ii. Using method of "risk-neutral probabilities":



The call option price is the simple expectation of payoffs using the risk-neutral probabilities:

$$C_0 = \frac{0.7121\times 6.41 + 0.2879\times 11.25}{1+6\%/2} = \$7.58$$

iii. Using method of "payoff-replicating portfolio":



To price the call option, we seek to match/replicate the option's payoff of $^{\$}6.41$ in the up-rate state and $^{\$}11.25$ in the down-rate state at t = 1. Suppose we form a portfolio that contains N_1 units of the 0.5-yr bond and N_2 units of the 1-yr bond, with a total cost of:

$$T_0 = N_1 \times B_1 + N_2 \times B_2 = N_1 \times {}^{\$}990.29 + N_2 \times {}^{\$}978.45$$

We choose the values of N_1 and N_2 such that at t = 1 (the option's expiry date), the portfolio's worth will *replicate* the payoff of the option in *each* state, that is:

$$N_1 \times (FV + C) + N_2 \times (V_{1,u} + C) = 6.41 \qquad \leftarrow \text{in the } up \text{ state at t = 1}$$

$$N_1 \times (FV + C) + N_2 \times (V_{1,d} + C) = 11.25 \qquad \leftarrow \text{in the } down \text{ state at t = 1}$$

i.e.,

$$N_1 \times (1020) + N_2 \times (986.41 + 20) = 6.41$$

$$N_1 \times (1020) + N_2 \times (991.25 + 20) = 11.25$$

In matrix format (for the purpose of an Excel spreadsheet set-up):

$$\begin{bmatrix} 1020 & 1006.41 \\ 1020 & 1011.25 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} 6.41 \\ 11.25 \end{bmatrix}$$

↑ (cum-coupon) ↑

Solving the system:

N_1 = – 0.9804 unites of bond 1 with 0.5-yr maturity in the replicating portfolio (short sell),

N_2 = 1.0000 units of bond 2 with 1-yr maturity in the replicating portfolio.

Law of One Price (no arbitrage): The payoff-replicating portfolio and the call option will yield identical payoff in each state at time 1. Therefore, at time 0, the price of the option must be the same as the value of the replicating portfolio, which is:

$$C_0 = N_1 \times B_1 + N_2 \times B_2 = -0.9804 \times 990.29 + 1.00 \times 978.45 = {}^{\$}7.58$$

Thus, we see that all three methods produce the same call option price of $^{\$}7.58$.

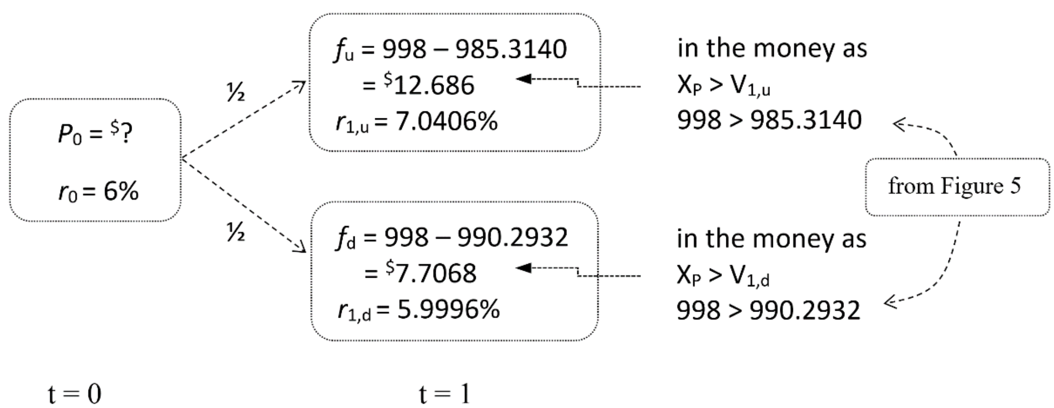
Indeed, with arbitrarily varied exercise prices of the call option, X_C, the three methods still produce the same option prices, as shown in Table 2.

Table 2
Three methods produce the same call option price for a given exercise price

Exercise price, X_C	**Call option price by Calibrated rate (CR) method**	**Call option price by Risk-neutral (RN) method**	**Call option price by Replicating portfolio (RP) method**
940	46.41	46.41	46.41
960	26.99	26.99	26.99
970	17.29	17.29	17.29
975	12.43	12.43	12.43
980	7.58	7.58	7.58
985	2.72	2.72	2.72

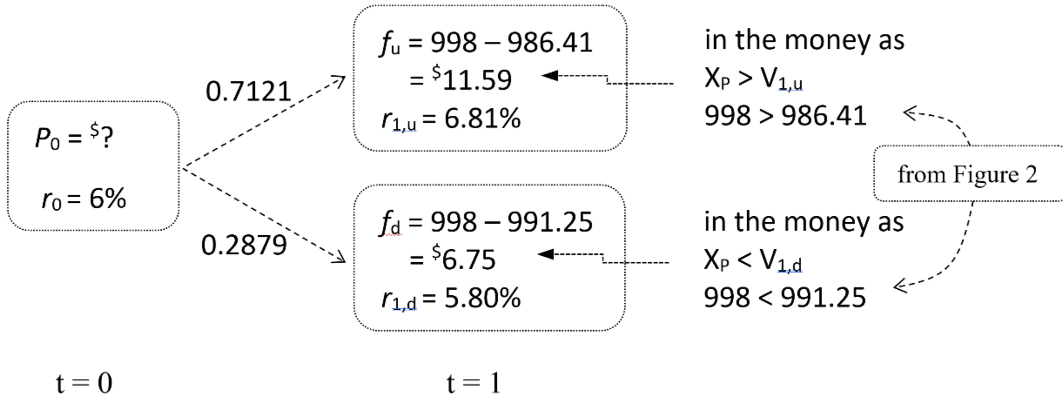
Example 2. Suppose a *put* option with exercise price $X_P = {}^{\$}998$ to sell the 1-yr coupon bond in Table 1, expiring in 0.5 year (at t = 1). Determine the price of the option using the three methods.

i. Using method of "calibrated forward rates":



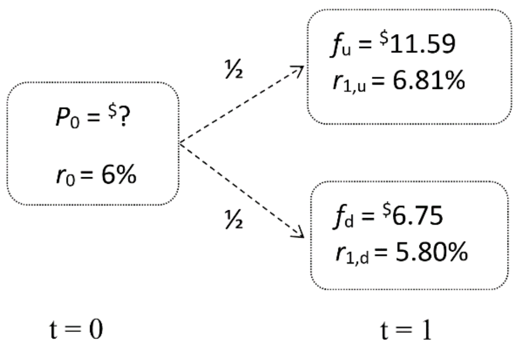
$$P_0 = \frac{{}^{\$}12.686 \times \frac{1}{2} + {}^{\$}\,7.7068 \times \frac{1}{2}}{1 + 6\%/2} = \$9.90$$

ii. Using method of "risk-neutral probabilities":



$$P_0 = \frac{^{\$}11.59 \times 0.7121 + ^{\$}6.75 \times 0.2879}{1 + 6\%/2} = \$9.90$$

iii. Using method of "payoff-replicating portfolio":



At time 1: $$\begin{bmatrix} 1020 & 1006.41 \\ 1020 & 1011.25 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} 11.59 \\ 6.75 \end{bmatrix}$$

↑ (cum-coupon) ↑

Solving the system:

N_1 = 0.9980 unites of bond 1 with 0.5-yr maturity in the replicating portfolio,

N_2 = –1.000 units of bond 2 with 1-yr maturity in the replicating portfolio (short sell).

The payoff-replicating portfolio and the put option will yield identical payoff in either the *down* state or the *up* state at time 1, so according to the law of one price, at time 0 the price of the option must be the same as the value of the replicating portfolio, which is:

$P_0 = N_1 \times B_1 + N_2 \times B_2 = 0.9980 \times 990.29 - 1.000 \times 978.45 = {}^{\$}9.90$

Again, all three methods produce the same put option price of $^{\$}9.90$.

Indeed, with arbitrarily varied exercise prices of the put option, X_P, the three methods still produce the same option prices, as shown in Table 3.

Table 3
Three methods produce the same put option price for a given exercise price

Exercise price, X_P	Put option price by Calibrated rate (CR) method	Put option price by Risk-neutral (RN) method	Put option price by Replicating portfolio (RP) method
992	4.07	4.07	4.07
998	9.90	9.90	9.90
1010	21.55	21.55	21.55
1025	36.11	36.11	36.11
1040	50.68	50.68	50.68
1050	60.38	60.38	60.38

Theoretical Logics Linking the Three Methods

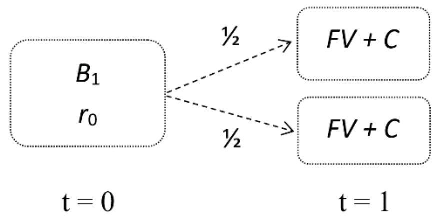
From the previous numerical illustrations, we see that all three methods for pricing interest rate derivatives, each involving its unique procedure, produce the same results. The root causes for such consistency are the inherent connections among these methods. Now we can provide the logical linkages.

To price an interest rate derivative whose cash flows vary across states and over time, we seek to form a portfolio of bonds to *replicate* the derivative's cash flows – provided that these bonds are available in the market, which is formally referred to as *tradable* or *hedgeable* in financial theory. The economic rationale for focusing on such payoff-replicating portfolios is the Law of One Price (or no arbitrage): two identical streams of future payoffs mirroring each other in *each state* and at *every time point* must be worth the same now – irrespective of the risk aversion of an investor facing the uncertainty of payoffs from each individual stream. Although risk aversion does affect bond prices and thereby the price of the replicating portfolio – akin to "absolute pricing," if payoffs of the replicating portfolio mirror those of another asset, e.g. a derivative, across all states at all times, then the replicating portfolio and the derivative must be worth the same. Risk aversion plays *no role in establishing such equality* of worth – akin to "relative pricing." Based on this equality, the price of the derivative is readily obtained from the value of the replicating portfolio. Payoff replication based on the Law of One Price is a fundamental, and the most intuitive, pricing tool.

As for "risk-neutral probability" method, it is derived from "replicating portfolio" method, after replacing the original probabilities with the *risk-neutral probabilities*, as shown formally below.

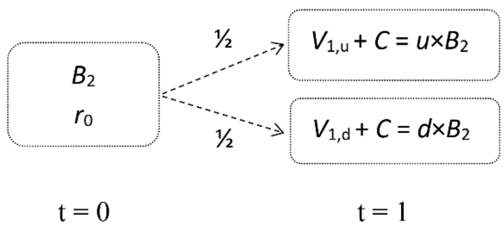
Derivation of *risk-neutral probabilities* from *replicating portfolio* framework

For the *riskless* bond 1 that pays sure cash flow of FV + C at maturity after 0.5 year:



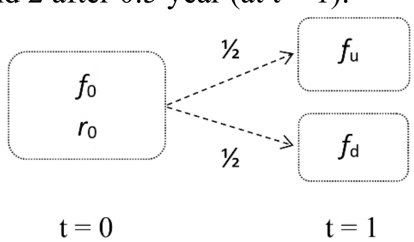
$$B_1 = \frac{FV + C}{(1 + r_0 / 2)}$$

For the *risky* bond 2 with multiplicative shocks (*u* and *d*) due to uncertain interest rate after 0.5 year:



Note: $u < d$, since interest rate in the up state lowers bond value, and vice versa.

For the *derivative* whose payoffs depend on the actual *market* interest rate and hence the value of the *risky* bond 2 after 0.5 year (at t = 1):



To replicate the payoffs, f_u and f_d, at t = 1, we use N_1 units of the 0.5-yr maturity, short-term, *riskless bond* 1, and N_2 units of the 1-yr maturity, long-term, *risky bond* 2, to form a payoff-replicating portfolio, such that:

$$N_1 \times (FV + C) + N_2 \times (V_{1,u} + C) = f_u \qquad \leftarrow \text{in the } \textit{up} \text{ state at } t = 1$$

$$N_1 \times (FV + C) + N_2 \times (V_{1,d} + C) = f_d \qquad \leftarrow \text{in the } \textit{down} \text{ state at } t = 1$$

Solving the system yields

$$N_1 = \frac{1}{FV + C} \times \frac{(V_{1,u} + C) \times f_d - (V_{1,d} + C) \times f_u}{V_{1,u} - V_{1,d}}$$

$$N_2 = \frac{f_u - f_d}{V_{1,u} - V_{1,d}}$$

The replicating portfolio's current value must be the same as the derivative's price, according to the Law of One Price or no arbitrage. Thus,

$$f_0 = N_1 \times B_1 + N_2 \times B_2$$

$$= \frac{(V_{1,u} + C) \times f_d - (V_{1,d} + C) \times f_u}{V_{1,u} - V_{1,d}} \times \frac{1}{(1 + r_0/2)} + \frac{f_u - f_d}{V_{1,u} - V_{1,d}} \times B_2$$

Since $V_{1,u} + C = u \times B_2$ and $V_{1,d} + C = d \times B_2$, we change the above expression of f_0 to

$$f_0 = \frac{u \times B_2 \times f_d - d \times B_2 \times f_u}{u \times B_2 - d \times B_2} \times \frac{1}{(1 + r_0/2)} + \frac{f_u - f_d}{u \times B_2 - d \times B_2} \times B_2$$

$$= \frac{1}{(1 + r_0/2)} \times \left(\frac{(1 + r_0/2) - d}{u - d} \times f_u + \frac{u - (1 + r_0/2)}{u - d} \times f_d \right)$$

Define:

$$Q_u = \frac{(1 + r_0/2) - d}{u - d}$$

$$Q_d = \frac{u - (1 + r_0/2)}{u - d}$$

The quantities, Q_u and Q_d, possess two interesting features:

a) $Q_u + Q_d = 1$

b) $Q_u, Q_d \geq 0$, if $u \leq (1 + r_0/2) \leq d$

The last condition is an equilibrium requirement to prevent any riskless arbitrage profit between the two bonds. To see this, note that the risky bond 2 at t = 1 earns a gross rate of return equal to $\frac{\boldsymbol{u \times B_2}}{\boldsymbol{B_2}} = \boldsymbol{u}$ in the up state of interest rate, and $\frac{\boldsymbol{d \times B_2}}{\boldsymbol{B_2}} = \boldsymbol{d}$ in the down state of interest rate, whereas the riskless bond 1 always earns a gross rate of return equal $\frac{\boldsymbol{FV+C}}{\boldsymbol{B_1}} = (1 + r_0/2)$ regardless of the state of interest rate. Thus, if $(1 + r_0/2) < u < d$, the risky bond would always outperform the riskless bond, even in the up state of interest rate, which cannot sustain in equilibrium. On the contrary, if $(1 + r_0/2) > d > u$, the riskless bond would always outperform the risky bond even in the down state of interest rate, which again cannot sustain in equilibrium.

The two features of a) and b) give quantities Q_u and Q_d all the properties to be treated as probabilities. Indeed, they are transformed probabilities, commonly referred to as "risk-neutral probabilities".

Thus, the price of the derivative can be expressed as:

$$f_0 = \frac{1}{(1 + r_0/2)} \times (Q_u \times f_u + Q_d \times f_d)$$

which becomes a simple *expectation under* the *risk-neutral probabilities* (rather than the original/real probability of ½) of present value of the derivative's payoffs *discounted at* the *riskless rate*. Interestingly, the original probabilities ½ no longer appear in the above expectation; they have been replaced by the risk-neutral probabilities. In fact, the principle behind the above pricing formula applies to *all* interest rate derivatives.

As mentioned already, although the original probabilities are absent in the formula for pricing a derivative, they do influence the price of the derivative – but only through indirectly determining the price of the risky bond 2, B_2, and thereby the payoffs to the derivative, f_u and f_d, both depending on B_2. B_2 already accounts for risk aversion to payoff uncertainty (or risk premium) of the risky bond 2 at t =1, as indicated by the original probabilities ½. However, once B_2 is *determined*, the effect of the original probabilities on f_0 is *fixed*, and we should not consider again how the original probabilities affect the derivative's price; otherwise, we would double count. This insight is a feat and one of the most useful discoveries in the theory of derivatives pricing under the theme of *risk-neutral pricing*.

Thus, we have established the theoretical connection between the payoff-replicating portfolio method and the risk-neutral probabilities method – the inherent logic for these two methods to always produce the same result when pricing interest-rate derivatives.

Logic connection between *calibrated rates* and *risk-neutral probabilities*

As alluded to in Note 1 earlier, the market value of the risky bond 2 at t = 1 depends on the up- or down-state of the actual interest rate. Given such uncertainty in bond value, market price is *lower* than the expectation of discounted value (the difference being the risk premium):

$$\frac{(½)(V_{1,u}+C)+(½)(V_{1,d}+C)}{1+r_0/2} > B_2 .$$

The adjustment from the *real* to the *calibrated* interest rates seeks precisely to make up the difference – the correctly calibrated rates ($r_{1,d}$ and $r_{1,u}$) affect $V_{1,d}$ and $V_{1,u}$ – and render the above relationship to an equation.

The adjustment from the *original* to the *risk-neutral* probabilities also seeks to make up the difference – the newly derived risk-neutral probabilities (p and 1 - p) replace the original probability of ½ – and render the above relationship to an equation.

Thus, the "calibrated rates" method and the "risk-neutral probabilities" method differ only algebraically: the former searches, for each period, the parameter of "down rate r_d" to match market price, while the latter searches, for each period, the parameter of "up probability p" to match market price. For each period, there is one degree of freedom in choosing a parameter (r_d or p) to match the market price that incorporates risk aversion. The trick is that once the r_d or p is found, they can be used to price other interest-rate derivatives as simple expectations of discounted value.

The "replicating portfolio" method stands out as being conceptually different from the other two; it relies on the fundamental Law of One Price, or no arbitrage. As shown by our formal demonstration in section 4.1, risk-neutral probabilities are derived from the replicating portfolio framework, while calibrated interest rates and risk-neutral probabilities are mere algebraic conversions of one another.

Given these inherent logical connections among the three methods, they certainly produce the same pricing result, and thus are equivalent for pricing purposes. For comparisons, Table 4 summarizes the appeals and inconveniences of each method.

Table 4
Comparative features of the three pricing methods

Pricing method	Pro	Con
calibrated forward rates (CR) method	Use of real probabilities; avoidance of challenging concept of risk-neutral probabilities.	Time consuming; tedious calculations with trials and errors.
risk-neutral probabilities (RN) method	Reduced amount of calculations.	Invokes the challenging concept of risk-neutral probabilities.
payoff-replicating portfolio (RP) method	Use of economic notion of Law of One Price.	Invokes intimidating matrix operations or system of linear equations.

Conclusion

Through numerical illustrations in a pedagogical setting, we have summarized three different approaches to pricing interest-rate contingent financial securities, such as call or put options on bonds, that are frequently used in finance courses, notably those dealing with fixed income.

We have further shown the inherent logics linking the three approaches to pricing interest-rate derivatives, which make them consistently produce the same price results. Thus we have established the differences, connections and equivalences of these highly popular pricing tools. Instructors and students can now appreciate the free choice of one approach over another in the class of fixed income, rather than feel puzzled or confused by the differences involved in each procedure.

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Overcoming Barriers to Teaching Machine Learning in Finance Courses Through Visual Tools

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This paper presents an innovative, project-based approach to teaching machine learning techniques to finance students using the visual, no-code platform Dataiku. The step-by-step procedure is demonstrated through an example project on predicting cryptocurrency returns, which equips students with hands-on experience in applying machine learning to financial problems. The visual, no-code approach enables students, even those without prior programming experience, to focus on understanding and applying machine learning concepts. This methodology is easily transferable to other finance educators looking to introduce machine learning in their courses. Student outcomes and feedback suggest that this approach effectively prepares students for the data-driven decision-making prevalent in the financial industry and serves as a foundation for future advanced data analytics education.

Keywords: Machine Learning, Visual Learning, Forecasting

Introduction

Machine learning (ML) and artificial intelligence (AI) have gained significant ground in analytical fields in recent years, transforming the way businesses operate and make decisions. The financial industry is no exception, with ML and AI being increasingly applied to enhance operational efficiency, improve customer experience, and mitigate risks (Pattnaik et al., 2024). As a result, it has become critically important for finance students to acquire knowledge and hands-on experience with these techniques to remain competitive in the job market and contribute to the industry's ongoing digital transformation (Aziz & Dowling, 2019).

However, learning ML and AI can be challenging for finance students who may lack programming experience, especially when it comes to working on data analysis projects. Traditionally, ML and AI courses often require proficiency in programming languages such as Python or R, which can be daunting for students without prior coding background (Brunhaver et al., 2020). This barrier can discourage finance students from engaging with these essential technologies, limiting their ability to leverage them in their future careers.

To address this challenge, researchers have explored the use of visual and user-friendly tools to introduce ML and AI concepts in a more accessible manner. Shmueli and Koppius (2011) advocated for the use of graphical user interfaces (GUIs) and visual programming environments, which allow students to focus on understanding the underlying principles of ML algorithms rather than grappling with complex coding syntax. Similarly, Witten et al. (2011) suggested the use of visual data mining tools, such as Weka, to provide an interactive and intuitive platform for students to experiment with ML techniques.

Building upon these ideas, this paper presents an innovative approach to teaching machine learning techniques to finance students using the visual, no-code platform *Dataiku*. By combining a project-based methodology with an intuitive, visual interface, this approach aims to equip finance students with practical experience in applying ML to financial problems, regardless of their programming background.

The project-based learning approach has been shown to be effective in enhancing student engagement and understanding of complex topics (Kolb & Kolb, 2005). In the context of teaching ML in finance, researchers have proposed the integration of ML techniques into finance courses through case studies and practical applications (Serrano-Cinca & Gutiérrez-Nieto, 2013; Brunhaver et al., 2020). They argued that exposing students to real-world financial problems and datasets can foster a deeper understanding of ML algorithms and their relevance in the industry.

Despite these advancements, there remains a gap in the literature on how to effectively integrate visual, no-code tools like *Dataiku* into finance curricula to teach ML techniques. This paper aims to fill this gap by demonstrating a step-by-step procedure for implementing a project-based approach using *Dataiku*, focusing on predicting cryptocurrency returns as an example. By providing a comprehensive roadmap for educators to follow, this work contributes to the growing body of research on innovative pedagogical approaches in financial education.

The remainder of the paper is structured as follows: Section 2 describes the project-based teaching methodology and the *Dataiku* platform; Section 3 presents the step-by-step procedure for the cryptocurrency returns prediction project; Section 4 discusses the implications of this approach for financial education; and Section 5 concludes the paper.

Project Based Teaching

The course uses a project-oriented approach focused on predicting cryptocurrency returns using ML algorithms. The software platform adopted is *Dataiku*©, which provides virtual recipes enabling users to implement ML techniques without coding. *Dataiku* offers free access for higher education and personal use. The platform also allows advanced users to write Python code to customize the workflow. The flexibility of combining no-code and future coding and no-code integration provides unique and valuable opportunity for the finance students who start with little or no experience of coding and data analysis while transaction to more advanced skill sets.

The step-by-step procedure involves literature review, data collection from sources like Glassnode, data processing and merging in *Dataiku*, sample selection, training ML models, making predictions, and verifying model performance.

As an example, a Random Forest model is used with explanatory variables easily selected and adjusted through the *Dataiku* visual interface. The platform also provides visual decision tree representations. After training, the model is deployed on a testing dataset and performance visualized.

This project approach allows students to apply ML techniques to financial problems, gaining hands-on experience with an intuitive, visual platform. The visualization and guided recipes facilitate learning for those new to programming and data analysis.

Comparing Visual, No-Code Approach with Traditional Teaching Methods

The visual, no-code approach to teaching machine learning in finance courses using platforms like *Dataiku* differs from traditional methods in several key aspects. Traditional methods often rely on teaching programming languages such as Python or R, which can be challenging for students without prior coding experience (Brunhaver et al., 2020). In contrast, the visual, no-code approach allows students to focus on understanding the underlying principles of machine learning algorithms without getting bogged down by complex coding syntax. This approach emphasizes hands-on application and experimentation through a project-based learning methodology, providing students with practical experience in applying machine learning to real-world financial problems.

However, it is important to acknowledge potential limitations of the visual, no-code approach. Students may not gain a deep understanding of the underlying programming concepts, which could limit their ability to customize models or handle more complex data preprocessing tasks. Therefore, while the visual, no-code approach is a great starting point, students should be encouraged to progress to traditional coding-based methods as they advance in their learning journey to gain a more comprehensive understanding of machine learning. Integrating both approaches can provide students with a well-rounded education, equipping them with the conceptual understanding and practical skills needed to succeed in the rapidly evolving field of financial technology.

Course Platform

The course uses a project-oriented approach focused on predicting cryptocurrency returns using ML algorithms. The software platform adopted is *Dataiku©*, which provides virtual recipes enabling users to implement ML techniques without coding. Dataiku offers free access for higher education and personal use, making it an accessible choice for educational institutions and students. A detailed documentation of the package can be found at www.Dataiku.com.

One of the key advantages of using *Dataiku* in this course is its flexibility in combining no-code and code-based workflows. The platform allows beginners to start with visual recipes and pre-built components, enabling them to understand and implement ML techniques without the need for extensive coding knowledge. As students progress and gain more experience, they can gradually transition to writing Python code within the *Dataiku* environment to customize their workflows and build more advanced models.

This unique combination of no-code and future coding integration in *Dataiku* provides a valuable opportunity for finance students who may start with little or no experience in coding and data analysis. By using *Dataiku*, students can focus on understanding the fundamental concepts and best practices of machine learning in finance, while still having the option to expand their skills and explore more complex techniques as they advance in their learning journey.

Moreover, *Dataiku*'s user-friendly interface and collaborative features make it an ideal platform for project-based learning. Students can easily share their workflows, models, and insights with their peers and instructors, fostering a collaborative learning environment that encourages knowledge sharing and peer feedback.

Data Source and Feature Selection

The project utilizes data from Glassnode.com, a comprehensive data platform for cryptocurrency and blockchain analysis. Glassnode provides a wide range of on-chain and market data for various cryptocurrencies, including Bitcoin. The platform offers a user-friendly interface and API access, making it an ideal choice for educational projects and research.

For this project, we focus on predicting Bitcoin returns using lagged explanatory variables. The primary data collected from Glassnode includes:

1. Daily Bitcoin price data: The daily closing prices of Bitcoin are collected to calculate the daily returns, which serve as the target variable in the predictive model.

2. Change in new/active addresses: The daily change in the number of new and active Bitcoin addresses is collected as the main explanatory variable. This metric can potentially indicate changes in Bitcoin's adoption and usage, which may impact its price.

The raw data is collected from Glassnode's web interface. The data is then preprocessed and transformed to create the necessary features for the predictive model.

To select the most relevant features for the project, the professor facilitates extensive discussion among the students. The discussion covers various aspects of feature selection, including:

1. Economic significance: Students are encouraged to think critically about the economic factors that may influence Bitcoin returns and propose potential explanatory variables based on their understanding of the cryptocurrency market.
2. Data availability and quality: The discussion also focuses on the availability and quality of data for the proposed explanatory variables. Students learn to assess the reliability and completeness of the data sources and consider any limitations or challenges in data collection.
3. Statistical significance: Students explore the statistical relationships between the proposed explanatory variables and the target variable (Bitcoin returns) using techniques such as correlation analysis and scatter plots. This helps them identify the most promising features for the predictive model.
4. Domain knowledge: The professor encourages students to leverage their domain knowledge in finance and economics to justify their feature selection choices. This promotes a deeper understanding of the underlying factors driving cryptocurrency returns.

Through this guided discussion, students actively participate in the feature selection process, developing critical thinking skills and gaining practical experience in data-driven decision making. The professor's role in facilitating the discussion ensures that students consider various aspects of feature selection and make informed choices based on economic significance, data availability, statistical relationships, and domain knowledge.

The selected features, along with the daily Bitcoin returns, are then used to create the final dataset for the project. The dataset is split into training and testing subsets to evaluate the performance of the predictive model.

The project file which include data, procedure, flow and models can be downloaded via link: https://github.com/CinderZhang/FinML/blob/main/Dataiku/MYNAME_BLOCKCHAIN_TESTING.zip

Procedure

The procedure starting from literature review, data collection, data processing, sample selection, Machine learning model training, prediction, and verification. The project file, including datasets and workflow, can be downloaded and imported to your own *Dataiku* platform.

We try to conduct an analysis to see if any feature/factor has prediction power of Crypto return. In particular, we run the following model using Bitcoin return as the example.

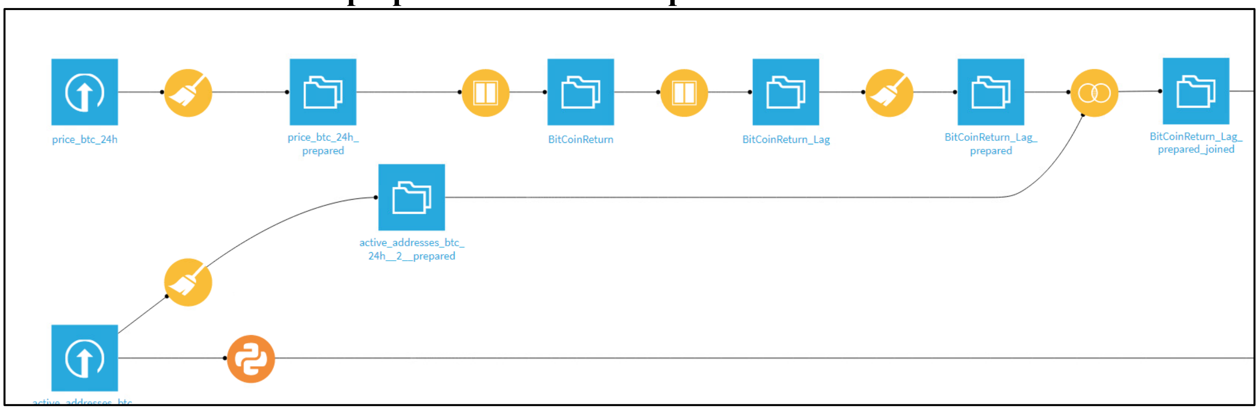
$$\boldsymbol{r_t = f(X_{t-1}) + \epsilon_t}$$

Where $\boldsymbol{r_t}$ is the Bitcoin return, $\boldsymbol{X_{t-1}}$ is the lagged explanation variable(s). The frequency is not specified as the students should explore alternative frequencies. We use the change of new/active address as the main interested variables in the example. The raw data is collected from

Glassnode.com. There are more information for the readers to learn more about what Glassnot.com offers via the link: https://studio.glassnode.com/catalog.

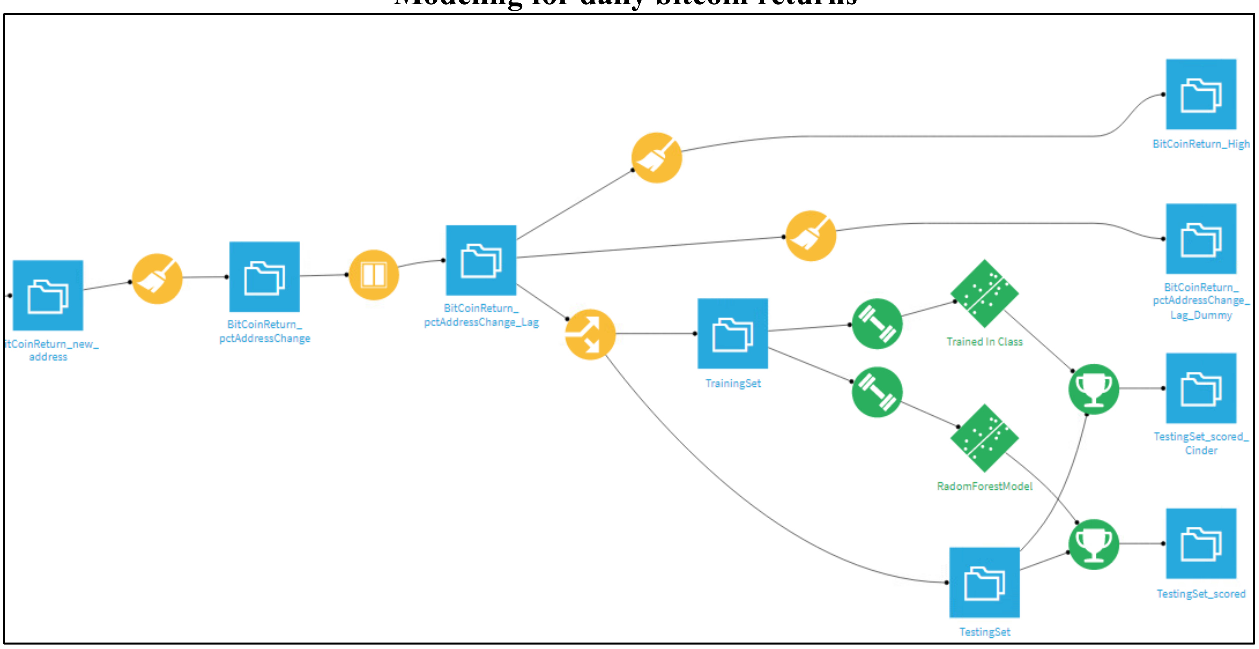
The following diagram shows how we use *Dataiku* to prepare the data for the analysis. As demonstrated below, we employ *Dataiku* to convert the raw pricing data and new address data into return and changes and merged the datasets. There is no single code needed for the process. However, during the process, the students visually observe the data transactions from raw to processed, such as from price to return; and merging datasets by common identifiers, *data* and *coin name* in this case. The diagram clearly demonstrates the procedure and the merging process visually.

Figure 1
Data preparation for bitcoin price and address data



After the heavy lifting of the data processing, we then start the fun part by letting *Dataiku* run prebuild models and examine the model performance as shown below.

Figure 2
Modeling for daily bitcoin returns



Using Random Forest model as an example, the explanatory variables can be easily selected and adjusted. In the course, students were introduced to various machine learning algorithms and had the opportunity to test multiple models in class and through take-home practices. The models explored included but not limited to Decision Trees, Random Forests, Support Vector Machines, and Gradient Boosting. Each model has its own strengths and weaknesses, and students gained hands-on experience in applying these algorithms to financial data using the *Dataiku* platform

For the cryptocurrency return prediction project, we chose to present the Random Forest model in this paper due to several key advantages:

1. Robustness to overfitting: Random Forest is less prone to overfitting compared to single decision trees. The algorithm introduces randomness by selecting a random subset of features at each split and building multiple trees on different bootstrap samples of the data. This randomness helps to reduce the variance of the model and improve its generalization ability.
2. Feature importance: Random Forest provides a built-in measure of feature importance, which helps to identify the most informative variables for predicting cryptocurrency returns. This information can be valuable for students to understand the factors driving the market and make more informed investment decisions.
3. Interpretability: Although Random Forest is an ensemble model, it still maintains some level of interpretability. The visual decision tree representations provided by *Dataiku* allow students to explore the structure of individual trees and gain insights into the decision-making process of the algorithm.

While Random Forest was selected as the primary model for this paper, it is important to note that students were encouraged to experiment with multiple algorithms and compare their performance. The hands-on experience gained through testing different models provided students with a comprehensive understanding of the strengths and limitations of each algorithm and equipped them with the skills to make informed choices when applying machine learning to financial problems.

By presenting the Random Forest model in this paper, we aim to showcase its effectiveness in predicting cryptocurrency returns and demonstrate how the visual, no-code approach using *Dataiku* can enable finance students to harness the power of machine learning without the need for extensive programming knowledge.

Figure 3
Feature selection

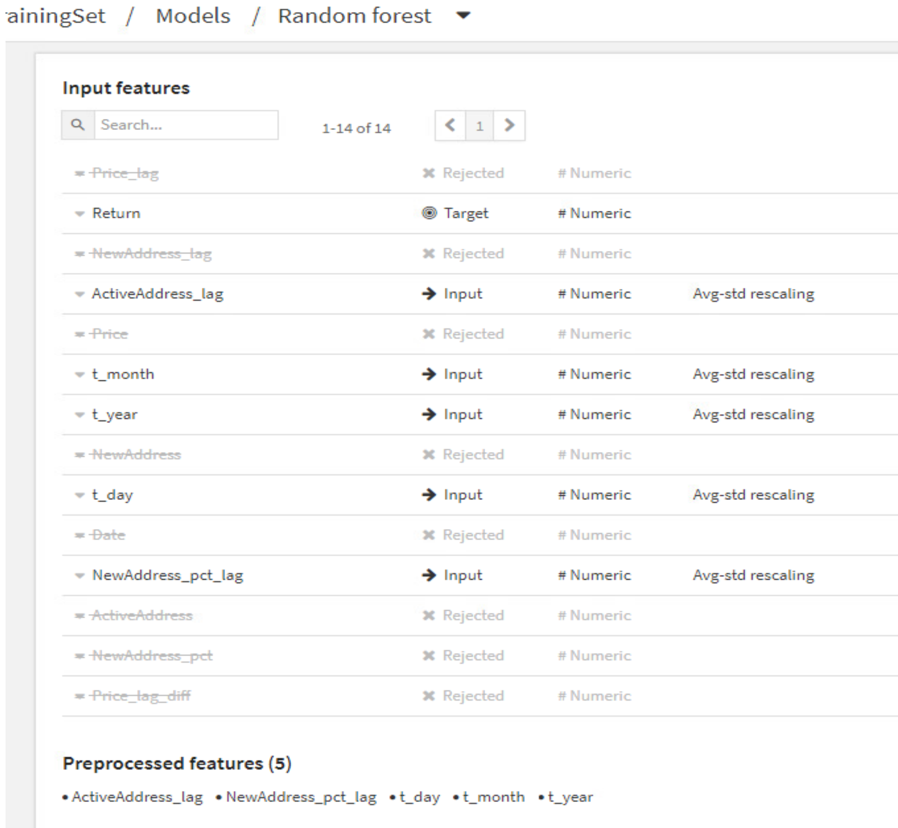
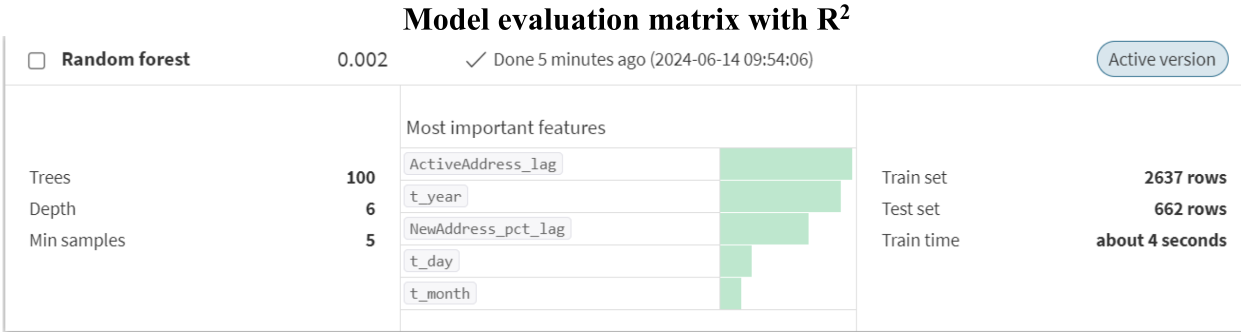
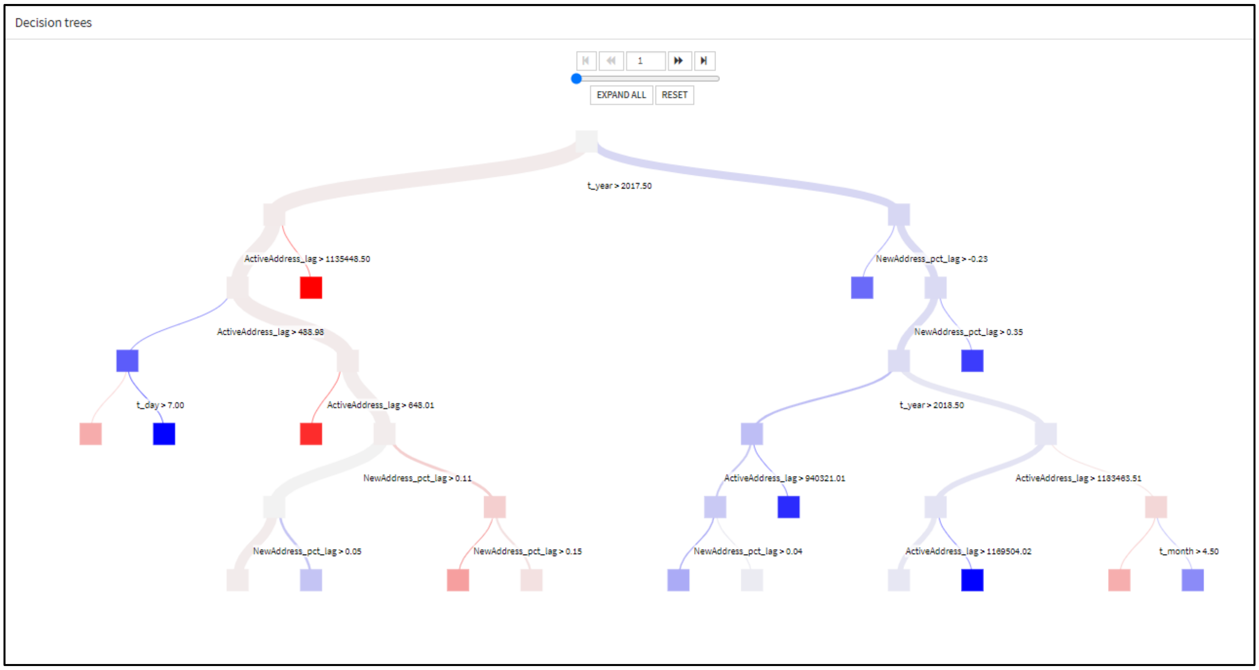


Figure 4
Model evaluation matrix with R^2



A visual decision tree is also presented in the platform.

Figure 5
One of the decision trees for the Random Forest model



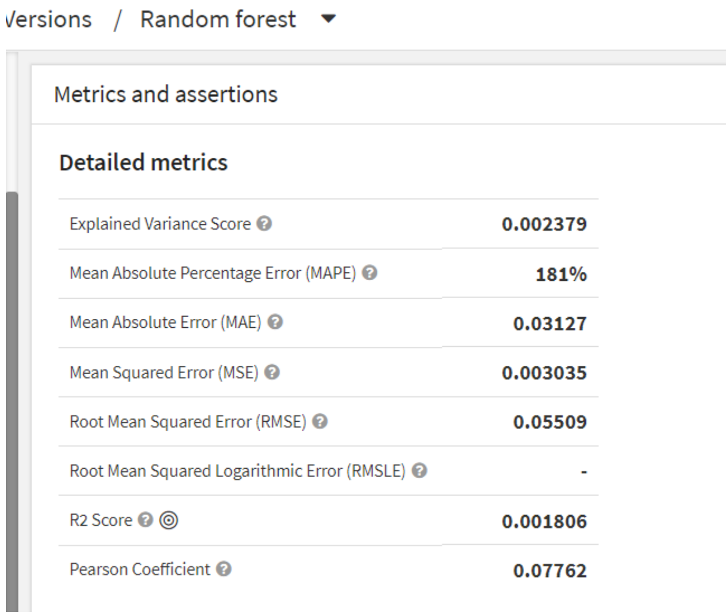
Evaluation Metrics

To assess the performance of the Random Forest model in predicting cryptocurrency returns, *Dataiku* provide several evaluation metrics commonly used for tasks:

1. Mean Squared Error (MSE): MSE measures the average squared difference between the predicted and actual values. It penalizes larger errors more heavily than smaller ones, making it sensitive to outliers.
2. Root Mean Squared Error (RMSE): RMSE is the square root of the MSE and provides an estimate of the standard deviation of the residuals. It is in the same units as the target variable, making it easier to interpret than MSE.
3. Mean Absolute Error (MAE): MAE measures the average absolute difference between the predicted and actual values. It is less sensitive to outliers compared to MSE and RMSE.
4. R-squared (R^2): R^2 represents the proportion of variance in the target variable that is predictable from the explanatory variables. It ranges from 0 to 1, with higher values indicating better model performance.

The diagram below shows the available evaluation metrics on *Dataiku*. Instructor may want to emphasize on R^2 as it available for most of models and helps to transit from statistical analysis to machine learning techniques. The 0.18% R^2 indicates that the model is not performing well in term of predicting bitcoin returns. The analysis definitely need to refined and developed further. This initial analysis can lead to long term projects for students to explore.

Figure 6
Alternative Evaluation Metrics



Versions / Random forest

Metrics and assertions

Detailed metrics

Explained Variance Score	0.002379
Mean Absolute Percentage Error (MAPE)	181%
Mean Absolute Error (MAE)	0.03127
Mean Squared Error (MSE)	0.003035
Root Mean Squared Error (RMSE)	0.05509
Root Mean Squared Logarithmic Error (RMSLE)	-
R2 Score	0.001806
Pearson Coefficient	0.07762

The Random Forest model is then deployed and tested in the testing data sample. A visual presentation of the performance is as follows.

Figure 7
Plot for testing dataset with the trained Random Forest model

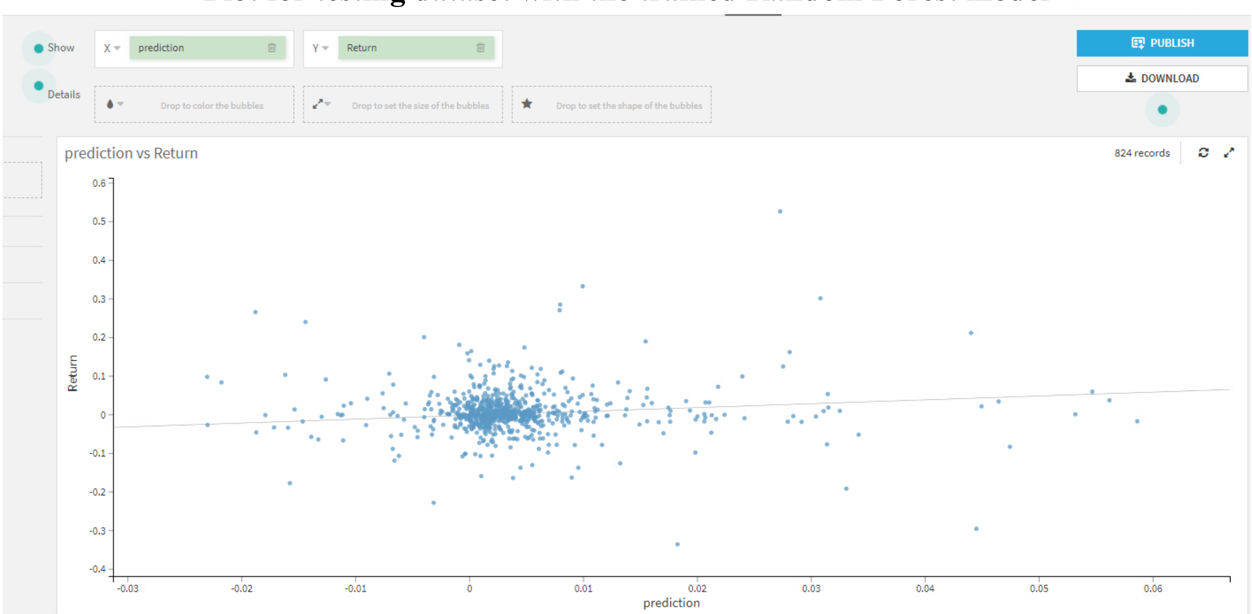
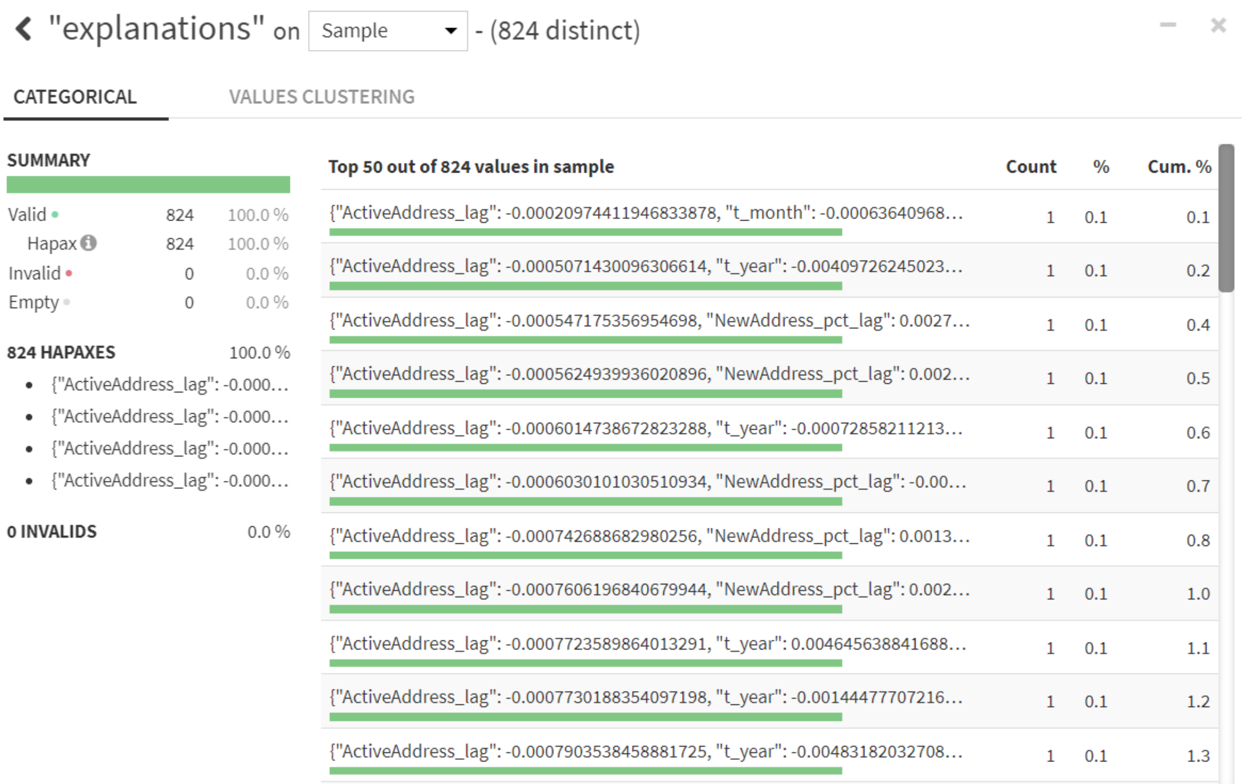


Figure 8
Detailed Explanations of each Observations



The technique introduces students to rationalize financial data analysis, that is, research and define the question, data collection, data processing, modeling, and testing. The visualized and intuitive procedure allows students to apply and test their financial knowledge such as market efficiency, factor models and so on.

The technique can be easily adopted by other instructors. *Dataiku©* is free to higher education institutions and for personal use. Financial data is readily available in many financial websites such as Yahoo Finance, Federal Reserve, FINRA, Glassnote, Kaggle and more. The procedure is easy to follow and intuitive to learn for whom with some financial data handling experience. A guide can be found on the author's GitHub page. The link for the summary of the resources can be found via the link: https://github.com/CinderZhang/FinML/blob/main/ML%20Resource.ipynb

Learning Outcomes and Student Feedback

The courses delivered promising outcomes, as evidenced by students securing data analytics related jobs at companies such as Goldman Sachs, Citi Group, Stevens Bank, StateStreet, Tyson Food Innovation Financing, and WalMart Global IT among others. Some part-time students were even able to apply the skills learned in class to their current workplace during the semester. This success demonstrates the practical value and relevance of the course content.

Throughout the journey, I grew alongside my students, learning from their experiences and feedback. Many students appreciated the connection between the knowledge and skills taught in class and their real-world applications. Using *Anthology Course Evaluations System* in the

University of Arkansas, comments from students were anonymously collected. Students commented the following:

> "The use of real-world examples and applications of the lessons throughout the course has helped me expand my knowledge past just passing a course and utilizing the information in real life" (*Anthology Course Evaluations System*).
>
> "Dr. Z did an awesome job of relating the course material to real-world application. I feel like I learned a lot about the future of technology in finance and how understanding python code and AI will give you an edge in the workplace" (*Anthology Course Evaluations System*).

However, when it came to implementing the learning in personal projects, some students found it challenging. One student commented the following:

> "Dr. Zhang clearly understands the subject matter and is enthusiastic about teaching this class. Our projects were, however, very challenging this semester compared to last. We attended a couple of very interesting workshops on *Dataiku* and Machine Learning / Artificial Intelligence. These were all great! Unfortunately, he provided limited assistance on the projects. The course would have been much more interesting if he assisted us more on the projects" (*Anthology Course Evaluations System*).

Upon realizing these shortcomings, I introduced a requirement for students to meet with me one-on-one for their projects during the semester. These Personal Project Review (PPR) sessions focused on discussing the objectives, progress, challenges, and follow-up plans for each student's project. The PPRs proved to be effective, as evidenced by the following student comments:

> "The PPRs were a great tool in furthering my understanding of the topics discussed in class. Being able to get real-time feedback on the projects was a valuable part of the course that led to me being able to grasp some of the more complex topics in the course" (*Anthology Course Evaluations System*).
>
> "I think that being face to face and meeting really helps me get a grasp on the concept more so than just doing it, turning it in, and getting a grade" (*Anthology Course Evaluations System*).

This comment demonstrates how the visual, no-code approach of *Dataiku* allowed students to focus on understanding and applying financial concepts rather than getting bogged down in coding details, enabling them to achieve impressive results like quickly computing accurate regressions for complex financial models. It also paves and eases the way for them to further their development into coding if they are interested.

Conclusion

This paper presents an innovative, project-based approach to teaching machine learning to finance students using the visual, no-code platform *Dataiku*. By providing a comprehensive roadmap and demonstrating the effectiveness of this approach through student outcomes and feedback, this work contributes to the growing body of research on pedagogical approaches in financial education. Importantly, this course serves as a gateway for students to learn coding in the future, as the visual, no-code approach prepares them to understand the underlying concepts and logic behind machine learning algorithms. As the financial industry continues to evolve, equipping students with machine learning skills through accessible, hands-on methods, and laying the foundation for further coding education will be crucial for their future success.

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Factor Models, Shortfall Constraints, and Efficient Frontiers: Exercises and Applications

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The mean-variance approach to creating efficient portfolios is a cornerstone of investment theory and practice. Efficient portfolios provide the highest expected return for a given level of portfolio risk. The mean-variance approach to construction of efficient portfolios uses the mean to measure expected returns and the standard deviation to measure risk. Implicitly, this assumes that the distribution of portfolio returns is symmetric around the mean return. This convention persists in textbooks and traditional instruction even though factor models are taught as superior approaches to measuring expected returns. Furthermore, modern portfolio managers often view risk in terms of shortfall constraints and measure only the volatility below a critical benchmark, rather than using the standard deviation. In this paper we present a comprehensive set of student exercises that integrate factor model estimates of expected returns, alternative views of portfolio risk, and efficient portfolios. We do not introduce new instructional topics, concepts, or theories. Rather, our goal is to integrate applications of concepts learned separately in traditional statistics, investments and portfolio management courses. Files with student assignments, supporting instructions, and student deliverable products for each assignment are provided in an instructional website (https://facultystaff.richmond.edu/~dnorth/EFpaper/).

Keywords: Efficient Frontiers, Shortfall Constraints, Factor Models, Semivariance

Introduction

Students and teachers of investments work with a variety of theories, models, and graphs that explore different risk and return concepts. Due to the complexity of the material, step by step instruction is necessary, resulting in coverage of statistical means, standard deviations, regression models, betas, factor models, market models, expected returns, and efficient frontiers over a number of textbook chapters. Students often struggle to connect all of these theories, models, calculations, and graphs in a way that allows application to real world data. In this paper we present exercises and Excel applications using readily available data to integrate key investment concepts leading to construction of alternative efficient frontiers. The exercises are separable allowing instructors to either make a series of assignments matched to topic coverage in different courses or use the exercise as a comprehensive capstone assignment in a portfolio analysis class. We provide a website (https://facultystaff.richmond.edu/~dnorth/EFpaper/) with links to instructional files that allow users to walk through all applications to include assignments, specific instructions, and examples of student deliverable products from each assignment.

We start with a review of the key theories and concepts supporting our exercises. We do not break new theoretical ground and the review will be similar to course material in investments classes. Next, we report on a series of student projects we assigned using data for a manageable set of stocks. Students first construct traditional mean-variance efficient frontiers as a starting point. They then modify their efficient frontier for factor model estimates of expected returns, rather than using the mean return. The last project requires students to use a downside risk measure based on deviations below a target return rather than the standard deviation. Our instructional website contains the specific student assignments, instructions for using Solver in Excel to find optimal portfolio weights, and the student deliverable products used for assessment of learning. Our overall objective is to provide hands-on experiences that both reinforce instruction on risk and return concepts, and most important, illustrate how these concepts are integrated in unifying applications. For each assignment, Appendix I provides the specific learning objectives, topical background coverage, and the formative and summative assessments. We conclude with an overview of student responses to a questionnaire about the assignments and suggestions for extensions of the paper.

Review of Risk and Return Topics – The Holy Grail

The review we provide in this section is commonly found in textbooks spread out over several chapters. It helps to remind students that all this material is based on a risk-return theme but there are different ways of thinking of expected returns and risk. The traditional risk-return tradeoff builds on a mean-variance model assuming a normal distribution. The expected return for the i^{th} security is the arithmetic mean (R_{ave}) over n periods, defined as equation (1).

$$E\,(R_i) = \sum_{k=1}^{n} R_k = R_{Ave} \qquad (1)$$

Risk is defined as the uncertainty of achieving the expected return (mean) and is measured by the standard deviation (σ) shown as equation (2).

$$\sigma_i = \left[\sum_{k=1}^{n} (R_k - R_{Ave})^2 / (n-1) \right]^{1/2} \qquad (2)$$

The coefficient of variation defined by equation (3) is a simple risk-return tradeoff measure.

$$CV = \text{risk / expected return} = \sigma / R_{ave} \qquad (3)$$

The preferred investment has the lowest CV. Markowitz (1952) extended the mean-variance approach to portfolios and modified the view of risk by illustrating the concept of diversification. Sensitivity to a common market factor is the only relevant risk affecting expected returns if investors hold portfolios, since unsystematic components cancel out as securities are added. The expected return for a portfolio $E(R_p)$ with m securities is the weighted average of past returns, where w_i is the weight for the ith security. Equation (4) represents the portfolio expected return.

$$E\,(R_p) = \sum_{i=1}^{m} w_i R_i \qquad (4)$$

The standard deviation of a portfolio given by equation (5) is more complicated.

$$\sigma_p = [\sum_{i=1}^{m} w^2_i \sigma^2_i + \sum_{i=1}^{m} \sum_{j=1}^{m} COR_{i,j}\, w_i\, w_j\, \sigma_i\, \sigma_j\,]^{1/2} \quad (5)$$

where w_i is the weight on the ith individual security in the portfolio
where w_j is the weight on the jth individual security in the portfolio
where σ_i is the standard deviation of the ith security in the portfolio
where σ_j is the standard deviation of the jth security in the portfolio
$COV_{i,j}$ is the covariance of the ith and jth securities in the portfolio = variance when i = j
$CORR_{i,j} = COV_{i,j} / (\sigma_i, \sigma_j)$ and can take values from +1 to -1.

Efficient Portfolios

Equation (5) shows that the portfolio standard deviation (σ_p) is not just a function of the standard deviations (σ_I) and weights (w_i). Correlations ($COR_{i,j}$) measuring how securities move relative to each other also play a key role. When the correlation between securities is less than 1 the portfolio standard deviation is less than the weighted average of the individual standard deviations. The only case where adding securities does not reduce the portfolio standard deviation occurs when all securities in the portfolio have a perfectly positive correlation (CORR = 1) with each other. Only non-diversifiable volatility (systematic) enters into the risk-return tradeoff, since the diversifiable component can be eliminated by holding multiple securities.

Coverage of Markowitz Efficiency follows the discussion of diversification. Of all portfolios that could be constructed from a given set of stocks, an efficient portfolio provides the minimum risk for a given expected return. A portfolio manager should hold an efficient portfolio with the highest reward to risk ratio defined as the Sharpe ratio (Sharpe,1966).

$$\text{Sharpe Ratio} = (R_p - R_f) / \sigma_p \quad (6)$$

Construction of the efficient frontier first requires expected return data for a given set of desired securities. Textbooks use the historic mean as the expected return and the standard deviation as a measure of risk. The traditional two-dimensional efficient frontier using the mean-variance criteria for expected returns and risk is a base-case for our first exercise, specified as the first assignment in the instructional website.

Expected Returns and Factor Models

Factor models explain expected returns as the risk free rate plus a risk premium determined by an asset's sensitivities to risk factors and the expected market premium for exposure to the risk factors. Our student assignments require estimation of expected returns with applications of Sharpe's (1964) single factor CAPM model and the three factor model of Fama and French (noted as F&F, 1993). Extensions of our assignment to other expected return models such as Haugen and Baker (1996), or Carhart (1997) are possible once our basic exercise is mastered.

<u>The Capital Asset Pricing Model</u>

The Capital Asset Pricing Model (CAPM) defines the market index (R_M) as the single factor driving systematic (non-diversifiable) risk of a security or portfolio. An investor can expect a rate of return commensurate with the risk free security (R_f) plus a market risk premium (RP). This premium is defined as the sensitivity of the security's return to movement of the market index (β_i) times the expected premium for taking market exposure, defined as $E[(R_M)– R_f]$. For the CAPM, specified in equation (7), the only measure specific to the security is its beta (β_i).

$$E(R_i) = R_f + \beta_{i,M}\ [E(R_M)– R_f\,] \tag{7}$$

The expected return from the CAPM requires inputs of the estimated beta, risk free rate, and expected equity market risk premium. Equation (8) is a regression model (called the market model) used to estimate beta (β_i) and an excess return called alpha (α_i), which is expected to be zero in an efficient market. Data for the market model consists of past monthly stock returns ($R_{i,t}$), the monthly risk free rate ($R_{f,t}$) and the monthly market index return ($R_{M,t}$) over the past five year period.

$$(R_i - R_f)_t = \alpha_i + \beta_{i,M}\ (R_M)_t + \text{error}_t \tag{8}$$

The expected return estimate comes from substituting into equation (7) the estimated beta from the regression using equation (8), a risk free rate at the time of estimation, and the expected equity market premium. We follow F&F by using the long run average equity market premium ($R_M – R_f$) obtained from the Ken French website (hereafter noted as KFW): http://mba.tuck.*Dartmouth.edu/pages/faculty/data_library.html.* Several more sophisticated forecasting models are available to estimate the equity market premium for extensions of our exercises (e.g., see: https://www.investopedia.com/investing/calculating-equity-risk-premium).

Fama and French Three Factor Model

F&F (1993) provide an alternative formulation for expected returns. They expand the CAPM by adding two additional factors beyond the market factor. A premium is required for a market capitalization factor (SMB), measured as the difference in the expected return for small minus large capitalized firms. The third premium (HML) is required for the difference between the expected return for high book to market and low book to market firms. F&F consider returns for added factors to be premiums for inherent risks of smaller and out-of-favor firms (high book to market ratio). Practitioners consider these returns to be premiums for persistent size and value inefficiencies. In any event, the point is to get the best prediction of returns and evidence suggests that adding factors improves predictions (lower adjusted R-squares). Equation (9) is the specification of the three factor F&F model.

$$E(R_i) = R_f + \beta_{i,M}\, E(R_M – R_f) + \beta_{i,S}\, SMB + \beta_{i,B/M}\, HML \tag{9}$$

The KFW site provides monthly data for the market index, small capitalization, and the high book to market premiums. With return data for the security ($R_{i,t}$) and the factor premium data, the alpha (α) and factor betas (β_M, β_S, and $\beta_{B/M}$) are estimated using regression equation (10).

$$(R_i – Rf)_t = \alpha_I + \beta_{i,M}\ (R_M – R_f)_t + \beta_{i,S}\ SMB_t + \beta_{i,B/M}\ HML_t + \text{error}_t \tag{10}$$

Expected returns are calculated by substituting the risk free rate at the time of estimation along with the regression estimates of the three factor betas and the long run factor premiums from the KFW site into the F&F model of equation (9).

Portfolio Risk – The Semi-standard deviation

The standard deviation is an appropriate measure of portfolio risk only when an investor's target is the mean return and returns have a symmetric distribution. In most cases a shortfall from a critical return target represents risk. For example, pension fund managers consider the key risk to be falling short of an actuarial return target. Also, Kahneman and Tversky (1979) find that investors suffer greater disutility from losses than the utility gained from commensurate gains, suggesting a focus on downside volatility as risk. To measure downside risk, students calculate a semi-deviation using a relevant target return (RT) rather than the mean. The target return may be the mean, median, zero, or a specific shortfall return, depending on the distribution and the manager's objective. Equation (11) specifies the semi-deviation.

$$\sigma_{semi,\, i} = [\sum_{i=1}^{v} (R_i - RT)^2 / v]^{1/2} \tag{11}$$

A number of different efficient frontiers exist depending on the choice of an expected return model and choice of the relevant portfolio risk. Nevertheless, investment courses focus only on the traditional mean-variance approach, missing an opportunity to integrate alternative views of risk and expected returns taught in prior chapters. Each variant of an efficient frontier requires students to work with expected return models, relevant risk measures, portfolio construction, and efficient frontier estimation. These concepts, presented over a number of textbook chapters, are integrated with real data in the assignments that follow.

Construction of the Traditional Mean-Variance Frontier

Our first assignment requires students to use historical data to construct a traditional efficient frontier. The assignment specifies a set of deliverable products that students turn in for assessment. Students construct the traditional mean-variance efficient frontier to establish basic concepts and applications before adding factor models and alternative risk measures in subsequent assignments. Students chose ten stocks from one of our student managed investment funds to construct efficient portfolios, although the project could begin with a set of stocks selected by the instructor or students. The number of stocks should be small to make the exercises manageable, especially for later assignments where regressions are needed for each stock to get expected values. In practice, the number of stocks would be larger to provide better diversification. For each stock, students first calculate the monthly total rates of return ($R_{i,t}$) over the prior five-year period. There are a number of readily available sources for these returns but *https://finance.yahoo.com* is likely to be the most accessible. Students calculate the total return $R_i = (P_1 - P_0)/P_0$ using adjusted monthly stock prices, where P_1 is the end of month price and P_0 is the prior end-of-month price. The end-of-month adjusted prices from *Yahoo* include stock splits and dividends. Monthly data coding may be confusing for students because prices listed at the beginning of the month are actually end of month prices. Once loaded into Excel, students generally have little difficulty in generating monthly returns.

Tables 1, 2, and 3 represent the first deliverable products from students. In Table 1 summary statistics for the chosen stocks are provided to include the mean, standard deviation, coefficient of variation, and Sharpe Ratio. The monthly risk free rate average of 0.082% for this part of the assignment came from the KFW site and was given to students. Students noted that stock #3 offered the best performance on a risk-adjusted basis while stock #8 had the highest monthly total rate of return.

Table 1
Monthly Five-Year Summary Statistics for Ten Selected Stocks

	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10
Mean	1.51%	1.75%	2.43%	1.01%	1.37%	1.92%	2.49%	2.81%	1.44%	2.20%
Std. dev.	5.74%	6.14%	6.21%	6.14%	4.83%	9.43%	7.19%	7.89%	3.95%	5.65%
Coef. of Var.	3.81	3.51	2.56	6.05	3.53	4.92	2.89	2.81	2.74	2.57
Sharpe ratio	0.25	0.27	0.38	0.15	0.27	0.19	0.34	0.35	0.34	0.37

Note: The monthly risk free rate average was 0.08%

Table 2 provides correlations for all combinations of the ten stocks allowing students to examine which stocks offer the best and worst potential diversification. For example, the lowest correlations tend to occur for stock #9 while #8 tends to have higher correlations with other stocks. Even so, none of the stocks are redundant since all correlations are well below 1.

Table 2
Correlation Matrix for Ten Selected Stocks over Five Years of Data

	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10
Stock1	1.000									
Stock2	0.375	1.000								
Stock3	0.267	0.503	1.000							
Stock4	0.358	0.586	0.426	1.000						
Stock5	0.357	0.525	0.378	0.337	1.000					
Stock6	0.090	0.208	-0.011	0.163	0.225	1.000				
Stock7	0.213	0.250	0.285	0.101	0.327	0.138	1.000			
Stock8	0.409	0.420	0.451	0.433	0.370	0.320	0.445	1.000		
Stock9	0.146	-0.271	0.081	-0.114	0.040	0.048	0.127	-0.107	1.000	
Stock10	0.254	0.559	0.337	0.335	0.624	0.182	0.427	0.349	-0.051	1.000

Students produce the variance-covariance matrix shown in Table 3, which completes the basic output required to construct the traditional efficient frontier. This is a good point to check on student output for Tables 1, 2, and 3 before this output is used to construct the basic efficient frontier.

Table 3
Variance-covariance Matrix for Ten Stocks over Five Years of Data

	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10
Stock1	0.33%	0.13%	0.10%	0.13%	0.10%	0.05%	0.09%	0.19%	0.03%	0.08%
Stock2	0.13%	0.38%	0.19%	0.22%	0.16%	0.12%	0.11%	0.20%	-0.07%	0.19%
Stock3	0.10%	0.19%	0.39%	0.16%	0.11%	-0.01%	0.13%	0.22%	0.02%	0.12%
Stock4	0.13%	0.22%	0.16%	0.38%	0.10%	0.09%	0.04%	0.21%	-0.03%	0.12%
Stock5	0.10%	0.16%	0.11%	0.10%	0.23%	0.10%	0.11%	0.14%	0.01%	0.17%
Stock6	0.05%	0.12%	-0.01%	0.09%	0.10%	0.89%	0.09%	0.24%	0.02%	0.10%
Stock7	0.09%	0.11%	0.13%	0.04%	0.11%	0.09%	0.52%	0.25%	0.04%	0.17%
Stock8	0.19%	0.20%	0.22%	0.21%	0.14%	0.24%	0.25%	0.62%	-0.03%	0.16%
Stock9	0.03%	-0.07%	0.02%	-0.03%	0.01%	0.02%	0.04%	-0.03%	0.16%	-0.01%
Stock10	0.08%	0.19%	0.12%	0.12%	0.17%	0.10%	0.17%	0.16%	-0.01%	0.32%

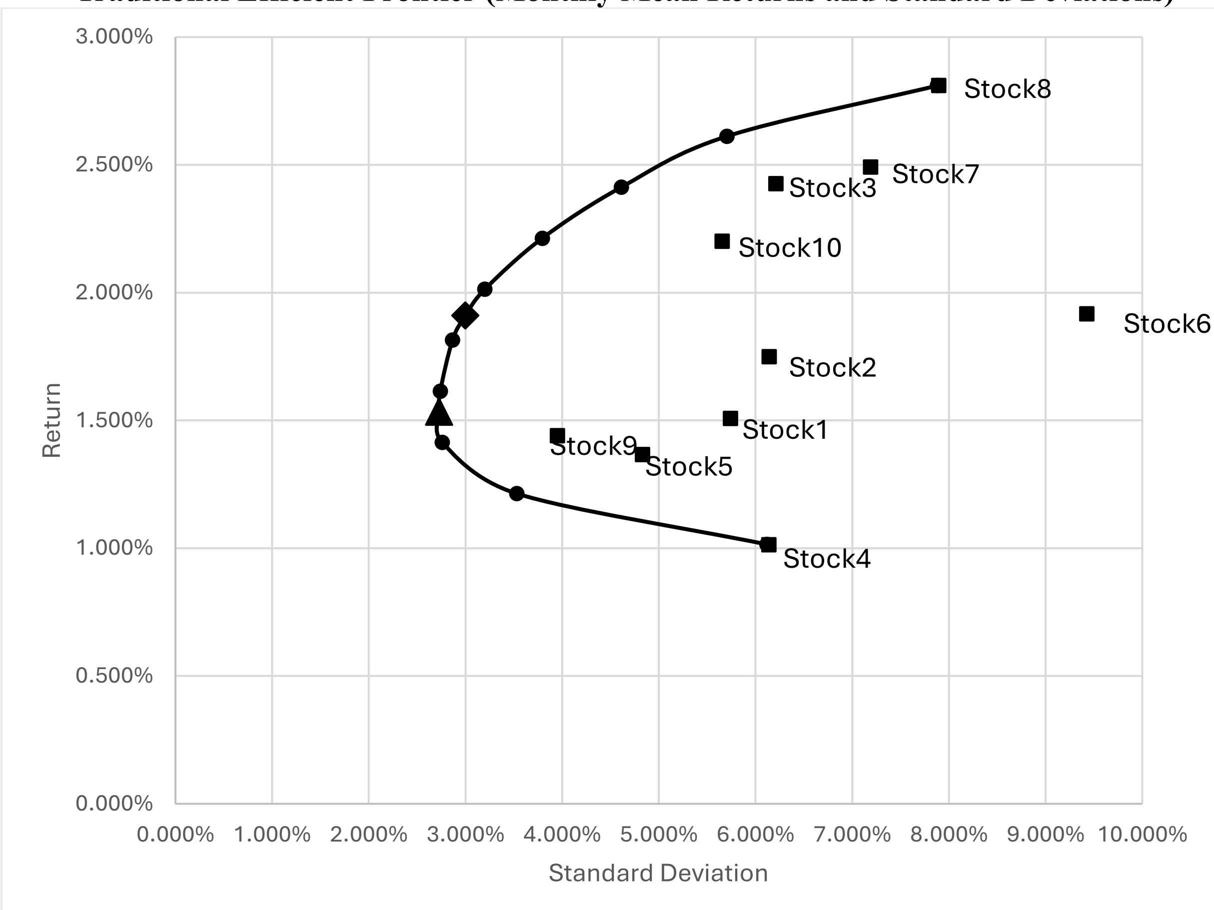
The next step in the assignment requires students to construct 10 portfolios containing different combinations of the 10 selected stocks. The 10 portfolios are created as a linear combination of monthly mean returns from the lowest to the highest possible portfolio return. For example, possible portfolio monthly returns ranged from 2.81% (all in stock 8) to 1.015% (all in stock 4) with a difference between each of the 10 portfolios of approximately 0.20%. Next, students used the Solver operation in Excel along with the mean stock returns and the variance-covariance matrix to find optimal weights for each stock in each portfolio that provide the minimum portfolio variance given each portfolio return. Our instructional website provides the specific assignment given to students along with full instructions for using Excel and Solver to find the optimal stock weights for each of the 10 portfolios. Table 4 represents the results of this analysis and the data necessary to then plot the efficient portfolio.

Table 4
Traditional Minimum Variance Weights and Portfolio Statistics

Portfolio	Portfolio weights										Portfolio statistics			
	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10	Sum weights	Std. dev.	Return	Sharpe ratio
1	0.0%	0.0%	0.0%	99.7%	0.0%	0.0%	0.0%	0.0%	0.3%	0.0%	100.0%	6.12%	1.02%	0.146
2	0.0%	0.0%	0.0%	51.6%	8.2%	0.0%	0.0%	0.0%	40.2%	0.0%	100.0%	3.53%	1.21%	0.309
3	1.8%	12.2%	0.0%	14.0%	15.8%	1.3%	0.0%	0.0%	54.8%	0.0%	100.0%	2.76%	1.41%	0.468
4	1.6%	15.7%	0.8%	5.6%	5.4%	2.0%	1.4%	4.1%	55.0%	8.4%	100.0%	2.74%	1.61%	0.544
5	0.0%	13.1%	7.0%	0.0%	0.0%	2.7%	2.3%	8.6%	51.9%	14.5%	100.0%	2.87%	1.81%	0.590
6	0.0%	0.4%	16.3%	0.0%	0.0%	3.7%	5.5%	12.8%	40.3%	21.0%	100.0%	3.20%	2.01%	0.591
7	0.0%	0.0%	23.1%	0.0%	0.0%	2.9%	11.5%	18.4%	23.4%	20.7%	100.0%	3.79%	2.21%	0.551
8	0.0%	0.0%	29.8%	0.0%	0.0%	2.0%	17.6%	24.1%	6.3%	20.2%	100.0%	4.61%	2.41%	0.496
9	0.0%	0.0%	29.9%	0.0%	0.0%	0.0%	22.9%	45.4%	0.0%	1.8%	100.0%	5.71%	2.61%	0.436
10	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.3%	99.7%	0.0%	0.0%	100.0%	7.88%	2.81%	0.341
Min.Var.	2.0%	15.2%	0.0%	8.3%	9.0%	1.9%	0.3%	2.2%	55.3%	5.9%	100.0%	2.73%	1.53%	0.517
Optimal	0.0%	6.9%	11.5%	0.0%	0.0%	3.2%	3.8%	10.6%	46.2%	17.7%	100.0%	3.00%	1.91%	0.596

Given the results in Table 4, students produce the efficient frontier plots shown as Figure 1 and identify the portfolios with the highest Sharpe ratio (large diamond marker) and the minimum risk portfolio (large triangle marker).

Figure 1
Traditional Efficient Frontier (Monthly Mean Returns and Standard Deviations)



Factor Models and Efficient Frontiers

Once students gain confidence with the concepts and data applications required to construct an efficient frontier, the emphasis shifts to estimating expected returns with factor models. The use of factor models to generate expected returns receives a lot of attention in investments courses but the extension to efficient frontiers is missing. Our second assignment addresses this omission. Students use market and factor premium data available in the KFW site to conduct basic regressions using Excel to obtain estimated inputs required to calculate factor model expected returns. Efficient frontiers using factor model expected returns are improvements, to the extent that factor models provide better estimates of expected returns than the mean. This point should be a key motivation for conducting the second exercise.

The first deliverable product from the extension to factor models requires students to download their monthly total rates of return over the past five years along with monthly risk free rates and the three equity market factor premiums (market, size, and value) from the KFW site. With this data students then use regression analysis based on the market model of equation (8) for the CAPM and equation (10) for the F&F model.

Table 5 provides the student output from the regressions to included alphas, betas, and the regression R-squares. There are ten regression outputs for each factor model. The single factor regressions are in Panel A and the F&F three factor regressions are in Panel B of Table 5.

Table 5
Factor Model Regressions over a Five Year Period

Panel A. Single Factor Model Regression Output for Individual Stocks

	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10
R-squared	0.25	0.51	0.42	0.32	0.57	0.12	0.19	0.43	0.01	0.47
Alpha (α_i) (%)	0.712	0.579	1.350	0.076	0.382	1.012	1.626	1.455	1.281	1.165
α_i p-value	0.289	0.318	0.038	0.911	0.372	0.396	0.065	0.075	0.018	0.039
Market Beta ($\beta_{i,M}$)	0.80	1.22	1.12	0.96	1.02	0.92	0.88	1.43	0.09	1.07
$\beta_{i,M}$ p-value	0.000	0.000	0.000	0.000	0.000	0.006	0.000	0.000	0.552	0.000
Mean return (%)	1.508	1.750	2.427	1.014	1.367	1.917%	2.491	2.811	1.440	2.201

Panel B. Three Factor Regression Output for Individual Stocks

	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10
R-squared	0.26	0.76	0.56	0.34	0.58	0.26	0.26	0.48	0.08	0.51
Alpha (α_i) (%)	0.767	0.915	1.014	0.015	0.489	1.002	1.304	1.227	1.084	1.353
α_i p-value	0.269	0.046	0.081	0.983	0.262	0.378	0.134	0.125	0.044	0.016
Market Beta ($\beta_{i,M}$)	0.787	1.265	1.332	1.043	0.975	0.686	0.947	1.409	0.168	0.966
$\beta_{i,M}$ p-value	0.000	0.000	0.000	0.000	0.000	0.035	0.000	0.000	0.261	0.000
Size Beta ($\beta_{i,S}$)	0.079	(0.180)	(1.014)	(0.383)	0.193	1.119	(0.320)	0.106	(0.393)	0.509
$\beta_{i,S}$ p-value	0.786	0.347	0.000	0.197	0.297	0.023	0.382	0.751	0.083	0.032
Value Beta ($\beta_{i,B/M}$)	0.073	1.107	0.093	0.215	0.103	(1.155)	(0.567)	(0.736)	(0.151)	0.007
$\beta_{i,B/M}$ p-value	0.780	0.000	0.671	0.417	0.534	0.009	0.088	0.017	0.455	0.974
Mean return (%)	1.508	1.750	2.427	1.014	1.367	1.917	2.491	2.811	1.440	2.201

Notes: Mean monthly risk free rate is 0.082%, mean monthly equity market risk premium is 0.889%, mean monthly SMB premium is -0.178, and mean monthly HML premium is -0.365%.

A brief review of the output in Table 5 was instructive for a few students, but many knew the basic interpretations. It was helpful to point out a few of the regression results. For example, the low R-squares reflect a low degree of explained variation (high unsystematic risk) for individual stocks. The higher adjusted R-square values for the F&F regressions in Table 5 imply that the F&F model explains more of the variation in returns and offers a better model for prediction. Differences in signs and in statistical significance for estimated betas of factors in the model reveal how differences size and market valuations affect returns.

Table 6 shows the calculated expected returns for each of the ten socks using the CAPM (Panel A) and the F&F (Panel B) models. The expected return calculations require substituting the estimated factor betas along with the long run factor premiums from the DFW site and the risk free rate at the time of estimation into the factor model equations. With this new data for expected returns and the traditional variance-covariance matrix, students use Solver again to find the optimal weights for each stock in each portfolio that minimized the portfolio variance.

Table 6
Expected Returns from CAPM and F&F Models for Selected Stocks

Panel A. Forecasted CAPM Returns for Selected Stocks

	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10
($\beta_{i,M}$) x (Mkt-RF)	0.80	1.22	1.12	0.96	1.02	0.92	0.88	1.43	0.09	1.07
Return Forecast (%)	0.558	0.785	0.728	0.643	0.672	0.623	0.600	0.898	0.170	0.704

Panel B. Forecasted F&F Returns for Selected Stocks

	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10
($\beta_{i,M}$) x (Mkt-RF)	0.787	1.265	1.332	1.043	0.975	0.686	0.947	1.409	0.168	0.966
($\beta_{i,S}$) x SMB	0.079	(0.180)	(1.014)	(0.383)	0.193	1.119	(0.320)	0.106	(0.393)	0.509
($\beta_{i,B/M}$) x HML	0.073	1.107	0.093	0.215	0.103	(1.155)	(0.567)	(0.736)	(0.151)	0.007
Return Forecast (%)	0.589	1.103	0.641	0.665	0.726	0.398	0.390	0.686	0.079	0.764

Note: Current risk free rate is 0.123%, long run equity risk premium is 0.541%, long run SMB risk premium is 0.228%, and long run HML risk premium is 0.304%.

Table 7 contains the estimated optimal stock weights along with portfolio expected returns and standard deviations when the factor model expected returns are used rather than mean returns. Panel A of Table 7 represents estimates from the single factor CAPM and Panel B contains estimates from the three-factor F&F model.

Table 7
Minimum Variance Weights and Portfolio Statistics using Forecasted Returns

Panel A. Optimal Stock Weights and Portfolio Statistics for CAPM Returns

	Portfolio weights										Portfolio statistics			
Portfolio	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10	Sum Weights	Std. dev.	Return	Sharpe ratio
1	0.0%	0.0%	0.0%	0.3%	0.0%	0.0%	0.0%	0.0%	99.7%	0.0%	100.0%	3.935%	0.171%	0.012
2	0.0%	7.7%	0.0%	7.1%	0.0%	0.0%	0.0%	0.0%	85.1%	0.0%	100.0%	3.281%	0.251%	0.039
3	0.5%	13.8%	0.0%	8.5%	2.3%	1.1%	0.3%	0.0%	70.2%	3.3%	100.0%	2.867%	0.332%	0.073
4	2.2%	15.1%	0.0%	8.4%	8.9%	2.0%	0.8%	2.0%	54.9%	5.7%	100.0%	2.726%	0.413%	0.106
5	3.2%	14.6%	3.8%	7.2%	14.0%	2.9%	1.0%	4.9%	40.3%	8.0%	100.0%	2.860%	0.494%	0.130
6	4.3%	13.7%	9.0%	5.7%	19.0%	4.0%	0.6%	7.7%	25.7%	10.2%	100.0%	3.218%	0.575%	0.140
7	5.4%	12.7%	14.3%	4.3%	23.9%	5.2%	0.4%	10.2%	11.0%	12.5%	100.0%	3.734%	0.656%	0.143
8	2.6%	16.3%	17.6%	0.5%	27.5%	4.9%	0.0%	16.7%	0.0%	13.9%	100.0%	4.363%	0.737%	0.141
9	0.0%	37.4%	6.5%	0.0%	4.3%	0.0%	0.0%	43.0%	0.0%	8.9%	100.0%	5.456%	0.818%	0.127
10	0.0%	1.0%	0.0%	0.0%	0.0%	0.0%	0.0%	99.0%	0.0%	0.0%	100.0%	7.840%	0.897%	0.099
Min.Var.	2.0%	15.1%	0.1%	8.5%	8.9%	2.0%	1.3%	1.7%	54.9%	5.5%	100.0%	2.726%	0.412%	0.106
Optimal	5.4%	12.8%	13.8%	4.3%	23.5%	5.1%	0.4%	10.1%	12.2%	12.3%	100.0%	3.689%	0.650%	0.143

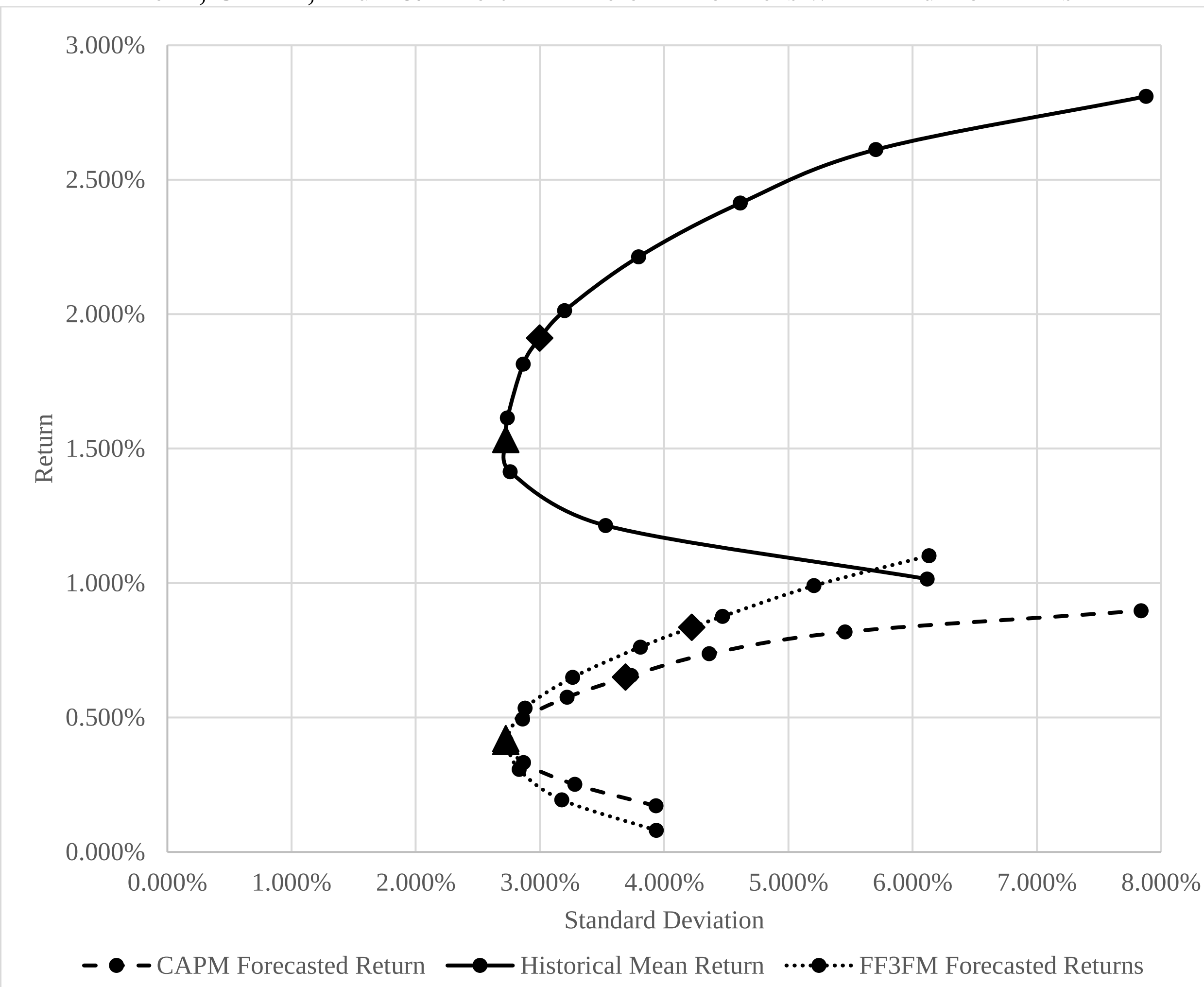
Table 7 (Continued)

Panel B. Optimal Stock Weights and Portfolio Statistics for F&F Returns

	Portfolio weights										Portfolio statistics			
Portfolio	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10	Sum weights	Std. dev.	Return	Sharpe Ratio
1	0.0%	0.0%	0.0%	0.0%	0.0%	0.4%	0.0%	0.0%	99.6%	0.0%	100.0%	3.936%	0.080%	-0.011
2	0.0%	0.0%	0.0%	13.0%	0.0%	4.5%	6.9%	0.6%	75.1%	0.0%	100.0%	3.177%	0.194%	0.022
3	1.4%	5.1%	0.0%	12.1%	5.4%	3.2%	3.8%	1.5%	62.9%	4.7%	100.0%	2.832%	0.307%	0.065
4	2.0%	16.2%	0.0%	8.1%	9.5%	1.8%	0.9%	1.9%	54.1%	5.5%	100.0%	2.727%	0.421%	0.109
5	2.8%	27.3%	0.0%	4.5%	13.3%	0.5%	0.0%	1.5%	44.4%	5.7%	100.0%	2.881%	0.535%	0.143
6	3.9%	38.5%	0.0%	1.0%	17.1%	0.0%	0.0%	0.4%	33.7%	5.3%	100.0%	3.264%	0.649%	0.161
7	4.3%	48.3%	0.0%	0.0%	20.7%	0.0%	0.0%	0.0%	21.9%	4.8%	100.0%	3.808%	0.762%	0.168
8	4.2%	57.6%	0.0%	0.0%	24.3%	0.0%	0.0%	0.0%	9.7%	4.2%	100.0%	4.470%	0.876%	0.168
9	1.6%	70.4%	0.0%	0.0%	25.8%	0.0%	0.0%	0.0%	0.0%	2.2%	100.0%	5.207%	0.990%	0.167
10	0.0%	99.8%	0.0%	0.0%	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%	6.133%	1.102%	0.160
Min.Var.	2.0%	15.1%	0.0%	8.5%	8.9%	2.0%	1.2%	1.8%	54.9%	5.5%	100.0%	2.726%	0.410%	0.105
Optimal	4.3%	54.2%	0.0%	0.0%	23.1%	0.0%	0.0%	0.0%	14.0%	4.4%	100.0%	4.224%	0.835%	0.169

Plots of the data from Table 7 as well as plots for the traditional efficient frontier appear in Figure 2. The minimum risk portfolios (larger diamond marker) and the optimal portfolios (highest Sharpe ratios indicated by larger triangle marker) appear as indicated for each frontier in Figure 2. For the sample of stocks chosen for the assignment, the traditional mean measure of expected portfolio return is higher than for either the CAPM or F&F frontiers. The highest Sharpe ratio on the traditional frontier has a higher expected return and lower standard deviation than the highest Sharpe ratio portfolios on either the CAPM or F&F frontiers. While these results are specific to our selected sample of stocks, Figure 2 illustrates that the model used to generate the portfolio expected returns makes a big difference in defining an efficient frontier. The optimal portfolio based on the Sharpe ratio is different for each frontier, illustrating the sensitivity of the frontier to the expected return choice.

Figure 2
Mean, CAPM, and F&F Return Efficient Frontiers with Traditional Risk



Efficient Frontiers with Downside Risk Measures

Our last student assignment introduces an alternative measure of portfolio risk designed for shortfall constraints and downside risk considerations. Students use a semi-deviation (square root of the semivariance) to measure portfolio risk rather than the standard deviation. Students generate the output in Table 8 by replicating their work for a traditional efficient frontier but with the portfolio semi-deviation rather than the standard deviation. Stock #3 again has the most attractive coefficient of variation (modified by using the semi-deviation as the risk measure) and the highest Sharpe ratio.

Table 8 illustrates the new semivariance matrix for the assignment, based on only downside deviations from the target return. In this exercise students use the mean as the target, but a factor model's expected return or a chosen shortfall return could be the target. The semivariance matrix represents new input replacing the traditional variance-covariance matrix in the analysis.

Table 8
Semi-Deviations and Modified Coefficient of Variation for Selected Stocks

	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10
Mean (%)	1.508	1.750	2.427	1.014	1.367	1.917	2.491	2.811	1.440	2.201
Semi-Deviation (%).	4.105	4.377	4.329	3.807	3.614	6.599	5.118	5.192	2.811	4.027
Coefficient of Var.	2.721	2.52	1.784	3.755	2.644	3.443	2.054	1.847	1.952	1.829
Sharpe ratio	0.347	0.381	0.542	0.245	0.355	0.278	0.471	0.526	0.483	0.526

Note: The monthly risk free rate average was 0.08%. The total number of return observations (n) is used in the calculation of the semi-deviation when calculating portfolio risk, since returns above the target contribute to diversification but not to the sum of deviations. The calculation of the semi-deviation for an individual security uses only the number of observations below the target for the divisor. Calculation of the variance-covariance matrix could use deviations from the expected return of a factor model rather than deviations from the mean.

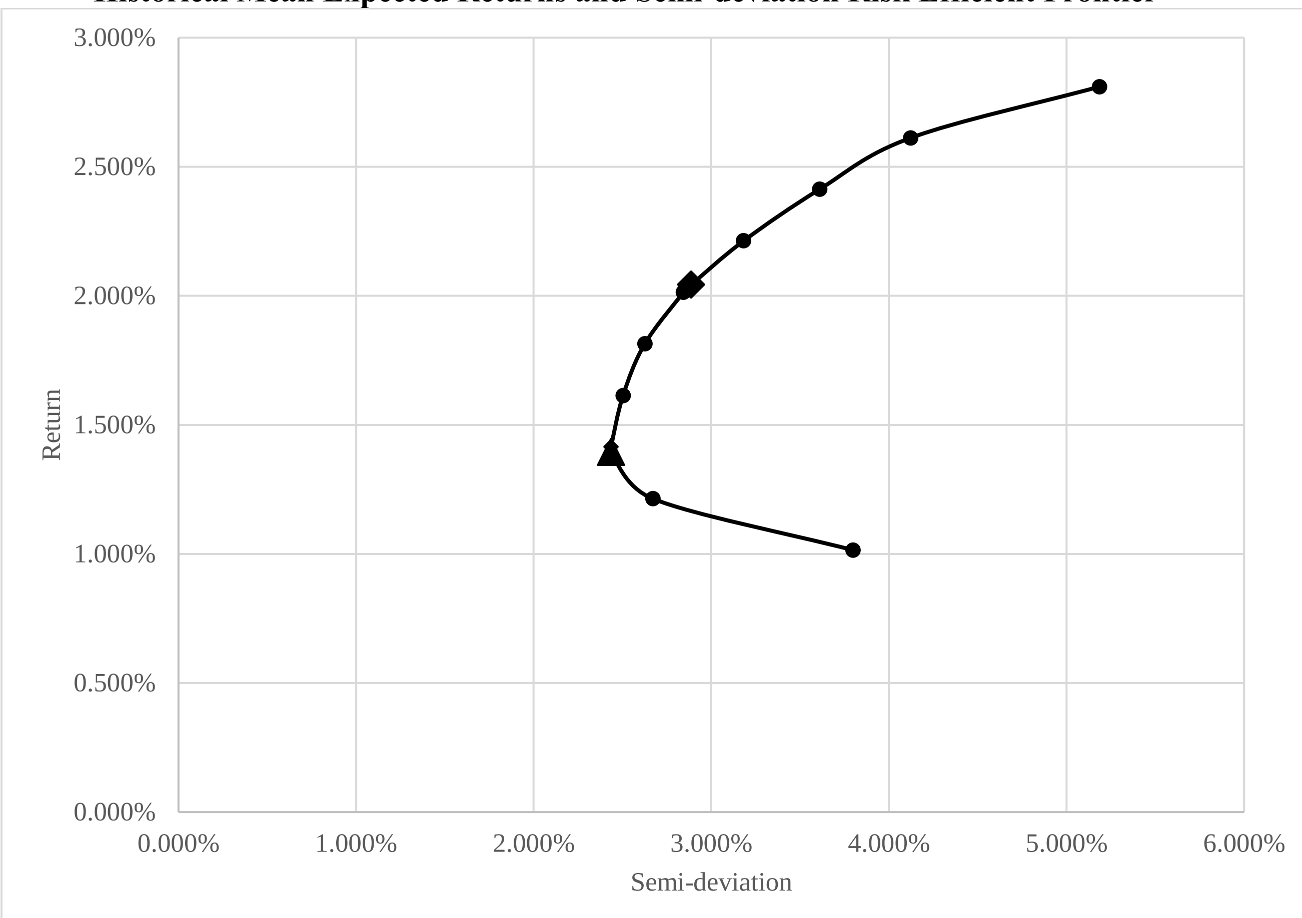
Table 9 provides the optimal weights for each stock in each portfolio using the new semi-variance matrix in the Solver application. This step should now be familiar to students given their work with the first two assignments. Additional background, if needed, is provided in our instructional files. The data from Table 9 provides the information needed to then construct a new efficient frontier based on a shortfall constraint.

Table 9
Minimum Variance Weights and Portfolio Statistics using Mean Expected Returns and Semi-deviation Risks

Portfolio	Portfolio weights: Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10	Portfolio statistics: Sum weights	Std. dev.	Return	Sharpe ratio
1	0.0%	0.0%	0.0%	99.7%	0.0%	0.0%	0.0%	0.0%	0.3%	0.0%	100.0%	3.798%	1.015%	0.235
2	0.0%	0.0%	0.0%	53.0%	0.0%	0.0%	0.0%	0.0%	47.0%	0.0%	100.0%	2.671%	1.214%	0.408
3	6.2%	6.6%	0.0%	11.8%	12.7%	0.0%	0.0%	0.0%	61.5%	1.2%	100.0%	2.436%	1.414%	0.530
4	6.4%	7.5%	8.8%	2.5%	6.0%	0.0%	0.3%	1.0%	60.1%	7.5%	100.0%	2.503%	1.614%	0.596
5	3.6%	0.0%	12.3%	0.0%	0.0%	0.0%	0.4%	5.9%	56.1%	21.6%	100.0%	2.626%	1.814%	0.644
6	0.0%	0.0%	18.1%	0.0%	0.0%	0.0%	5.7%	13.2%	42.9%	20.1%	100.0%	2.844%	2.013%	0.665
7	0.0%	0.0%	24.4%	0.0%	0.0%	0.0%	11.9%	20.5%	26.6%	16.6%	100.0%	3.183%	2.213%	0.657
8	0.0%	0.0%	30.7%	0.0%	0.0%	0.0%	18.0%	27.7%	10.4%	13.2%	100.0%	3.611%	2.413%	0.634
9	0.0%	0.0%	31.9%	0.0%	0.0%	0.0%	23.9%	44.2%	0.0%	0.0%	100.0%	4.123%	2.612%	0.604
10	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.3%	99.7%	0.0%	0.0%	100.0%	5.185%	2.810%	0.518
Min.Var.	5.6%	5.4%	0.0%	13.8%	13.7%	0.0%	0.3%	0.0%	61.2%	0.0%	100.0%	2.436%	1.394%	0.522
Optimal	0.0%	0.0%	19.0%	0.0%	0.0%	0.0%	6.6%	14.3%	40.5%	19.6%	100.0%	2.887%	2.042%	0.665

Students delivered Figure 3 with the new efficient frontier based on the data in Table 9. Again, the optimal portfolio (highest Sharpe ratio with large diamond marker) and minimum semi-deviation portfolio (large triangle marker) appear in the figure. The frontier in Figure 3 is not directly comparable to the other frontiers in Figure 2, since the risk measures (X-axes) are fundamentally different (standard deviation versus semi-deviations).

Figure 3
Historical Mean Expected Returns and Semi-deviation Risk Efficient Frontier



While our outputs are specific to the stocks chosen for the assignment, it is helpful to review differences in stock weights for the optimal portfolios depending on the combination of expected return and risk measures. Table 10 summarizes the optimal portfolio stock weights for each of the constructed efficient frontiers. The least concentrated portfolio occurs with the CAPM expected return and traditional standard deviation measure of risk. Weights vary wildly depending on the frontier construction. For example, stock 2 has a 54.2% weight for the efficient frontier with F&F expected returns and a traditional standard deviation measure of risk but has a 0% weight for a traditional mean return and a semi-deviation measure of risk.

Table 10
Optimal Stock Weights and Portfolio Statistics for Best Sharpe Ratios and Different Efficient Frontiers

Panel A. Optimal Stock Weights

Expected Return	Portfolio Risk	Portfolio weights									
		Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10
Mean Return	Historical Std. Dev.	0.0%	6.9%	11.5%	0.0%	0.0%	3.2%	3.8%	10.6%	46.2%	17.7%
CAPM Return	Historical Std. Dev.	5.4%	12.8%	13.8%	4.3%	23.5%	5.1%	0.4%	10.1%	12.2%	12.3%
F&F Return	Historical Std. Dev.	4.3%	54.2%	0.0%	0.0%	23.1%	0.0%	0.0%	0.0%	14.0%	4.4%
Mean Return	Semi-deviation	0.0%	0.0%	19.0%	0.0%	0.0%	0.0%	6.6%	14.3%	40.5%	19.6%

Panel B. Optimal Portfolio Statistics

Expected Return	Portfolio Risk	Portfolio statistics			
		Sum weights	Std. dev.	Return	Sharpe ratio
Mean Return	Historical Std. Dev.	100.0%	2.999%	1.911%	0.596
CAPM Return	Historical Std. Dev.	100.0%	3.689%	0.650%	0.143
F&F Return	Historical Std. Dev.	100.0%	4.224%	0.835%	0.169
Mean Return	Semi-deviation	100.0%	2.887%	2.042%	0.665

Summary and Student Responses

We used assignments from this paper in a senior-level Securities and Portfolio Analysis Course to integrate portfolio concepts with data analysis. Students completed all relevant required prerequisites in statistics, investments, financial management, economics, and accounting. Proficiency in Excel is required in our business school and lower level finance courses require Excel applications. We chose to use the assignments in this paper as a comprehensive exercise, rather than a step-by-step approach. Each step of the assignment culminates in construction of a different efficient frontier. Another sequence that some instructors might prefer is to first start with estimation of all the different factor models followed by construction of different risk measures and ending in construction of different efficient frontiers. Users of our exercises should open our instructional files to see each of the three assignments complete with instructions and deliverable products.

In any given sequence, students should be challenged to explain their output and link their findings to concepts covered in class. Mastering this assignment signifies an ability to work with the relevant concepts, data, and interpretations. A discussion of the most relevant efficient frontier is a good way to end the assignment. Ultimately, the best frontier is the one that offers the best estimate of the expected return and relevant portfolio risk. Different efficient portfolios occur based on different views of relevant risk and expected returns.

We administered a questionnaire following completion of all assignments. The results of the questionnaire are provided in Table 12. Responses to questions #2, #3, #6, and #8 suggest that students were comfortable executing the mechanical aspects of the exercise, given the supporting material in our instructional files. In almost every case, students had some prior experience in downloading total rates of return for individual stocks and conducting regressions. The key extension was calculation of semi-deviations and using Solver in Excel.

Table 12
Efficient Frontier Student Survey Summary
Please answer the survey questions using a scale of 1 (strongly disagree) to 5 (strongly agree.)

Questionnaire	Likert Scale	% with a 4 or 5
1. The exercise helped me integrate what I learned about asset pricing models, portfolio risk measures, and efficient portfolios.	*3.96*	*86.6%*
2. The relevant data for the exercise came in user-friendly formats.	*4.49*	*100%*
3. The instructions were clear and easy to follow.	*4.43*	*85.7%*
4. The exercise improved my ability to use multifactor asset pricing models to get expected returns.	*4.1*	*81.6%*
5. The exercise helped me understand alternatives to the simple standard deviation as a measure of portfolio risk.	*3.86*	*79.6%*
6. The exercise helped me appreciate the complexities of constructing the relevant efficient frontier for a portfolio manager.	*4.27*	*91.9%*
7. Overall, the exercise was too complicated and confusing.	*2.02*	*5%*
8. The exercise requires Excel skills beyond what can be expected of undergraduate seniors.	*1.56*	*5%*
9. I would recommend using the exercise again for the Portfolio Management class.	*3.93*	*91.8%*

Finally, we were somewhat surprised that students had not already made connections between asset pricing models, risk measures, and efficient frontiers in prior courses. When addressed separately on exams, students demonstrated a good understanding of each concept. An integration of the concepts in a portfolio management context was new and more challenging. We are looking at our foundation courses to build more integration of concepts and applications. Our goal is to achieve proficiency in applying investment concepts to portfolio management rather than assess differences in skills. For this reason, we do not place a high percentage of the course grade for the project. Rather, we make successful completion of each assignment a proficiency requirement that must be repeated until the final work is correct. Our formative assessments are based on successful completion of the assignment and by how much additional support a student requires to reach completion. Other approaches are possible depending on the instructor's goals and objectives. In Appendix I we outline our specific objectives in each assignment along with our assessments of learning. Overall, we are pleased with the outcomes from our use of the project as a capstone in a portfolio management course.

Our assignments in this paper can easily be extended to additional asset pricing models, inputs of subjective or forward looking returns, and different shortfall constraints. We are planning several extensions of this project to include additional assessments of learning. One example is a pre-course exercise asking students to provide a step-by-step explanation of how they would construct efficient frontiers. A post-course repetition of the same exercise reveals the extent of learning how asset pricing models and risk measures lead to different frontiers. The pre-course questionnaire would also reveal the extent to which students have been able to integrate risk and return relationships learned in prior courses.

Finally, several research ideas are linked to our paper. The best efficient frontiers require the best estimates for expected returns. Time series models allowing updates, such as one-step-ahead forecasts, may offer better predictions than asset pricing models using ex-post data over long periods of changing market conditions. Surveys of portfolio managers with respect to inputs used in practice for constructing efficient portfolios would also provide a contribution to our understanding of active portfolio management relative to efficient portfolio construction from factor models. These contributions would allow significant improvements in our understanding of efficient portfolio construction.

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Appendix I. Learning Objectives and Assessment for the Factor Models, Shortfall Constraints and Efficient Frontiers

Learning Objectives	Topics	Formative Assessment*	Summative Assessment*
1.) Students apply basic risk, return and efficient frontier concepts learned in investment class.	Mean Standard deviation, Correlations, Diversification, Markowitz efficiency	Students download data on selected stocks and equity market returns to construct a basic efficient frontier from mean-variance measures of risk and return. Students turn in outputs of their estimations and a graph of their efficient frontier. (Assignment 1)	Students must continue working on the assignment until they have passed a proficiency requirement.
2.) Students apply and integrate knowledge of single and 3-factor asset pricing models by constructing new efficient frontiers using factor models.	All prior topics Capital Asset Pricing Market Model Three Factor Model	Students extend their first assignment to now include different expected returns using factor models. Students turn in outputs of the estimation and construction of new efficient frontiers to include a graph based on the new expected return estimates. (Assignment 2)	Students must continue working on the assignment until they have passed a proficiency requirement.
3.) Students compare and contrast three different efficient frontiers based on different expected return models.	No new topics.	Class discussion of the student outputs from Assignments 1 and 2.	
4.) Students compare and contrast efficient portfolios based on different expected return measures and a shortfall measure of risk	Target return Semi-variance Semi-std. deviation	Students first construct semi-standard deviations for selected stocks and a semi-standard deviation covariance matrix. Students then construct new efficient frontiers using the semi-standard deviation measure of risk and three different expected return measures. Students turn in the outputs of the different efficient portfolios and a graph using the three different expected return measures. (Assignment 3).	Students must continue working on the assignment until they have passed a proficiency requirement.
5.) Students explain why and how there are different efficient frontiers for different investors.			Final exam question on steps in constructing an efficient frontier.

******Assessments measure how well a student learned what we wanted them to learn. Formative assessments evaluate student learning at intermediate points, while summative assessment occurs at the end of the course or module to sum up student learning from the overall project.*

A Framework for the Integration of CFA and CFP Exam Preparation into an Undergraduate or MBA Course of Study

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In this paper, we first present a structured literature review of the extant peer-reviewed, academic research on the Chartered Financial Analyst (CFA) or Certified Financial Planner™ (CFP®) designations. This review motivates the potential benefits for students to work towards one or both designations. We then propose a framework and curriculum for integrating a CFA or CFP® program into an undergraduate or MBA curriculum. We use as our case study a large, private university in the Western United States. Using 16 years of data on CFA enrollment and pass rates, the students who completed the proposed curriculum earned an average 65% pass rate on the CFA Level I examination compared to a national average over the same years of 41%, a statistically significant difference beyond the one percent level. We also present enrollment and depletion data for the CFP® capstone class for its first four semesters.

Keywords: certification, certified financial analyst (CFA), certified financial planner (CFP), financial services, and business curriculum innovation

Introduction

Academic research on the Chartered Financial Analyst (CFA) and the Certified Financial Planner (CFP) programs spans from Hamilton and Marshall (1987) through Grieb, Noguera, and Trejo-Pech (2021). In this paper, we extend this thread of literature by presenting a framework for providing a CFA prep program and a registered CFP program at the university level without a heavy professor burden (i.e., typically one professor can run a program).

Earning a professional designation can set a candidate apart, especially as they leave their undergraduate or graduate training. Often students ask which would be the best designation for them to pursue. In one of the author's syllabi, the following section addresses this question.

> We have two prep classes – one for the CFA charter and the other for the CFP® certification. I have taught the CFA prep class for five years and now Dr. Holmes, who originally developed it in 2010, teaches it. Dr. Holmes is an expert at the CFA

> material and is a CFA charter holder. I hold both the CFA charter and CFP® designation, so am familiar with both. Some students ask which exam is best to take if there is only bandwidth to take one (which is most likely the case). It completely depends on what career path you are targeting. If you want to be an investment banker, hedge/mutual fund manager, or do financial engineering aspects of private equity, such as designing optimized covariance portfolios or financial derivative structures for clients, the CFA would probably be best. The CFA program is a series of three exams. You can pass one as an undergraduate and the other two after graduating. Passing the CFA Level 1 is a gigantic signal to employers.
>
> If you are more interested in assisting families, individuals, and businesses in designing their financial plan as a professional wealth advisor, the CFP® would most likely be better than the CFA. As you can see in the topics below, the CFP® curriculum covers material not covered in the finance junior core, such as trusts, estate planning, education planning, and insurance planning, among others. The CFA on the other hand, is highly correlated with our junior core (but more advanced and comprehensive).

The example university explicitly designs its finance core curriculum so it correlates highly with the CFA body of knowledge. Just one professor heads both the CFA prep program and the CFP prep program. The CFA prep program consists of one capstone class after the finance junior core. The CFP curriculum is built around five fundamental accounting and finance classes that are part of the finance major, with the addition of a personal finance course and the capstone financial analysis class.

In the remainder of this paper, we provide a review of the existing literature to motivate the potential benefits of a CFP or CFA program at the university level. We then discuss in detail the curriculum framework for the case study university. We finish by discussing the results over a 10-year period for the CFA course and the continued development of the CFP program.

Literature Review and Development

CFA Certification and Exams

The CFA charter is a designation issued by the CFA Institute, which can serve as the educational capstone of a finance student intending to enter the world of investment banking, hedge fund management, and other technical finance fields. The CFA charter is seen as "one of the most prestigious and respected designations in the financial service industry" (Chen and Chen, 2005). The CFA Institute identifies its charter as useful for individuals entering the fields of portfolio management, commercial and investment banking, risk management, wealth management, and others (CFA Institute, 2022b). The typical base salary for a portfolio manager with a CFA charter is $126,000, with an additional $50,000 in bonuses and other compensation (CFA Institute, 2022).

The primary requirements to earn a CFA charter are the three CFA exams, which are referred to simply as Level I, Level II, and Level III (CFA Institute, 2022a), and 4,000 hours of work experience. The pass rate for each exam ranges from 42% (Level I) to 54% (Level III), but since each exam requires a passing score on the previous exam, each exam level causes attrition of approximately 50% of the potential CFA candidates. Levels I and II are multiple-choice only, while

Level III is a mix of multiple-choice and essay questions. The levels are designed to measure the aspiring CFA professional's knowledge of the CFA Candidate Body of Knowledge (CBOK), a detailed description of which is given by Broome (1980), although we note that the CBOK has been regularly updated since Broome's description. A less detailed but more up-to-date version of the CBOK may be accessed on the CFA Institute's website.

CFP Certification and Exams

The requirements for the CFP designation are less focused on exams and more focused on gaining exposure and experience in the world of financial planning. "Many studies have shown that the average American is not competent to make even simple financial decisions" (Warschauer, 2002), and so the role of a financial planner is to assist individuals in making these important decisions. A survey conducted by Bae and Sandager (1997) found that 92% of 227 individuals surveyed preferred a financial planner with a CFP certification to one who did not, suggesting that the average individual recognizes that CFP certification is associated with greater subject matter knowledge and professionalism.

Preparation to become a CFP begins in the classroom, where one must earn a bachelor’s degree and complete coursework in a CFP Board-registered program (CFP Board, 2022). Next, the individual takes a two-part, multiple-choice exam totaling six hours. As well as passing the exam, the individual must complete 6,000 hours of professional experience either before the exam or during the five years after the exam, unless the individual’s work qualifies as “apprenticeship experience,” in which case the requirement is decreased to 4,000 hours. After completing the work experience and several ethics requirements including a background check, the individual is awarded the CFP certification.

Differences between CFA and CFP

The similarities and differences between the CFA and CFP credentials have been explored in depth by Terry and Vibhakar (2006). Since neither certification is required of individuals to be hired in most finance-related positions (in contrast, for example, to accounting's CPA designation), a budding professional would choose to pursue one or both certifications to demonstrate knowledge and competency above and beyond what is required to enter the profession. However, there are several key differences between the CFA and CFP certifications.

CFA/CFP exam pass rates can vary from year to year and across different levels of the exam. Keep in mind that pass rates are influenced by various factors, including the overall preparedness of candidates, the curriculum provided by the educational institution, and the rigor of the CFA/CFP exam itself. It is essential to consider these factors when interpreting pass rate data (CFA Institute, 2023). It is typical for business schools to take pride in their programs having received high distinction for a higher stage pass rate. With the CFP exam, however, pass rates more consistently seem to hover just over of half exam test takers (CFP Board, 2023).

The CFP exam is based on the entire learning process of becoming a financial planner, including approved education requirements, an exam, and work experience requirements. The CFA exam, on the other hand, does not require a specific curriculum of study before sitting for the exam. The CFP certification is generally considered to be a “broad” certification, covering many topics in financial planning (Terry and Vibhakar, 2006) and enabling the CFP certificant to give "comprehensive expert advice" to individuals in all stages of their financial lives (Smith, Vibhaker, and Terry, 2008). Topics covered on the CFP exam include general financial planning principles, investment planning, retirement savings and income planning, risk management and insurance planning, tax planning, estate planning, professional conduct and regulation, and psychology of financial planning. The CFA charter, in contrast, is much more specialized, covering the topics of ethics and professional standards, quantitative methods, economics, financial statement analysis, corporate finance, equity investments, fixed income, derivatives, alternative investments, and portfolio management and wealth planning (Larson et al., 2006; CFA Institute, 2022c).

Whereas the CFP exam involves some applied mathematics, such as computing the time value of money, the CFA exam is more computationally heavy. Due to the greater depth and more involved computations, the vast majority of individuals surveyed by Terry and Vibhakar found the exam portions of the CFA charter more difficult than those of the CFP certification (Terry and Vibhakar, 2006; Moy, 2011). One of the authors, who passed the CFA exams and the CFP exam after earning a finance Ph.D. and teaching for several years, found the opposite. His training and teaching experience had prepared him for all three levels of the CFA exam and virtually not at all for the CFP exam.

At many schools, Finance faculty have broad discretion to select textbooks that include publisher-enhanced materials that integrate CFA/CFP exam prep content. Some of the most well-known companies (such as Kaplan, Princeton Review, and Wiley for example) invest heavily to this end. A dedicated army of sales representatives and robust technical support teams are common in this lucrative industry. An annual update for expected content revision is required for changes in professional standards, rules, and laws. In addition, more seasoned faculty with previous involvement with CFA/CFP exam prep are considered mentors in their programs integrating prep content into their individual course content through assignments, quizzes, and projects. More junior faculty without certifications will often begin their implementation of a course with the syllabus of a more seasoned faculty member with prior exam prep experience.

Students at many universities historically self-report specific areas of disconnect between course prep and post-test outcome performance. For example, (Terry & Vibahakar, 2006) found significant differences in CFA and CFP post-test performance concerning candidates who reported an average number of hours studying to complete, the number of months to complete exams, and the perceived level of difficulty ("very difficult" vs "somewhat difficult") in a survey. Candidates report on average that both CFA and CFP prep programs failed to prepare them in niche areas (such as advisory, nonprofit, and small business consulting among others). Whereas both faculty and students indicated that having an instructor with the credential can better assist their preparation, a relatively small number of department faculty typically hold the designation. The information environment in the CFA/CFP prep program improves when the instructor knows from personal experience via a certification.

Both the CFA and CFP credential allow candidates to display a professional credential and gain professional association status. There is a unique incentive for all college faculty, Wall Street practitioners, financial service consultants, veterans, exchange students, and graduate students to pursue success on the test. The CFA and CFP pools of candidates are large and diverse (Bracker & Shum, 2011, Chan, Shum, & Thapa, 2016, Gray, 2023, Moy, 2011). According to these studies, some programs require refined curriculum and development; others have underserved demographics (also see Beverly, 2019). Additional research describes barriers to race and gender performance among prospective professionals (Beverly, 2019). In addition, most faculty and students report that their program candidates are better informed when the instructor holds both credentials, which is not often the case (Chan, Shum, & Thapa, 2016).

Benefits of CFA/CFP Designations

Arman and Shackman (2012) find that the CFP designation is associated with higher pay than non-CFP individuals, especially in roles where compensation is largely tied to performance. This fact suggests that employers have greater confidence in the abilities of CFP-holding individuals and/or that individuals who are CFPs perform better than those who are not. Note that this is not the case for all certifications: individuals with ChFC (Chartered Financial Consultant) and PFS (Personal Financial Specialist) certifications are on average no better off than their peers. The cited study did not examine the effects of the CFA designation (Arman and Shackman, 2012).

Because CFA-holding individuals often work with publicly traded securities and give public recommendations, as opposed to CFP individuals who tend to offer private recommendations to individuals, families, and small businesses, it is easier to analyze the performance of CFA individuals' recommendations than those of CFP professionals. Hanna and Lindamood (2010) conclude that "valid empirical estimates of the value of financial planning advice are not practical." However, the same is not true for the financial impact made by CFAs. Kang, Li, and Su (2018) who analyzed the performance of recommendations made by sell-side analysts with and without CFA certifications conducted one such study on CFA impact. They found that sell-side analysts with CFA certifications gave better-performing recommendations than their non-CFA peers by 4.7% per year in abnormal returns and 0.058 in the information ratio (Kang, Li, and Su, 2018). De Franco and Zhou (2009) find that CFAs give timelier forecasts than those without a charter, but the difference in the accuracy of those forecasts was statistically insignificant. Shukla and Singh (1994) also found positive benefits of the CFA designation in managed equity fund performance. Brockman and Brooks (1998) employ a Granger causality model and conclude that the rise in the number of individuals with the CFA designation (beginning in 1963) was a contributing factor to the growth of the S&P 500 in that period.

Enablers of Successful CFA/CFP Programs

Archuleta et al. (2019) interviewed individuals at different stages of acquiring the CFP certification and found that individuals who received greater support and encouragement from their employers were more motivated to acquire the CFP designation. One individual who had expressed interest in the CFP designation but had not yet begun the process said, "I'd say that if I were working with a firm who also viewed the CFP as the gold standard, I'd probably be a little more motivated" (Archuleta et al., 2019, p. 329). Another individual, who had received the CFP certification, also included employer encouragement as a reason for why they chose to pursue the designation: "It was the expectation that I would study, sit, and pass the CFP before I would be considered for promotion" (p. 328). As shown, an individual who received the CFP certification gave employer expectations as a reason for why they underwent the effort to become certified, and another individual who had not received the CFP certification stated that they would be more interested in doing so if their employer valued the certification more.

Archuleta et al. (2019) also found that "Faculty support of certification was another influence for pursuing CFP certification. Most interviewees shared that faculty promoted the CFP mark as the 'gold standard' in the profession" (p. 331). However, Archuleta et al. (2019) note that the incentives for business school faculty members often lie in increasing attendance in the professors' programs, not necessarily in increasing the number of students who go on to receive a certification in the field. Therefore, we suggest that an effective CFA/CFP program should put some focus on motivating the program's professors to promote designations like the CFA and CFP.

When Charlton and Johnson (1999) examined the coverage of finance curricula compared to the CFA Candidate Body of Knowledge (CBOK) in 1999, they found significant shortfalls in several areas; especially in ethics (see Table 4). Fortunately, 18 years later in 2017, Grieb, Noguera, and Trejo-Pech (2022) found that these shortfalls had been improved, at least in programs associated with the CFA Institute's University Recognition Program. Thus, we postulate that the passage of time contributes positively to the efficacy of CFA programs, as long as the program receives progress indicators such as CFA test score reports of students who graduated from the program.

A more recent paper by the same authors (Grieb, Noguera, Trejo-Pech, 2022), finds other benefits reaped by CFA programs when those programs go through the process of becoming a formal program of the CFA institute. Those benefits include staying current on industry issues, which helps ameliorate concerns from both inside and outside academia that the academic curriculum has drifted away from what is necessary in the workplace (Clinebell, 2002), which has in the past proven to be a valid concern (Hamilton and Marshall, 1987). As Muhtaseb (2009) states, "The finance field quickly advances over time. The CFA curriculum provides an opportunity to learn about major developments in the field from a practitioner's point of view." In a poll conducted by Bauman (1975), it was found that "Over two-thirds of the candidates reported that the program helped them significantly to improve their professional competency." Other benefits found by Grieb et al. (2022) include study and CFA exam preparation materials for students, such as free sample exams, as well as feedback mechanisms such as an analysis of pass rates of graduates from the program (Grieb, Noguera, and Trejo-Pech, 2022).

Obstacles to Successful CFA/CFP Programs

It is difficult to recruit business students into a pre-CFA or pre-CFP program if they have never heard of the designation before, and unfortunately, this appears to be the case for many students. Out of 16 individuals that Archuleta et al. (2019) interviewed who had chosen to major in financial planning, only two of them (12.5%) had heard of the CFP certification before declaring their major, and of the two that had heard about it, one learned about it from their own research, and one learned about it from their father. It is telling that not even one of these 16 individuals heard about the CFP certification from professors in their undergraduate business/finance courses or career guidance counselors. One use of our paper is to post it for students so they can become familiar with both designations.

When Chen and Chen (2005) surveyed Taiwanese finance students about their attitudes towards the CFA certification, the recommendation that they made before all others was, "The AIMR [now CFA Institute] should lower its CFA exam fees" (p. 87). For further consideration, the amount presented to the students in the Chen and Chen study for the total exam costs was $550, which, after adjusting for inflation, only covers the cost of one of the three required exams today, which are now $900 at the cheapest reservation date (CFA Institute, 2022a). Even multiplying that number by three is conservative: it assumes one passes each exam the first time, which is a daunting proposition for all but the most gifted students. This sum does not include the cost of preparation classes or study materials that are seen as nearly mandatory, even for the less-intensive CFP test (Archuleta et al., 2019). Thus, the number that Chen and Chen (2005) gave to students estimating the exam cost was extremely conservative, and even that number, students concluded, was too high.

One might make the argument that high exam fees are justifiable because they keep the number of exam takers down, reducing crowding and maintaining the CFA charter's rarity. However, this is a difficult position to justify ethically, as the high fees, particularly impact newly graduated students and individuals who do not have employers willing to cover the exam fees, the individuals most in need of a CFA certification to assist them in entering the labor market.

Hussain (2020) asserts that the CFA Charter does not create a monopoly in the field of financial analysis, because a CFA charter is not required to practice, and such a restriction has never been the intention of the CFA Institute. Finally, individuals who possess the CFP designation often lean more toward tradition and peer consensus than academically published evidence when making recommendations to their clients (Buie and Yeske, 2011). Perhaps with additional university professors offering CFP and CFA prep classes, these students will be exposed to more academic reports and articles and learn to incorporate such analyses in their professions.

Examples of Pedagogy and Curriculum

At the case study university, the syllabus states under the assessment section that if the student does not report her or his score to our CFA program administrator then they receive minus 100 percent (i.e., fail the course). Before incorporating this policy, many students would not report their scores when they became available in the summer because several months had passed since the class had ended. With this policy, we can collect accurate records (as will be seen in Table 1).

Another unique part of the curriculum is that if a student passes the actual exam (CFA or CFP), then a reimbursement scholarship from our department's financial service institute will be issued to cover the entire expense of taking the exam. All students do to claim the scholarship is to email a screenshot of their CFA pass and our administrator cuts them a check. As discussed above by Chen and Chen (2005), the fees for the CFA can be a major hindrance to students. Currently, the Level I CFA enrollment fee is $450, the early registration fee is $700, and the standard registration fee is $1,000. Our institute will reimburse up to the standard registration fee of $1,450 if a student passes the actual CFA exam, as long as she or he registered for the exam while they were still a registered student at the university. A similar scholarship is offered for the CFP students, who face an examination fee of between $825 and $1,025 depending on how early they register. The scholarships are sustained through our financial institute.

The curriculum of both capstone courses is designed as a self-study class. However, a detailed schedule of required quizzes and exams, with specific cut-offs requires students to stay on schedule. Thus, students self-study, but they are paced by the professor. When we explain the format to our students, we explain our role as the professor is more like a personal trainer who makes sure the client stays on schedule, but it is up to the client to work hard.

Finally, the curricula highlight a section titled "Really Important Topics" which is derived from topics that are commonly found either on the actual CFA/CFP exam or that we have seen patterns in our students struggling in these areas over the years.

Outcomes of the Prep Programs

CFA Outcomes

At our case study university, the first program started in 2007 when Dr. Andrew Holmes began a CFA Level 1 prep course as an advanced finance elective. The prep course follows the Kaplan Schweser preparation program and the results for a decade of tests can be seen in Table 1. The prep course is typically taken after the prerequisite and quantitative prep classes and what is known as the finance junior core. The three prerequisites for the finance major are Principles of Economics, Principles of Accounting, and Principles of Finance. The quantitative prep classes include Principles of Statistics, Finite Mathematics, Spreadsheet Skills and Business Analysis, and Business Calculus. The finance core consists of Principles of Accounting II, Professional Development for Finance, Advanced Financial Management, Investments, Money, Banking & Business, and Financial Derivatives. For the two finance electives required to graduate, students in the CFA track are encouraged to take Fixed Income Analysis and the CFA Preparation class. Comparing this curriculum to the CFA Level I curriculum listed in an earlier section shows a close correlation in topics. This correlation is by design and is an integral part of the CFA Prep program. The reason we recommend the fixed income class is that this topic is typically not stressed sufficiently in a typical finance curriculum to prepare students for the CFA section on fixed income.

Table 1 reports the quantitative outcomes of the CFA prep's decade of operation. The first column reports the professor who taught that year's section. The performance of the students when parsed by a professor is not statistically significant. All three professors have used the same prep package through Schweser, so this is a three-professor sample that suggests the preparation success is not professor-specific. Unlike the CFP, any major can take the CFA without a specific curriculum. As a result, over the years we have had students majoring in finance, accounting, economics, strategy, engineering, and MBA take the course and then sit for the actual exam. Finance majors overwhelmingly perform better than any other major, including MBAs. The finance major curriculum listed above is more technical than the MBA finance track curriculum and thus the finance undergraduates are better prepared to enter the CFA capstone class. The other majors listed above all significantly underperform finance majors as they have not taken the finance junior core.

The school pass rates range from 37% to 100% with the national pass rates ranging from 35% to 46%. Annual differences between school pass rates and national pass rates range from 2% (2008) to 57% (2018) with mean (median) university pass rates of 65% (64%) and national mean (median) pass rates of 41% (42%). The university prep program outperforms the national pass rate every year. We test for statistical significance by first conducting a two-sample F-test for variances. The school sample and national sample have unequal variances with a p-value less than 0.0001. Next, we conduct a two-tailed t-test with unequal variances and reject the null of equal means with a p-value less than 0.0001. On average, our program outperforms the national average by 25% and the national median by 22%. If we included only finance undergraduates who took the CFA prep class and the CFA exam, the difference is nearly double. The statistical and nominal difference results provide evidence that the school test-takers significantly outperform the national test-takers.

Table 1
CFA Level I Exam Enrollment Statistics and Pass Rates

Instructor	Year of Prep Class	Number of Students	School Pass Rate	National Pass Rate	School minus National
Holmes	2007	17	67%	40%	27%
Holmes	2008	15	37%	35%	2%
Holmes	2009	35	60%	46%	14%
Holmes	2010	42	60%	42%	18%
Holmes	2011	55	68%	39%	29%
Holmes	2012	38	56%	38%	18%
Brau	2013	27	57%	38%	19%
Mitton	2014	16	56%	42%	14%
Holmes	2015	11	64%	42%	22%
Brau	2016	18	78%	43%	35%
Brau	2017	33	69%	43%	26%
Brau	2018	9	100%	43%	57%
Brau	2019	15	80%	41%	39%
Brau	2020	10	Covid - no exam		N/A
Holmes	2021	11	Covid 5 test dates		N/A
Holmes	2022	15	No June test		N/A
Mean		23	65%	41%	25%
Median		17	64%	42%	22%
2-sample unequal variance from F-test p-value					0.000002
P-value t-test difference in means					0.000064

CFP Outcomes

The second part of the paper discusses the CFP program, which began in January 2021 at the university when Dr. Jim Brau obtained registered status for the CFP curriculum through the CFP Board. Unlike the CFA, which can be taken without completing a registered educational program, the CFP requires a CFP Board-approved registered program. The university officially became CFP-registered in December 2020, with the first CFP capstone course being offered in Winter semester 2021. The program consists of seven classes: Principles of Finance; Principles of Accounting; Principles of Accounting II; Investments; Money, Banking, and Business; Personal Finance; and Financial Planning. The CFP program is offered as a track in the finance major at the university and the sequence is 21 semester credit hours, three greater than the CFP minimum of 18 semester credits. The CFP capstone course follows the Wiley CFP prep program.

Table 2
CFP Class Enrollments

Winter 2021	Day before Drop	Day after Drop	Finished
Finance	10	9	9
Accounting	10	9	9
Strategic Management	6	4	4
Business Management	3	3	3
Human Resource Management	3	3	3
Economics	1	1	1
Entrepreneurial Management	1	0	0
Exercise Science	1	1	1
Global Supply Chain Management	1	1	1
Pre-Business	0	1	0
Non-degree seeking	0	1	1
Total	36	33	32
Fall 2021	**1st Day**	**Day after Drop**	**Finished**
Finance	16	13	12
Business Management	6	2	0
Strategic Management	4	1	1
Entrepreneurial Management	3	2	1
Global Supply Chain Management	2	0	0
Human Resource Management	2	0	0
Accounting	1	1	1
Professional Accounting	1	0	0
Total	35	19	15
Winter 2022			
Finance	1	1	1
Fall 2022			
Finance	10	10	Ongoing

Table 2 provides the class enrollments for the CFP prep class since its advent. The first semester it was offered, it was still listed in the university bulletin as a general business elective. Many students thought it would be the same class that had been offered the prior year before the course was reinvented into the CFP course. Most of these students needed the course to graduate. Originally there were over 100 students registered on the first day of class when the prior course structure was expected – personal finance, a less rigorous course. All but 36 students transferred to other electives after the instructor sent out the syllabus and several emails explaining the new course. Of the 36, three additional students transferred out on the last day of the drop-add period. All these students passed the course, but only one of the finance students has registered to take the actual CFP exam.

For the Fall 2021 semester, the course was still available as an elective to all business majors. The first day of class showed 35 students registered, most non-finance majors. After the syllabus was distributed and several emails sent to the class, 19 students remained to complete the course, 13 of whom were finance majors. Two of these finance majors have registered for the actual CFP exam.

The course was not offered in the Winter 2022 semester due to instructor scheduling, but one student requested to take the course as an independent study so he could sit for the CFP exam. This student completed the course and is registered for a future CFP exam, bringing the total to four students who have registered or plan to register for a future CFP exam. The final semester in the table, Fall 2022, is the current semester. Again, the course was originally scheduled only for Winter 2023 for this academic year, but 10 students requested to take it as an individual study because they graduate at the end of this semester. All 10 plan to register for the CFP. We acknowledge a lack of time series data and small sample sizes as a limitation of the study; however, we feel it important to share these programs now so other universities can start their own programs. Future research such as long-term surveys of students vis-à-vis career impact is currently underway.

Going forward, the course is only offered to Finance Majors in the Private Wealth and Financial Planning track who plan to sit for the CFP exam. The course is formally scheduled for the Winter 2023 and Winter 2024 semesters, and student interest anecdotally seems high. None of the other majors has a track at this point for their students to sit for the exam, but some, such as the accounting major, are considering it. If other majors obtain CFP Board Registered status, their students will be eligible for the CFP capstone class.

Summary and Conclusions

We have provided a detailed literature review that includes every peer-reviewed article we could locate that discussed the CFA and CFP in a university setting. The literature review motivates the possible positive net present value of hosting a CFA and/or CFP program for university students. After covering the extant literature, we provided examples of syllabi for the two capstone prep courses and discussed the class curricula for both designations. We concluded with descriptive data for both the CFA and CFP programs at the case study university.

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Black-Scholes-Merton Option Pricing Sensitivity Analysis: A Dynamic Excel Approach

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This paper presents a unique approach using Excel to teach the sensitivity of European Call and European Put option prices, to changes in the values of the variables employed in the Black-Scholes-Merton option-pricing model equations. This method shows students the direction of the Call and Put price changes, the magnitude of these changes, and introduces the concept and application of Implied Volatility. This approach uses a feature in Excel called "Spinners" to dynamically adjust each of the variables in the Black-Scholes-Merton European Call price and European Put price equations. This eliminates the need for tedious, difficult, and time-consuming calculations and enables the professor to demonstrate the impact of changing the values of the equations' variables individually on the Call and Put prices. In addition, being an excel based model makes it more compatible with presentation tools while eliminating any internet requirement. This paper is the first to present an Excel based model that facilitates the instructors in teaching applications of the Black-Scholes-Merton option-pricing model in their classrooms.

Keywords: Option pricing, Black-Scholes-Merton, Excel, Teaching, Learning, Finance

Introduction

This paper presents a unique Excel approach to facilitate teaching and understanding the Black-Sholes-Merton model for option pricing sensitivity. The authors understand that online applications (apps) may perform similar functions, but these models only handle simple calculations while the authors' model can deal with complicated aspects of risk, such as the Greeks (Hull, 2016). In addition, this model is compatible with Microsoft Office and, thus, can be integrated into PowerPoint presentations, using the hyperlink tool without any internet requirement. These features make this approach easy to use especially in areas where internet service is weak or unavailable.

Options are used in trading, hedging, and speculation both in domestic and international markets. According to Option Clearing Corporation, from February 2019 to February 2020 option contracts in United States are up 60.1 percent with total exchange-listed options cleared contract volume of 558,802,415 in February 2020 from 349,117,021 in February 2019. Option trading is also rising in Europe, Latin American and Asia Pacific (https://www.fia.org/articles/fia-releases-annual-trading-statistics-showing-record-etd-volume-2018). This makes option pricing an important topic to cover in finance classes in both graduate and undergraduate programs. These classes include, but are not limited to, derivative securities, investment analysis, and international finance. The most commonly used model for option pricing is the Black-Scholes-Merton model. The formulas shown below are for the Price of a European Call Option (C^E) and a European Put Option (P^E).

$$\boldsymbol{C^E = S_0N(d_1) - Xe^{-rT}N(d_2)} \qquad (1)$$

and

$$\boldsymbol{P^E = Xe^{-rT}N(-d_2) - S_0N(-d_1)} \qquad (2)$$

Where

S_0 = Underlying Stock Price
X = Exercise Price
$N(.)$ = Cumulative Distribution Function (CDF) of the Standard Normal Distribution
T = Time to Expiration
r = Current Annual Risk-Free Rate

These equations are easy enough to explain but require extensive calculations. For example, d_1 and d_2, the arguments of the Normal CDF, are complicated terms and these need to be calculated first and then inserted into the main equations for C^E and P^E.

$$\boldsymbol{d_1 = \frac{[\ln(S_0/X) + (r - \sigma^2/2)T]}{\sigma\sqrt{T}}} \qquad (3)$$

$$\boldsymbol{d_2 = d_1 - \sigma\sqrt{T}} \qquad (4)$$

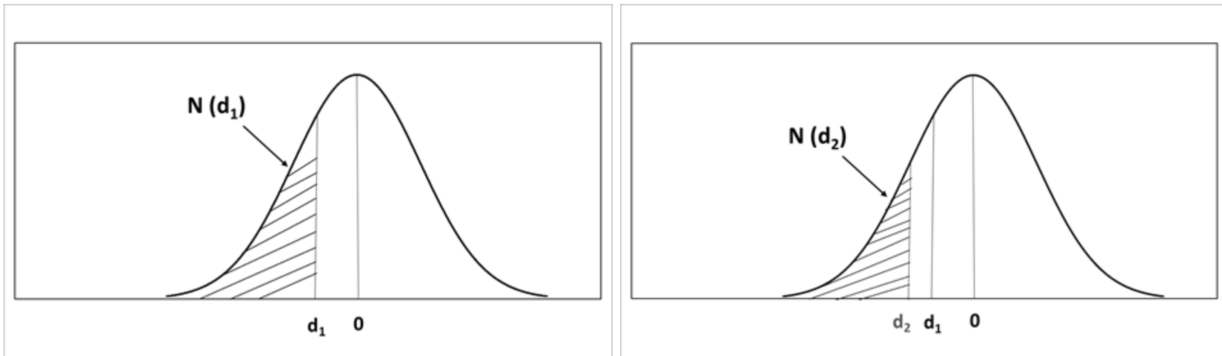
Where

σ = Standard Deviation of historical stock returns

$N(d_1)$ and $N(d_2)$ are probabilities equal to the areas under the standard normal density function to the left of d_1 and d_2 respectively (Figures 1a and 1b), which have to be either looked up in a table, obtained from Excel or calculated with a calculator that gives CDFs for a standard normal density function.

Figure 1a
Normal Cumulative Distribution Function (d_1) Figure 1b. Normal Cumulative Distribution Function (d_2)

Chart for European Calls and Puts



The conventional way to present the Black-Scholes-Merton Equations and their applications

Applying the Black-Scholes-Merton model requires extensive and complicated hand calculations for finance students. As will be discussed below, the "Greeks," which calculate rates of change, also require complicated calculations.

After putting these equations on the board, students are asked to hand-calculate C^E and P^E for a set of five arbitrarily chosen values for the variables {S_0, X, r, T, and σ}. It takes them anywhere from 5 – 10 minutes of class time to complete this task. In a class of 10 students one often gets 6 different answers. So, now, asking them to change one of the variables (for example, increase S_0) to see what happens to C^E and P^E is nearly unthinkable. And yet, sensitivity analysis requires looking at the effect on C^E and P^E as the values of each of the five variables is increased or decreased.

Most textbooks simply present a static chart (Table 1), and, without any demonstration, give a brief explanation of why the call and put prices change as they do.

Table 1
Effects of change in variables on European Call and Put Option Prices.

Variable	European Call (C^E)	European Put (P^E)
Current Stock Price (S_0)	+	-
Strike Price (X)	-	+
Time to Expiration (T)	+	+
Volatility(σ)	+	+
Risk-Free Rate (r)	+	-

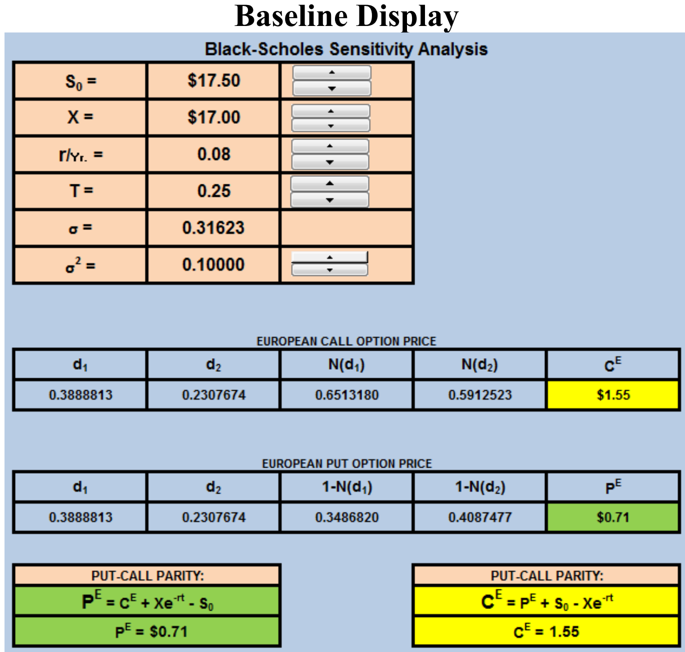
The Program

This approach uses a feature of Excel in which each of the five variables listed above is tied to its own "spinner," which allows the presenter to instantaneously increase or decrease any one of the variable's values and examine the effect on (C^E) and (P^E), thus dynamically presenting what is statically presented in Table 1.

Using several snapshots of the various Excel spreadsheet displays, this paper illustrates the step-by-step results that emerge as a classroom presentation would unfold. The authors believe that a representative selection of these snapshots will be adequate to demonstrate the method, the program, and the approach.

To begin with, in a single Excel display (Figure 2), typical values for S_0, the price of the underlying stock, X, the option's exercise price, r, the risk-free rate per annum, T, the time to expiration of the option, and σ, the volatility of the underlying stock price are listed along with the resulting European Call (C^E) and European Put (P^E) prices. In this model, the underlying stock can be substituted with other securities, such as foreign currencies, futures contracts, and real estate assets, among others, to calculate option values. N(.) is the cumulative standard normal distribution function, that is, the cumulative probability under the standard normal curve from negative infinity up to the argument in (.). Note, Put-Call Parity is also illustrated at the bottom of the display. These five variables {S_0, X, r, T, and σ} are set to those values, with their respective spinners, that the class was asked to use in their hand-calculator calculation.

Figure 2
Baseline Display



Black-Scholes Sensitivity Analysis

S_0 =	$17.50
X =	$17.00
r/Yr. =	0.08
T =	0.25
σ =	0.31623
$σ^2$ =	0.10000

EUROPEAN CALL OPTION PRICE

d_1	d_2	$N(d_1)$	$N(d_2)$	C^E
0.3888813	0.2307674	0.6513180	0.5912523	$1.55

EUROPEAN PUT OPTION PRICE

d_1	d_2	$1-N(d_1)$	$1-N(d_2)$	P^E
0.3888813	0.2307674	0.3486820	0.4087477	$0.71

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = $0.71	C^E = 1.55

Next, using the Excel spinners, each of these five variables is first increased and then decreased, in small steps, one at a time. The resulting values of C^E and P^E are instantaneously recalculated and observed by the viewer.

First, one can observe how the call and put prices (C^E and P^E) change as the price of the underlying asset (S_0) is increased and decreased. These changes happen instantaneously, and can be repeated multiple times in a few minutes (Figures 3 and 4).

Figure 3

C^E and P^E when S_0 is Increased to $18.50

Black-Scholes Sensitivity Analysis

S_0 =	$18.50	
X =	$17.00	
r/Yr. =	0.08	
T =	0.25	
σ =	0.31623	
$σ^2$ =	0.10000	

EUROPEAN CALL OPTION PRICE

d_1	d_2	$N(d_1)$	$N(d_2)$	C^E
0.7403359	0.5822220	0.7704519	0.7197914	$2.26

EUROPEAN PUT OPTION PRICE

d_1	d_2	$1-N(d_1)$	$1-N(d_2)$	P^E
0.7403359	0.5822220	0.2295481	0.2802086	$0.42

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = $0.42	C^E = 2.26

Figure 4

C^E and P^E when S_0 is Decreased to $16.50

Black-Scholes Sensitivity Analysis

S_0 =	$16.50	
X =	$17.00	
r/Yr. =	0.08	
T =	0.25	
σ =	0.31623	
$σ^2$ =	0.10000	

EUROPEAN CALL OPTION PRICE

d_1	d_2	$N(d_1)$	$N(d_2)$	C^E
0.0167413	-0.1413726	0.5066785	0.4437878	$0.97

EUROPEAN PUT OPTION PRICE

d_1	d_2	$1-N(d_1)$	$1-N(d_2)$	P^E
0.0167413	-0.1413726	0.4933215	0.5562122	$1.13

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = $1.13	C^E = 0.97

Next, one can observe how the call and put prices (C^E and P^E) change as the exercise price (X) is increased and decreased (Figures 5 and 6).

Figure 5
C^E and P^E when X is Increased to $18.00

Black-Scholes Sensitivity Analysis

S_0 =	$17.50	
X =	$18.00	
r/Yr. =	0.08	
T =	0.25	
σ =	0.31623	
σ^2 =	0.10000	

EUROPEAN CALL OPTION PRICE

d_1	d_2	$N(d_1)$	$N(d_2)$	C^E
0.0273798	-0.1307341	0.5109216	0.4479928	$1.04

EUROPEAN PUT OPTION PRICE

d_1	d_2	$1-N(d_1)$	$1-N(d_2)$	P^E
0.0273798	-0.1307341	0.4890784	0.5520072	$1.18

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = $1.18	C^E = 1.04

Figure 6
C^E and P^E when X is Decreased to $16.00

Black-Scholes Sensitivity Analysis

S_0 =	$17.50	
X =	$16.00	
r/Yr. =	0.08	
T =	0.25	
σ =	0.31623	
σ^2 =	0.10000	

EUROPEAN CALL OPTION PRICE

d_1	d_2	$N(d_1)$	$N(d_2)$	C^E
0.7723051	0.6141912	0.7800331	0.7304555	$2.19

EUROPEAN PUT OPTION PRICE

d_1	d_2	$1-N(d_1)$	$1-N(d_2)$	P^E
0.7723051	0.6141912	0.2199669	0.2695445	$0.38

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = $0.38	C^E = 2.19

Next, one can observe how the call and put prices (C^E and P^E) change as the annual risk-free rate (r) is increased and decreased (Figures 7 and 8).

Figure 7

C^E and P^E when r is Increased to 10%

Black-Scholes Sensitivity Analysis

S_0 =	\$17.50
X =	\$17.00
r/Yr. =	0.1
T =	0.25
σ =	0.31623
σ^2 =	0.10000

EUROPEAN CALL OPTION PRICE

d_1	d_2	$N(d_1)$	$N(d_2)$	C^E
0.4205041	0.2623902	0.6629414	0.6034897	\$1.60

EUROPEAN PUT OPTION PRICE

d_1	d_2	$1-N(d_1)$	$1-N(d_2)$	P^E
0.4205041	0.2623902	0.3370586	0.3965103	\$0.68

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = \$0.68	C^E = 1.60

Figure 8

C^E and P^E when r is Decreased to 6%

Black-Scholes Sensitivity Analysis

S_0 =	\$17.50
X =	\$17.00
r/Yr. =	0.06
T =	0.25
σ =	0.31623
σ^2 =	0.10000

EUROPEAN CALL OPTION PRICE

d_1	d_2	$N(d_1)$	$N(d_2)$	C^E
0.3572586	0.1991447	0.6395509	0.5789252	\$1.50

EUROPEAN PUT OPTION PRICE

d_1	d_2	$1-N(d_1)$	$1-N(d_2)$	P^E
0.3572586	0.1991447	0.3604491	0.4210748	\$0.74

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = \$0.74	C^E = 1.50

Continuing, it can be observed how the call and put prices (C^E and P^E) change as the time to expiration (T) is increased and decreased (Figures 9 and 10).

Figure 9

C^E and P^E when T is Increased to 4 Months

Black-Scholes Sensitivity Analysis

S_0 =	$17.50	
X =	$17.00	
r/Yr. =	0.08	
T =	0.333333333	
σ =	0.31623	
$σ^2$ =	0.10000	

EUROPEAN CALL OPTION PRICE

d_1	d_2	$N(d_1)$	$N(d_2)$	C^E
0.3961177	0.2135435	0.6539909	0.5845485	$1.77

EUROPEAN PUT OPTION PRICE

d_1	d_2	$1-N(d_1)$	$1-N(d_2)$	P^E
0.3961177	0.2135435	0.3460091	0.4154515	$0.82

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = $0.82	C^E = 1.77

Figure 10

C^E and P^E when T is Decreased to 2 Months

Black-Scholes Sensitivity Analysis

S_0 =	$17.50	
X =	$17.00	
r/Yr. =	0.08	
T =	0.166666667	
σ =	0.31623	
$σ^2$ =	0.10000	

EUROPEAN CALL OPTION PRICE

d_1	d_2	$N(d_1)$	$N(d_2)$	C^E
0.3923658	0.2632663	0.6526060	0.6038273	$1.29

EUROPEAN PUT OPTION PRICE

d_1	d_2	$1-N(d_1)$	$1-N(d_2)$	P^E
0.3923658	0.2632663	0.3473940	0.3961727	$0.57

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = $0.57	C^E = 1.29

Finally, it can also be noted how the call and put prices (C^E and P^E) change as the volatility of the underlying stock (σ) is increased and decreased (Figures 11 and 12).

Figure 11
C^E and P^E when σ^2 is Increased to .11

Black-Scholes Sensitivity Analysis

S_0 =	\$17.50
X =	\$17.00
r/Yr. =	0.08
T =	0.25
σ =	0.33166
σ^2 =	0.11000

EUROPEAN CALL OPTION PRICE

d_1	d_2	$N(d_1)$	$N(d_2)$	C^E
0.3783216	0.2124903	0.6474041	0.5841377	\$1.60

EUROPEAN PUT OPTION PRICE

d_1	d_2	$1-N(d_1)$	$1-N(d_2)$	P^E
0.3783216	0.2124903	0.3525959	0.4158623	\$0.76

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = \$0.76	C^E = 1.60

Figure 12
C^E and P^E when σ^2 is Decreased to .09

Black-Scholes Sensitivity Analysis

S_0 =	\$17.50
X =	\$17.00
r/Yr. =	0.08
T =	0.25
σ =	0.30000
σ^2 =	0.09000

EUROPEAN CALL OPTION PRICE

d_1	d_2	$N(d_1)$	$N(d_2)$	C^E
0.4015836	0.2515836	0.6560047	0.5993185	\$1.49

EUROPEAN PUT OPTION PRICE

d_1	d_2	$1-N(d_1)$	$1-N(d_2)$	P^E
0.4015836	0.2515836	0.3439953	0.4006815	\$0.66

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = \$0.66	C^E = 1.49

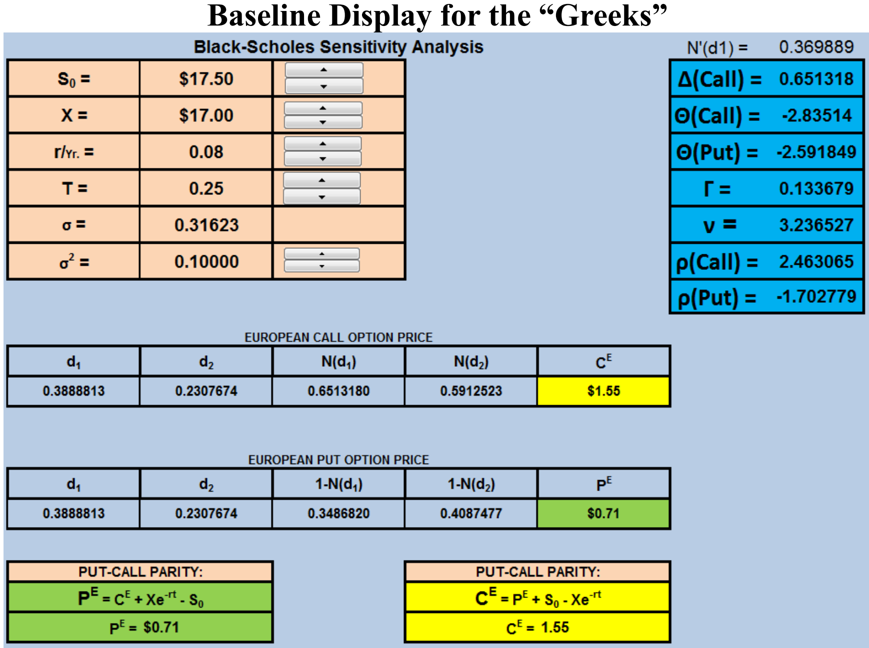
At each step on the above sequence of changes, this pedagogical tool allows for a discussion of the sensitivity of C^E and P^E to each of the five variables in the two equations and also a discussion about whether and why changes in C^E and P^E are directly or inversely related to changes in the particular values of the variable under discussion. The lecture can focus on sensitivity analysis instead of getting bogged down with tedious calculations, which would have to be repeated at least 10 times to cover just one increase and one decrease in value per variable.

This program also illustrates Put-Call Parity equations in the same single Excel display and an enhanced, dynamic discussion of Put-Call Parity is possible. With this pedagogical tool, if desired, one can also discuss how d_1, $N(d_1)$, d_2, and $N(d_2)$ change. After one numerical example is discussed, the instructor would then use the spinner connected to S_0 to manipulate the underlying stock price in small increments as the students watch the option prices change. The values of d_1, d_2, $N(d_1)$, $N(d_2)$, $1 - N(d_1)$, and $1 - N(d_2)$ are also calculated and shown in this single Excel display, allowing the instructor to tie in a discussion of the first of the "Greeks" Δ(call) and Δ(put), discussed in the next section, as S_0 is changed.

The "Greeks"

In more advanced courses, instructors often get into a discussion of the equations which deal with aspects of risk. These equations are normally referred to as the "Greeks." John C. Hull in the 9th edition of, Fundamentals of Futures and Options Markets [1] devotes an entire chapter to a discussion of the "Greeks." He states that "Each Greek letter measures a different dimension to the risk of an option position." [1] Hull focuses on five Greek letters in particular: Delta, Theta, Gamma, Vega and Rho. This paper illustrates the idea with one numerical example. In addition, the authors use dynamic manipulation of the appropriate variables to solidify the viewers understanding without the necessity of a multitude of tedious calculations that becomes much more apparent when illustrated live on an Excel spreadsheet (Baseline display shown in Figure 13).

Figure 13
Baseline Display for the "Greeks"



The equations for the "Greek" letters are shown below. Each of these has its own relatively complex equation, with Delta being the simplest.

Delta (Δ)

"Delta" is the rate of change of the option price with respect to the price of the underlying asset.

$$\Delta(\boldsymbol{Call}) = \boldsymbol{N}\,(\boldsymbol{d_1}) \qquad (5)$$

$$\Delta(\boldsymbol{Put}) = \boldsymbol{N}\,(\boldsymbol{d_1}) - \mathbf{1} \qquad (6)$$

Theta (Θ)

"Theta" is the rate of change of the value of the option with respect to the passage of time, all else remaining the same.

$$\boldsymbol{\theta}(\boldsymbol{Call}) = -\left[\frac{\boldsymbol{S_0 N'(d_1)\sigma}}{\mathbf{2}\sqrt{\boldsymbol{T}}}\right] - \boldsymbol{rXe^{-rT}N\,(d_2)} \qquad (7)$$

$$\boldsymbol{\theta}(\boldsymbol{Put}) = -\left[\frac{\boldsymbol{S_0 N'(d_1)\sigma}}{\mathbf{2}\sqrt{\boldsymbol{T}}}\right] + \boldsymbol{rXe^{-rT}N\,(-d_2)} \qquad (8)$$

Where

$\boldsymbol{N'(d_1)} = (1/\sqrt{\mathbf{2\pi}}\,)\boldsymbol{e^{-d_1^2/2}}$

Gamma(Γ)

"Gamma" is the rate of change of the option's "Delta" with respect to the price of the underlying asset.

$$\boldsymbol{\Gamma} = \frac{\boldsymbol{N'(d_1)}}{\boldsymbol{S_0\sigma\sqrt{T}}} \qquad (9)$$

Vega (V)

"Vega" is the rate of change of the option's value with respect to the implied volatility of the underlying asset.

$$\boldsymbol{V} = \boldsymbol{S_0\sqrt{T}N'(d_1)} \qquad (10)$$

Rho (ρ)

"Rho" is the rate of change of the value of the option with respect to the interest rate.

$$\boldsymbol{\rho}(\boldsymbol{Call}) = \boldsymbol{XTe^{-rT}N(d_2)} \qquad (11)$$

$$\boldsymbol{\rho}(\boldsymbol{Put}) = \boldsymbol{-XTe^{-rT}N(-d_2)} \qquad (12)$$

Again, rather than have the students compute these "Greeks" in class, the authors have added a table to the spreadsheet which, displays the value of each of them instantaneously, just as was done for C^E and P^E. Figure 13 is an example of what that display looks like.

Now, to illustrate just one example, let's focus on DELTA! Note again that *"Delta" is the rate of change of the option price with respect to the price of the underlying asset (Eqs. 5 and 6)* So, with Delta = .65, a \$1.00 increase in S_0, will result in an increase in C^E of approximately \$0.65 and a \$1.00 decrease in S_0, should result in a decrease in C^E of approximately \$0.65. Using the "Spinners," the class can easily see these results displayed. This discussion would proceed in a similar fashion for the other "Greeks" and the other variables and much time is saved by applying this pedagogical tool.

Implied Volatility

The volatility of the stock price, one of the five parameters in the Black-Scholes-Merton equations, cannot be directly observed. Volatility can be estimated from historical stock prices, but not without technical issues. An alternative approach is to use the volatility implied by an option price observed in the market. This is referred to as *implied volatility*.

Since Implied Volatility is one of the important features of the Black-Scholes-Merton discussion, the authors included one additional feature in the spreadsheet, which allows one to vary σ^2 in much smaller increments (Figures 14a and 14b). When the sensitivity is set to "100," σ^2 increases/decreases in the 1/100s position.

When the sensitivity is set to "1000," σ^2 increases/decreases in the 1/1000s position. The idea is to increase or decrease σ^2 and stop when C^E in the display reaches the value of C^E observed in the market. With a higher sensitivity setting, one can get closer to the actual observed market price of the option. The corresponding value of σ is the implied volatility!

Figure 14a
Implied Volatility with Sensitivity Set to 100

Black-Scholes Sensitivity Analysis

S_0 =	$17.50
X =	$17.00
r/Yr. =	0.08
T =	0.25
σ =	1.40712
σ^2 =	1.98000

100 = Sensitivity Setting

EUROPEAN CALL OPTION PRICE

d1	d2	N(d1)	N(d2)	C^E
0.4214090	-0.2821533	0.6632718	0.3889130	$5.13

EUROPEAN PUT OPTION PRICE

d1	d2	1-N(d1)	1-N(d2)	P^E
0.4214090	-0.2821533	0.3367282	0.6110870	$4.29

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = $4.29	C^E = 5.13

Figure 14b
Implied Volatility with Sensitivity Set to 1000

Black-Scholes Sensitivity Analysis

S_0 =	$17.50
X =	$17.00
r/Yr. =	0.08
T =	0.25
σ =	0.45497
σ^2 =	0.20700

1000 = Sensitivity Setting

EUROPEAN CALL OPTION PRICE

d1	d2	N(d1)	N(d2)	C^E
0.3290860	0.1015997	0.6289546	0.5404628	$2.00

EUROPEAN PUT OPTION PRICE

d1	d2	1-N(d1)	1-N(d2)	P^E
0.3290860	0.1015997	0.3710454	0.4595372	$1.16

PUT-CALL PARITY:	PUT-CALL PARITY:
$P^E = C^E + Xe^{-rt} - S_0$	$C^E = P^E + S_0 - Xe^{-rt}$
P^E = $1.16	C^E = 2.00

Conclusion

The presentation of Black-Scholes-Merton sensitivity analysis is often confusing to students. The authors believe that this dynamic Excel approach makes this discussion much easier to grasp and retain. The Excel-based method for Black-Scholes-Merton option pricing sensitivity analysis discussed herein eliminates the need for tedious, difficult, complicated and time consuming calculations. The same Excel-based method enables the professor to demonstrate, and the students to visualize, the impact of changing the variables' values on the Call and Put prices by making many changes in a short period of time.

For future research, the authors are working on a model to include options on dividend paying stocks and options on futures for advance courses. The authors will make the Excel files used in this paper available to readers upon request.

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Higher Level Excel Skills for Students: Dynamic Reports

Chris R. McNeil
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Students majoring in finance can potentially improve their initial job prospects by developing Excel skills that are valued by many employers. This paper highlights intermediate to advanced Excel skills used in the construction of dynamic reports – reports on sales, expenses, or other items of interest that include easily adjustable parameter selections that update a report's output. Adjustable parameters, for example, might include time period, product line, department, or division within a firm. Within corporate finance, well-constructed dynamic reports are an important means to facilitate prompt delivery of new information to managers and to support decision making focused on improving firm performance. The paper includes an assignment for students that illustrates dynamic report related Excel skills and describes an Excel VBA program that generates simulated data for updating the assignment across semesters. The assignment, Excel files, the VBA program, and instructional streaming video are available from this article's companion website (https://sites.google.com/appstate.edu/higher-level-excel-skills).

Keywords: Excel skills, Dynamic reports

Introduction

Several papers within the finance academic literature emphasize the importance of students developing strong Excel skills (e.g., Spiech 2005; Payne and Tanner 2011; Zhang 2020; Renz and Vogel 2021; and Wann 2021). In seeming recognition of the importance of these skills, many finance textbooks include spreadsheet activities for students. The purpose of this paper is to highlight the intermediate to advanced Excel skills for constructing dynamic reports, to provide a student assignment incorporating dynamic reports, and to offer an Excel VBA/macro program that can be used to generate simulated data for related student assignments. Possessing the expertise to build dynamic spreadsheet reports is among the higher-level Excel skills valued by many potential employers of students majoring in finance. In addition, the development of higher-level Excel skills can enhance a student's critical thinking skills as students work through challenging spreadsheet related problems.

Dynamic reports are useful within the practice of finance by increasing the efficiency of converting data into information needed to support wise decision making. Within corporate finance, for example, analysis of sales and expenses are routinely conducted as managers evaluate firm performance and consider actions to improve future results. New data accrues with the passage of time, prompting a frequent need for updated reporting and analysis. Given the frequency of updated reporting and analysis and the importance of responding quickly to unexpected changes in such things as sales and expenses, managers need prompt conversion of new data into information. Dynamic reports facilitate quick and efficient delivery of new critical information to a firm's managers.

What are dynamic reports? A dynamic report includes parameters (such as time period, geographic region, or product line) that a user can easily adjust and that are immediately reflected in a report's outputs about sales activity, expenses or some other item of interest to a firm. For example, as time passes and new data becomes available, a well-built dynamic report within Excel enables an analyst to quickly produce an updated report that incorporates the new data by changing input parameters for time period covered. In addition, a dynamic report commonly includes the ability to drill down into the data, moving, for example, from firm level information to division level results. Figure 1 shows an image of a dynamic report in which a user can select the year, month, geographic area, and product line for a report.

Figure 1
Dynamic Report Example

Report Parameters

Location	All
Product	All
Year	2022
Month	May
Months YTD	5

Monthly Sales
For All Locations
For All Products
Through May for the Years 2021 and 2022

Month	Sales 2021	Sales 2022	% Change	Sales YTD 2021	Sales YTD 2022	% Change
Jan	$4,692,163	$5,112,789	8.96%	$4,692,163	$5,112,789	8.96%
Feb	$5,110,404	$5,360,288	4.89%	$9,802,566	$10,473,077	6.84%
Mar	$6,986,136	$7,265,578	4.00%	$16,788,703	$17,738,654	5.66%
Apr	$7,968,412	$8,792,331	10.34%	$24,757,115	$26,530,985	7.17%
May	$10,921,392	$11,934,634	9.28%	$35,678,507	$38,465,620	7.81%

Constructing a dynamic report within Excel requires intermediate to advanced Excel skills. Typical Excel features and functions employed include such things as tables, data validation, concatenation, VLOOKUP (XLOOKUP in newer versions of Excel), INDEX/MATCH, SUMIFS, DATEVALUE, EOMONTH, ISNUMBER, IF, pivot tables, and conditional formatting. Appendix A contains a brief description of each of these Excel features along with some others. In addition, charts and pivot charts are often used to visualize reporting output.

The next section briefly summarizes the steps preceding the construction of dynamic reports. Section 3 contains a description of a student assignment that includes simulated sales data and dynamic report examples. Section 4 provides an overview of an Excel program that generates new simulated data, enabling instructors to vary data for assignments across classes or semesters. Section 5 concludes.

Steps Leading up to the Building of a Dynamic Report

Before discussing methods of constructing dynamic reports within Excel with students, it is important to share the general steps preceding that construction. The first step is to determine questions of interest to the firm and its managers. For example, what is the firm's current sales performance, which divisions or products are overperforming or underperforming, are expenses within budget? Second, we should determine the criteria and measures for evaluating the questions of interest. If the firm has related key performance indicators (KPIs) already in place, these KPIs should be included in the evaluation criteria/measures. Third, we would determine the data needed and available to provide insight into the questions of interest. The availability of data may require revisiting the prior step. A fourth step would be to obtain and import the data into a software application (i.e., Excel for the purposes of this paper) within which to build the dynamic reports. Fifth, the analyst reviews the raw data, considers checks for accuracy, and cleans the data as needed. While not an aspect of focus in this paper, it is beneficial to note to students that Excel has useful tools for importing and cleaning data. One of these newer tools is Get & Transform Data, otherwise known as Power Query, which can handle big and small data much more efficiently than the import data tool in earlier versions of Excel. Sixth, the analyst and manager should consider how to best present insights from the data. For example, how much information should a report include to avoid information overload and how to make important insights from the data more easily digestible. Seven, construct the dynamic reports within Excel and thoroughly check the reports to ensure their accuracy. Eight, for newly developed reports, share the reports with select individuals for feedback and edit the reports based on that feedback before distributing widely. After generating the reports with updated information, it is time for analysis. What does the data tell us? A thorough analysis often gives rise to additional questions.

Student Assignment

The assignment presented here takes the form of a mini-case in which students have two primary tasks: First, the construction of dynamic reports based on daily sales data; and, second, the preparation of an executive summary highlighting key points from their analysis of that data. The target audience is intermediate to advanced level finance students who have been provided an introduction to the features and functions used to construct dynamic reports within Excel. Excel offers two general means to build dynamic reports – formulas and pivot tables. This paper concentrates on the formula approach since it offers greater flexibility and a richer environment for Excel skill development. The text of the assignment to be provided to students appears in Appendix B. Accompanying materials for students include simulated daily sales data and two related datasets, spreadsheet templates, and "old" hardcopy reports. Spreadsheet templates speed review of student work and feedback to students. Old hardcopy reports based on prior period sales data enable students to assess their own work within Excel by back testing, similar to real world best practice. Students could be also be instructed to use pivot tables to spot check their reports for accuracy. An instructor could increase the assignment's level of difficulty by withholding the spreadsheet templates and old hardcopy reports and requiring students to develop their own criteria and reports for analysis of the data. Accompanying instructor materials available from this article's companion website (*https://sites.google.com/appstate.edu/higher-level-excel-skills*) include dynamic reports completed within Excel, an Excel VBA program to generate new simulated daily sales data that can be used to update the student assignment across semesters, and streaming video demonstrating construction of dynamic reports within Excel and execution of the VBA program to generate new data.

The setting for the assignment is a medium sized firm with four locations/regions and seven product lines, where each location sells the same products. Some number of months have passed during the current year and the protagonist in the assignment is first tasked with automating a number of existing monthly sales reports so that future monthly reports and analysis can be delivered to management more quickly. The locations can alternatively appear in a modified case as geographic regions, divisions, units, or even sales people, depending upon how the data is labeled and described in the assignment. An instructor can use the Excel VBA program briefly referenced above to vary both the number of locations/regions and products that appear within the sales data. A description of the Excel VBA program appears in a later section of this paper.

Assignment Datasets

The datasets for the version of assignment presented in this paper include: (a) daily sales data by location and product line for the first seven months of the current year, plus the thirty-six months of the prior three years; (b) sales goals by location and month for the current year; and (c) a very abbreviated dataset for the products. (The product dataset only lists product key and product name. In practice, product datasets typically include several additional attributes.) The daily sales data contains over 36,000 observations, giving students exposure to semi-big data. The daily sales data fields include Date, Location Key, Product Key, and Revenue, which is in dollars, as shown in Figure 2.

Figure 2
Simulated Daily Sales by Location and Product

Date	Location Key	Product Key	Revenue
7/31/2023	L-73913	PD-698632	11075.23
7/31/2023	L-76803	PD-698632	5539.75
7/31/2023	L-32047	PD-698632	9179.38
7/31/2023	L-54425	PD-698632	7604.17
7/31/2023	L-73913	PD-280805	6460.56
7/31/2023	L-76803	PD-280805	5904.67
7/31/2023	L-32047	PD-280805	10497.74
7/31/2023	L-54425	PD-280805	14707.93
7/31/2023	L-73913	PD-593537	11918.46
7/31/2023	L-76803	PD-593537	6301.78
7/31/2023	L-32047	PD-593537	10535.92
7/31/2023	L-54425	PD-593537	16675.15
7/31/2023	L-73913	PD-964850	15332.87
7/31/2023	L-76803	PD-964850	10670.52
7/31/2023	L-32047	PD-964850	21474.71
7/31/2023	L-54425	PD-964850	22254.09

Note: Figure 2 shows top rows within a simulated dataset for daily sales by location and product for four locations and seven products over 43 months ending July 2023. Total observations equal 36,622.

Location Key and Product Key denote unique identifiers for location and product. In practice, datasets for sales and other activities typically use unique identifiers for items like location/region and product, with separate datasets housing details for the items (e.g., location name, number of employees, number of stores, manager's name, etc. . .). Excluding location name and product name from the daily sales data requires students to use the VLOOKUP Excel function or something similar to add location name and product name to the sales data for their analysis, similar to what might be required in practice. The goals dataset contains the variables Location Key, Location Name, Month, and Sales Goal (see Figure 3). The product dataset is sparse, containing only Product Key and Product Name.

All three datasets are located in an Excel file as separate Excel tables named tbSales, tbGoals, and tbProds. Excel tables are useful in building formulas and, as time passes, when new data needs to be added to the dataset. To add rows of new data to an existing Excel table, we can simply copy and paste the new data immediately below the last row of the table. If the table does not expand automatically, move to a cell within the table and then select Table Tools Design → Resize Table. A more efficient approach is to use Excel's Power Query tool to append new data. If desired, an instructor could convert one or more of the datasets to text files and require that students import the datasets into Excel.

Figure 3
Simulated Monthly Sales Goals by Location

Location Key	Location	Month	Sales Goal ($s)
L-76803	Chicago	1/31/2023	904552
L-73913	Boston	1/31/2023	1267667
L-54425	New York City	1/31/2023	1524302
L-32047	Dallas	1/31/2023	1569652
L-76803	Chicago	2/28/2023	990922
L-73913	Boston	2/28/2023	1271931
L-54425	New York City	2/28/2023	1820197
L-32047	Dallas	2/28/2023	1438046
L-76803	Chicago	3/31/2023	1350968
L-73913	Boston	3/31/2023	1705380
L-54425	New York City	3/31/2023	2372061
L-32047	Dallas	3/31/2023	2055137

Note: Figure 3 shows top rows within a simulated dataset for monthly sales goals by location for the assumed current year of 2023 for four locations.

<u>The Assignment's Dynamic Reports</u>

For the student assignment, an instructor can choose from six dynamic reports shown in this paper, create their own, and/or task students with developing reports from scratch. Table 1 lists select Excel features and functions used in each of the six reports appearing in this paper. Data validation, table references, SUMIFS, concatenation, and EOMONTH, for example, are used extensively. In addition, students must use VLOOKUP or a similar function to add location and product names to the sales data prior to building any report. The reports in an Excel file and streaming video of their construction are available from this article's companion website (*https://sites.google.com/appstate.edu/higher-level-excel-skills*). Reports 1 and 5 appear in Figures 4 and 5 with accompanying discussion in the paper. To view the other four reports, please see Appendix C.

Dynamic Report 1

Figure 4's Report 1 (Sales YTD vs Goal) is the most succinct of the six reports, making it a good starting report for students. Textboxes within the Figure display select formulas for the reader's review. These formulas are not usually provided to students with the assignment. The "report parameters" in cells D18, D20, and D22 contain dropdown selections (using Excel's data validation feature) for Location, Year, and Month that factor into the report's output, making the report dynamic. A user can adjust these report parameters to vary report output to show all locations or a single location and can change the time period covered. The report parameters are reflected in the report's descriptive labels (cells C29, C30, C32, and C33) using concatenation.

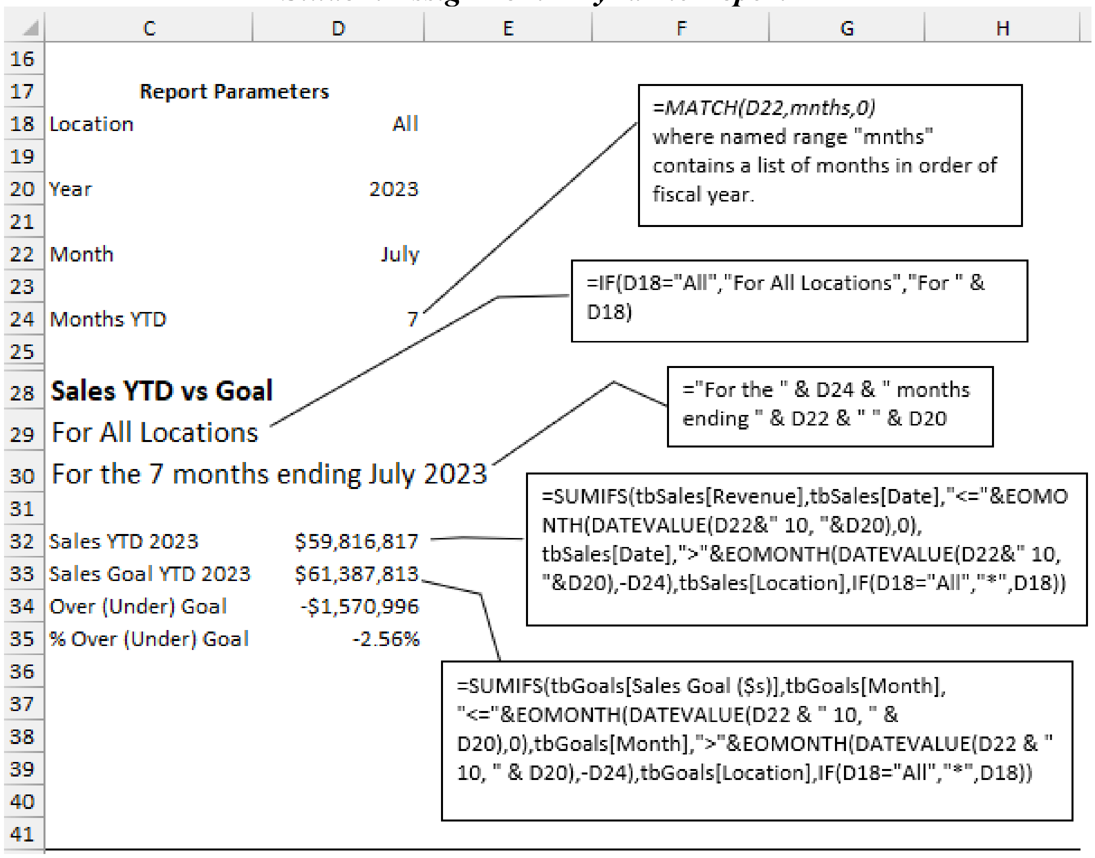
Table 1
Select Excel Features/Functions Used in Dynamic Reports 1 through 6

Excel Feature/Function	Report 1	Report 2	Reports 3 & 4	Reports 5 & 6
Concatenation	x	x	x	x
Custom Formatted Dates				x
Data Validation	x	x	x	x
DATEVALUE	x	x	x	x
Dynamic Dates as Row Labels				x
EOMONTH	x	x	x	x
IF	x		x	x
ISNUMBER				x
MATCH	x	x	x	x
ROWS				x
SUMIFS	x	x	x	x
Table References	x	x	x	x
Wildcard Character "*"	x		x	x

The formula in cell D32 of Figure 4 illustrates one advantage of using table references within formulas. In that SUMIFS formula, tbSales is the table of daily sales by location and product. Revenue, Date, and Location are fields/columns within tbSales. Using table references makes the resulting formula much easier to read (compared to using traditional cell references like $E$9:$E$36632). Additional advantages of table references are that it speeds up the building of formulas and the formulas automatically incorporate any new observations added to tbSales as new data becomes available. Within the SUMIFS formula, the EOMONTH function is employed to sum daily sales only for the time period specified. The DATEVALUE function within EOMONTH is used to convert the year and month selections in cells D20 and D22 to a numeric calendar date. To switch between summing for all locations or an individual location, the SUMIFS formula includes an IF function for the location criteria. When the report parameter selection for Location (D18) is "All," the IF function's result for the location criteria is an asterisk ("*"), Excel's wildcard character. The wildcard asterisk means that the SUMIFS will sum over any and all

Location values in the daily sales data (the date criteria within the SUMIFS still applies). Conditional formatting not shown here could be added to D35 (the -2.56% over (under) goal figure) to emphasize values over or under goal.

Figure 4
Student Assignment - Dynamic Report 1



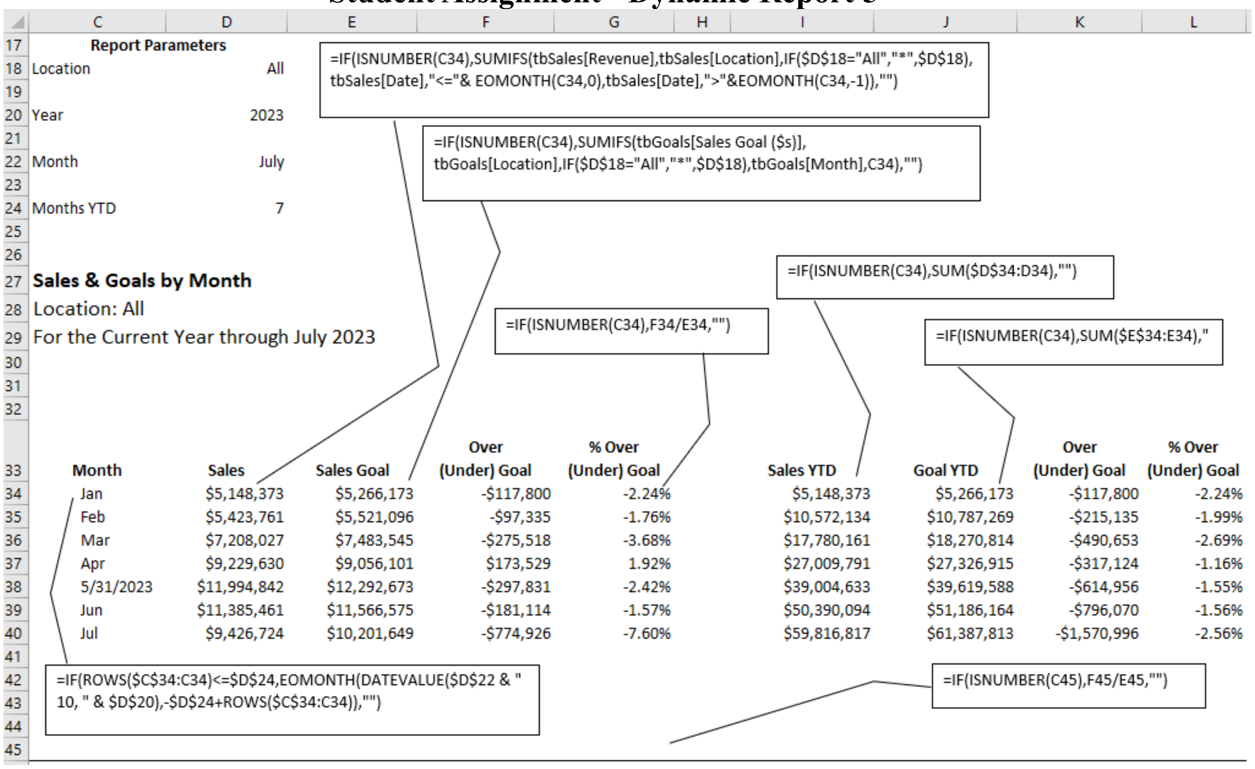
Dynamic Report 5

Figure 5 contains dynamic Report 5 (Sales & Goals by Month) for the student assignment. (Reports 2, 3, 4, and 6 are shown in Appendix C.) Similar to Figure 4's Report 1, Report 5 contains dropdown selections for Location, Year, and Month (cells D18, D20, and D22) that make the report's output dynamic. Among the six reports, Report 5 and Report 6 are the most challenging to construct since these two reports include dynamic calendar dates as row headers (starting in C34 of Figure 5). That is, the report displays results for each month but only through the month selected within the current year (July in D22). To reduce the level of difficulty of Reports 5 and 6, students could be instructed to build the reports to display all twelve months of the current year as row headers regardless of the input selected for month.

In Figure 5, the formula in C34 for the first row header for month (value of "Jan") can be challenging for students to follow. Students should first understand that the desired value in C34 is the month end calendar date for the first month of the firm's current year in order to facilitate calculating Sales and Sales Goal for the month in cells D34 and E34. Thus, underlying the value of “Jan” in C34 is the numeric calendar date 1/31/2023. Custom formatting has been applied to C34 to display the abbreviation for the month (formatting set to date for C38 for the reader).

Breaking down the formula in C34 into parts, as listed below, can help students better understand the workings of the formula, noting that the formula is built to be copied down through C45 to accommodate up to 12 months of current year values. Building formulas that can be copied in order to produce related values is a best practice in that it speeds up the construction of a report and can make the overall construction of a report easier for others to follow. The breakdown of C34’s formula is a bit detailed, so the reader might wish to simply skim the following bullets.

Figure 5
Student Assignment - Dynamic Report 5



Month	Sales	Sales Goal	Over (Under) Goal	% Over (Under) Goal	Sales YTD	Goal YTD	Over (Under) Goal	% Over (Under) Goal
Jan	$5,148,373	$5,266,173	-$117,800	-2.24%	$5,148,373	$5,266,173	-$117,800	-2.24%
Feb	$5,423,761	$5,521,096	-$97,335	-1.76%	$10,572,134	$10,787,269	-$215,135	-1.99%
Mar	$7,208,027	$7,483,545	-$275,518	-3.68%	$17,780,161	$18,270,814	-$490,653	-2.69%
Apr	$9,229,630	$9,056,101	$173,529	1.92%	$27,009,791	$27,326,915	-$317,124	-1.16%
5/31/2023	$11,994,842	$12,292,673	-$297,831	-2.42%	$39,004,633	$39,619,588	-$614,956	-1.55%
Jun	$11,385,461	$11,566,575	-$181,114	-1.57%	$50,390,094	$51,186,164	-$796,070	-1.56%
Jul	$9,426,724	$10,201,649	-$774,926	-7.60%	$59,816,817	$61,387,813	-$1,570,996	-2.56%

- Breaking down the formula in C34 of Figure 5, first note that the value of C34 must correspond to the first month of the firm’s current year. We have also chosen for it to be an end-of-month value (i.e., 1/31/2023).
- Within C34’s formula, the DATEVALUE function converts the year and month entries in D20 (2023) and D22 (July) into the numeric calendar date of 7/10/2023, where the 10 is an arbitrary value used by the instructor.
- The EOMONTH function changes the DATEVALUE output to an end-of-month date.
- The second argument within the EOMONTH function is the number of months from 7/30/2023. The desired value for C34 is 1/31/2023 (first month of the fiscal year), 6 months before 7/30/2023. So, EOMONTH’s second argument for C34 must be -6. To get the -6, the formula employs -$D$24 + ROWS($C$34:C34), which equals -7 + 1. More on the ROWS function next.
- The ROWS function above counts the number of rows within the range provided ($C$34:C34 in the formula). The range $C$34:C34 contains only one row. Thus

ROWS($C$34:C34) equals 1. When the formula in C34 is copied to C35 through C45, the ROWS result incrementally increases by 1 for each row further below C34, making the EOMONTH output progress from 1/31/2023 to 2/28/2023 to 3/31/2023 etc. . . as we move from C34 to C35 to C36 etc. . .

- For the report to only display the months corresponding with the number of months year-to-date in D24 (7 in the Figure 5), an IF is used in the formula of C34. In words, the logical test of the IF (i.e., ROWS($C$34:C34)<=$D$24) is whether the number of rows below row 33 is less than or equal to the number of months year-to-date to display noted in D24. When the logical test is true, we get the end-of-month value. When false, we get "", a blank cell. Since C34 is copied to C35 through C45, the ROWS result incrementally increases by 1 as we move down column C, reaching a value of 12 for C45. Thus, for a D24 Months YTD value of 7, months 1 through 7 are shown, while months 8 through 12 do not appear in the report.

Formulas in cells D34 through L34 of Figure 5's Report 5 are built so that they could be copied down each column through row 45. For the report to only display results for each month appearing in column C, each formula in D34 through L34 includes an IF with logical test ISNUMBER(C34). As its name suggests, the ISNUMBER function returns true when the argument entered is a number and false otherwise. When visible, the values in C34 through C45 are numeric calendar dates even though the values appear as monthly abbreviations. Thus, when the month in column C is visible, the IF returns the calculation entered for "true," otherwise, the IF returns "" (a blank cell).

In the next section, we present an overview of the Excel VBA used to generate the simulated data that is the focus of the assignment's dynamic reports.

Program To Generate Simulated Sales Data

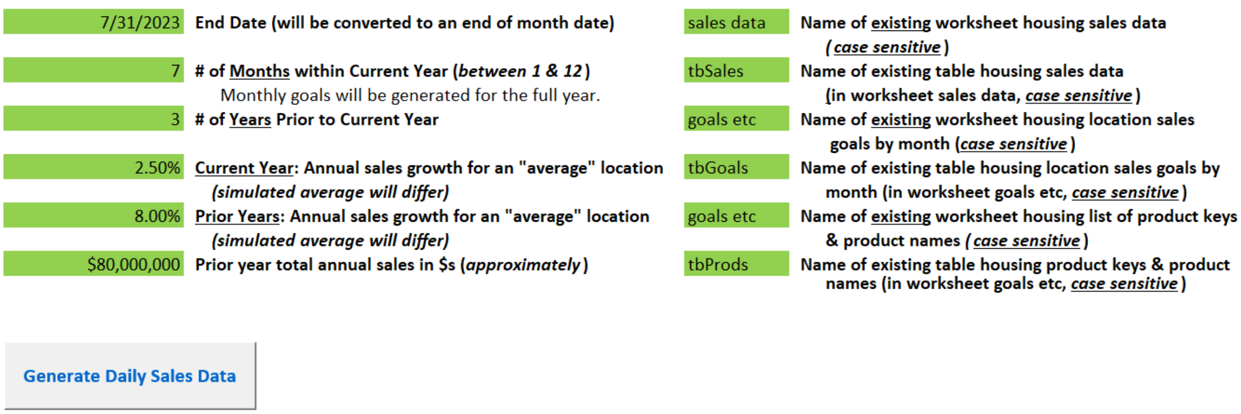
We use an Excel VBA/macro program to generate new simulated data to address the need for updated datasets for student work within and across semesters, since locating suitable datasets for student work can be time consuming. Varying the data for the assignment across semesters better ensures that students do not simply copy work from earlier semesters. The VBA program replaces and repopulates the three datasets described earlier in the example student assignment (see section three (Student Assignment), subsection Assignment Datasets). The three datasets are: (a) daily sales data by location and product; (b) sales goals by location and month for the current year; and (c) the product dataset. The program is housed within an Excel file and is available from this article's companion website (https://sites.google.com/appstate.edu/higher-level-excel-skills), along with a streaming video demonstration. The imagined setting for the data is a firm with some number of locations/regions and products, where each location sells the same products. Locations can alternatively be thought of as geographic regions, units, or even sales people. Some number of months have passed during the current year. The daily sales dataset includes the fields Date, Location Key, Product Key, and Revenue in dollars as previously shown in Figure 2. No VBA programing by the instructor is required.

Before running the VBA program, the instructor retains or updates as desired the following items, as also shown in Figure 6.

- End date for the simulated data.
- Number of months of data within the current fiscal year.
- Number of years of data prior to the current fiscal year.

- Annual sales growth rate for an average location during the current fiscal year.
- Annual sales growth rate for an average location during the prior years.
- Existing worksheet and table names for the three data sets (daily sales, sales goals, and product list).

Figure 6
Generating Simulated Data



7/31/2023	End Date (will be converted to an end of month date)	sales data	Name of existing worksheet housing sales data (*case sensitive*)
7	# of Months within Current Year (*between 1 & 12*) Monthly goals will be generated for the full year.	tbSales	Name of existing table housing sales data (in worksheet sales data, *case sensitive*)
3	# of Years Prior to Current Year	goals etc	Name of existing worksheet housing location sales goals by month (*case sensitive*)
2.50%	Current Year: Annual sales growth for an "average" location (*simulated average will differ)*	tbGoals	Name of existing table housing location sales goals by month (in worksheet goals etc, *case sensitive*)
8.00%	Prior Years: Annual sales growth for an "average" location (*simulated average will differ)*	goals etc	Name of existing worksheet housing list of product keys & product names (*case sensitive*)
$80,000,000	Prior year total annual sales in $s (*approximately*)	tbProds	Name of existing table housing product keys & product names (in worksheet goals etc, *case sensitive*)

Before selecting the VBA control button shown in Figure 6, the instructor can also provide a new list of location and product names. The number of locations and products can differ from that shown in the earlier example student assignment. To make the simulated data more interesting for the honing of students' analytical skills, the instructor can set differing sales growth rates across locations, products, and time periods (not shown in Figure 6). For example, current year sales growth rates for one or more locations or products could be selected to be much lower than the remaining locations or products. The sales data can include a seasonal component if the instructor desires. To vary the setting, an instructor could relabel revenue and products as expenses and expense types (e.g., salaries, travel, entertainment, etc. . .).

The program usually takes a few minutes or less to run. However, several consecutive runs of the program can slow the execution time. Closing and reopening Excel usually solves this issue. If the issue continues after reopening Excel, a system restart is probably needed. The VBA program has been used with Excel 2019 and Excel 365 (two pc versions of Excel). The program has not been used on a MAC.

Conclusion

The development of higher-level Excel skills can improve the job prospects of finance students while also enhancing student problem solving skills. This paper provides a student assignment focused on building intermediate to advanced Excel skills through the construction of dynamic reports and working with semi-big data. Excel features/functions utilized include data validation, table references, SUMIFS, DATEVALUE, EOMONTH, concatenation, VLOOKUP, and IF, among others. The paper also offers a VBA program that generates simulated data for the student assignment, allowing an instructor to more easily update the assignment across semesters. Files and streaming video associated with this paper are available from the article's companion website (*https://sites.google.com/appstate.edu/higher-level-excel-skills*).

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Appendix A
Description of Selected Excel Features and Functions

Excel Feature or Function	Description
Concatenation	Concatenation within Excel is the creation of a text string within a cell that includes text values from other cells. Concatenation can be used to make report titles and column/row headers dynamic.
Conditional Formatting	Excel's conditional formatting feature is a data visualization tool that allows us to apply formatting to a cell or cells dependent on cell values. E.g., we could apply green fill to cells corresponding to a firm's locations with the highest sales growth.
Data Validation	Data validation restricts the values that can be entered into cells to which data validation has been applied. E.g., one could use data validation to limit a cell's value to a region name within a firm. Data validation can be used to create a dropdown list of acceptable values for a cell.
DATEVALUE	Excel's DATEVALUE function converts a calendar date stored as text into a numeric calendar date. When working with dates in Excel, it is usually better for dates to be stored as numbers instead of text (regardless of how formatted).
EOMONTH	The EOMONTH Excel function returns an end of month date value. Inputs are a valid numeric calendar date and number of months from the input calendar date. E.g., =EOMONTH(10/15/2023,-1) returns 9/30/2023.
IF	Excel's IF function performs a logical test (e.g., is C10>D10) and returns one value if true and another value if false.
IFERROR	The IFERROR function is typically used in conjunction with a second function within a cell formula, such as VLOOKUP. IFERROR is a way to handle output errors from the second function. When the second accompanying function returns an error, IFERROR can be used to return an alternative text or number value (such as "missing" or -99) instead of an Excel error such as #NA or #VALUE.
INDEX/MATCH	INDEX/MATCH denotes two Excel functions used together as an alternative to VLOOKUP in order to pull corresponding data as a new column into one dataset from a second dataset.
ISNUMBER	ISNUMBER returns true when the function's input value is numeric. E.g., =ISNUMBER(D50) returns true if D50 contains a numeric value and false otherwise. When used, ISNUMBER is typically appears in combination with another function such as IF.
Pivot Tables	Pivot Tables are a tool in Excel that summarizes data. E.g., potential outputs for sales data include sum by month and product or average order quantity by customer.
ROWS	ROWS returns the number of rows in a range. E.g., =ROWS(E11:E18) returns the value of 8.
SUMIFS	SUMIFS is a conditional summing function. E.g., SUMIFS can be used to sum sales for a specific region and time period from a dataset containing multiple regions and time periods.
Table	A user has the option to designate a range of cells containing data as a table in which each column is a field and each row a record. E.g., for sales data, fields/columns might include date, sales in $s, product sold, geographic area where sold, and customer. A table facilitates the

	referencing of fields within Excel formulas and the addition of new data as it becomes available.
VLOOKUP	VLOOKUP is an Excel function that can be used to pull corresponding data as a new column into one dataset from a second dataset. For example, we may wish to add customer locale to sales data.
Wildcard Character "*"	The asterisk ("*") can serve as a wildcard to substitute for other characters when used in Excel functions such as SUMIFS and VLOOKUP. E.g., to sum across either one or all products depending on an input parameter in cell D5, we could use =SUMIFS(tbSales[Revenue],tbSales[Product],if(D5="All","*",$D$5). In this formula, the SUMIFS sums across all products when D5 = "All", otherwise the SUMIFS sums only for the product name entered in D5.

Appendix B
Student Assignment Mini-Case

Jordan leaned back at his desk and reflected on the morning's events. He had been wrapping up a project for the Finance Team and was looking forward to spending the weekend with friends traveling to a nearby music festival featuring some great bands. A slack message from Cheryl Brand, Finance Team manager, had popped up on Jordan's laptop asking him to see Cheryl in her office, pushing Jordan's thoughts of weekend plans aside.

Just finishing up six months on the job, Jordan had been a quick learner, including developing higher level Excel skills as encouraged by the Finance Team manager. Cheryl very much wanted the Team to boost the efficiency of its work so that her group could respond more quickly to management requests for financial analysis. The Team worked hard and often faced time crunches in responding to top management requests. One way to improve efficiency was to incorporate greater automation into the Team's Excel work, the primary software application used by the Finance Team. However, it had been a challenge for the Team to find time to work on that automation.

When Jordan met with Cheryl this morning, Cheryl asked Jordan to automate six of the sales reports that the Finance Team prepares monthly and shares with upper management along with an analysis in the form of an executive summary. Cheryl directed Jordan to build what she called "dynamic reports" in Excel following the layout of the past sales reports prepared by the Team and using the daily sales data on hand from the recent month end. Sarah, another member of the Finance Team, had pulled together the contents of an Excel file named Monthly Sales Reports that Cheryl gave Jordan. However, Sarah had been pulled to work on another project that was running behind schedule and no longer has time for the dynamic report project. The Excel file contained daily sales data among other things. Cheryl also provided Jordan hard copies of some old monthly sales reports (*similar to Figures 4 through 5 that appear earlier in this paper, but without formula images*) so that Jordan could back test his spreadsheet work. Cheryl asked Jordan to make this project a top priority so that future monthly sales reports and analysis could be delivered to top management more quickly.

Jordan looked forward to putting his Excel skills to good use on this project. Reviewing the content of the Excel file from Cheryl, he noticed several worksheets. In the worksheet "sales," he saw that the daily sales data had columns labeled Date, Location Key, Product Key, and Revenue. So, the sales data has daily sales by location and product. The data took up over 30,000 rows. He recalled from some of his recent training that he could use Excel's filter feature to check out the data and make sure that there were no missing items or unusual values. Jordan also noticed that the daily sales data was in a table named tbSales, which would make formula construction easier. From other Excel work, he knew that Location Key and Product Key were unique identifiers for the company's four locations and seven product lines. He would need to add location name and product name to the sales data from somewhere.

The worksheet "goals etc," had two tables of data. A table named tbGoals contained monthly sales goals for the current year for each location. The tbGoals table includes columns for Location Key, Location (i.e. location name), Month, and Sales Goal ($s). Jordan thought he might be able to add location name to the tbSales data from tbGoals. The second table in the worksheet was named tbProds, which listed the company's seven product lines and had only two columns – Product Key and Product (i.e., product name). Cheryl or Sarah must have deleted columns that Jordan would not need.

The Excel file had six additional worksheets labeled Report 1, Report 2, Report 3, Report 4, Report 5, and Report 6. Each worksheet matched a report that Cheryl had given to him in hard copy form. None of the five worksheets contained any formulas, as Jordan determined by using Home → Find & Select → Formulas.

Tasks

1. Use Excel's filter feature to check the data in each of the three tables for errors and missing values.
2. Add product name and location name as new columns in the table containing the daily sales data. Use formulas.
3. Construct dynamic reports within the Excel file provided for each of the six reports. See the Excel file for additional instructions.
4. Check the accuracy of your dynamic reports by comparing your reports to the old hardcopy reports that Cheryl provided and by using pivot tables to spot check key numbers.
5. Prepare an executive summary analysis of year-to-date sales performance. The executive summary should be no more than one page (standard 8.5 x 11 paper, one inch margins, 11 point Times New Roman font, 1.5 line spacing). Key points can be listed in bullet form. Include a list of questions in need of further analysis.

Appendix C
Student Assignment Dynamic Reports 2, 3, 4, and 6

Select formulas are shown within textboxes (not provided to students) within each report. Each report has dropdown selections beneath Report Parameters that make the report dynamic. The item tbSales within formulas is the table of daily sales by location and product. Revenue, Date, and Location are fields/columns within tbSales. The item tbGoals within formulas is the table of monthly sales goals by location. Sales Goal ($s), Month, and Location are fields/columns within tbGoals. Prior to building the reports, Location (i.e., location name) and Product (i.e., product name) columns were added to tbSales using the VLOOKUP function. Reports 1 and 5 not shown in this Appendix appear in the paper as Figures 1 and 5.

Report 2

Report Parameters

Year	2023
Month	July
Months YTD	7

```
=SUMIFS(tbSales[Revenue],tbSales[Date],"<="&EOMONTH(DATEVALUE($D22&" 10, "&$D20),0),
tbSales[Date],">"&EOMONTH(DATEVALUE($D22&" 10, "&$D20),-$D24),tbSales[Location],D32)
```

```
=SUMIFS(tbGoals[Sales Goal ($s)],tbGoals[Month],
"<="&EOMONTH(DATEVALUE($D22 & " 10, " &
$D20),0),tbGoals[Month],">"&EOMONTH(DATEVALUE($D
22 & " 10, " & $D20),-$D24),tbGoals[Location],G32)
```

Sales YTD vs Goal by Location
For the 7 months ending July 2023

Location	Boston	Chicago	Dallas	New York City	Total
Sales YTD 2023	$14,635,043	$10,763,988	$15,826,124	$18,591,663	$59,816,817
Sales Goal YTD 2023	$14,611,365	$10,618,454	$17,498,569	$18,659,426	$61,387,813
Over (Under) Goal	$23,678	$145,535	-$1,672,445	-$67,763	-$1,570,996
% Over (Under) Goal	0.16%	1.37%	-9.56%	-0.36%	-2.56%

Report 3

Report Parameters

Product	All
Year	2023
Month	July
Months YTD	7

```
="Through " & D23 & " for the Years " & D21-3 &
" through " & D21
```

```
=SUMIFS(tbSales[Revenue],tbSales[Location],$C34,tbSales[D
ate],"<="&EOMONTH(DATEVALUE($D$23&" 10, "&$D$21),-
12),tbSales[Date],">"&EOMONTH(DATEVALUE($D$23&" 10,
"&$D$21),-$D$25-12),
tbSales[Product],IF($D$19="all","*",$D$19))
```

```
=SUMIFS(tbSales[Revenue],tbSales[Location],$C34,tbSal
es[Date],"<="&EOMONTH(DATEVALUE($D$23&" 10,
"&$D$21),0),tbSales[Date],">"&EOMONTH(DATEVALUE(
$D$23&" 10, "&$D$21),-
$D$25),tbSales[Product],IF($D$19="all","*",$D$19))
```

Sales YTD by Location
For All Products
Through July for the Years 2020 through 2023

Location	Sales YTD 2020	Sales YTD 2021	Sales YTD 2022	Sales YTD 2023	% Growth 2021	% Growth 2022	% Growth 2023
Boston	$12,105,828	$13,055,592	$14,185,791	$14,635,043	7.85%	8.66%	3.17%
Chicago	$9,055,565	$9,779,888	$10,309,178	$10,763,988	8.00%	5.41%	4.41%
Dallas	$14,664,261	$15,605,976	$16,988,901	$15,826,124	6.42%	8.86%	-6.84%
New York City	$15,624,036	$16,827,895	$18,115,948	$18,591,663	7.71%	7.65%	2.63%
Total	$51,449,690	$55,269,352	$59,599,818	$59,816,817	7.42%	7.84%	0.36%

Report 4

	C	D
18	Report Parameters	
19	Location	All
20		
21	Year	2023
22		
23	Month	July
24		
25	Months YTD	7

=IF(D19="all","For All Locations","For " & D19)

=SUMIFS(tbSales[Revenue],tbSales[Product],C34,tbSales[Date],"<="&EOMONTH(DATEVALUE($D$23&" 10, "&$D$21),-24),tbSales[Date],">"&EOMONTH(DATEVALUE($D$23&" 10, "&$D$21),-$D$25-24),tbSales[Location],IF($D$19="all","*",$D$19))

=SUMIFS(tbSales[Revenue],tbSales[Product],C34,tbSales[Date],"<="&EOMONTH (DATEVALUE($D$23&" 10, "&$D$21),0), tbSales[Date],">"&EOMONTH(DATEVALUE($D$23&" 10, "&$D$21),-$D$25), tbSales[Location],IF($D$19="all","*",$D$19))

28 **Sales YTD by Product**

29 For All Locations

30 Through July for the Years 2020 through 2023

="Sales YTD " & $D$21 - 4 + COLUMNS($D$32:D32)

	Products	Sales YTD 2020	Sales YTD 2021	Sales YTD 2022	Sales YTD 2023		% Growth 2021	% Growth 2022	% Growth 2023
34	Accessories	$5,271,279	$5,740,530	$6,296,464	$6,184,813		8.90%	9.68%	-1.77%
35	Brakes	$5,522,621	$5,989,635	$6,543,685	$7,179,734		8.46%	9.25%	9.72%
36	Frames	$8,470,414	$8,730,605	$9,061,862	$8,355,598		3.07%	3.79%	-7.79%
37	Gears	$11,481,872	$12,509,058	$13,717,695	$13,754,977		8.95%	9.66%	0.27%
38	Seats	$9,075,402	$9,603,601	$10,203,055	$11,270,579		5.82%	6.24%	10.46%
39	Speciality Gears	$5,526,847	$6,095,385	$6,715,137	$6,706,352		10.29%	10.17%	-0.13%
40	Tire Rims	$6,101,257	$6,600,537	$7,061,920	$6,364,764		8.18%	6.99%	-9.87%
41	Total	$51,449,690	$55,269,352	$59,599,818	$59,816,817		7.42%	7.84%	0.36%

Report 6

	C	D
18	Report Parameters	
19	Locations	All
20		
21	Product	All
22		
23	Year	2023
24		
25	Month	July
26		
27	Months YTD	7

=IF(ISNUMBER(C37),SUMIFS(tbSales[Revenue],tbSales[Location],IF($D$19=" All","*",$D$19),tbSales[Product],IF($D$21="all","*",$D$21),tbSales[Date],"< ="& EOMONTH(C37,-12),tbSales[Date],">"&EOMONTH(C37,-1-12)),"")

=IF(ISNUMBER(C37),SUMIFS(tbSales[Revenue],tbSales[Location],IF($D$19=" All","*",$D$19),tbSales[Product],IF($D$21="all","*",$D$21),tbSales[Date],"< ="& EOMONTH(C37,0),tbSales[Date],">"&EOMONTH(C37,-1)),"")

=IF(ISNUMBER(C37),SUM($E$37:E37),"")

30 **Sales by Month Compared to Prior Year**

31 For All Locations

32 For All Products

33 Through July for the Years 2022 and 2023

	Month	Sales 2022	Sales 2023	% Change		Sales YTD 2022	Sales YTD 2023	% Change
37	Jan	$5,112,789	$5,148,373	0.70%		$5,112,789	$5,148,373	0.70%
38	Feb	$5,360,288	$5,423,761	1.18%		$10,473,077	$10,572,134	0.95%
39	3/31/2023	$7,265,578	$7,208,027	-0.79%		$17,738,654	$17,780,161	0.23%
40	Apr	$8,792,331	$9,229,630	4.97%		$26,530,985	$27,009,791	1.80%
41	May	$11,934,634	$11,994,842	0.50%		$38,465,620	$39,004,633	1.40%
42	Jun	$11,229,685	$11,385,461	1.39%		$49,695,304	$50,390,094	1.40%
43	Jul	$9,904,514	$9,426,724	-4.82%		$59,599,818	$59,816,817	0.36%

=IF(ROWS($C$37:C37)<=$D$27,EOMONTH(DATEVALUE($D$25 & " 10, " & $D$23),-$D$27+ROWS($C$37:C37)),"")

=IF(ISNUMBER(C48),I48/H48-1,"")

Incorporating Acquisition Entrepreneurship into the Finance Curriculum

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While entrepreneurship is becoming more popular and integrated in business education, one niche area that could also be more widely introduced is Acquisition Entrepreneurship (AE). Simply put, AE is becoming an entrepreneur by way of purchasing an already functioning business, rather than by founding a startup. Prior literature finds that the number of AE deals has increased fourfold over the past decade, largely driven by baby boomers nearing retirement and looking to sell their businesses. This article describes AE and the value of including it in finance curricula. Given the increased focus of experiential learning in business education along with the rapidly evolving student demographics and economic climate, AE is an area that would be worthwhile for finance departments and business schools to include in their curriculum.

Keywords: Acquisition entrepreneurship, entrepreneurship through acquisition, valuation, experiential learning

Introduction

The field of entrepreneurship has received increasingly greater attention over the past four to five decades (Jones and Wadwhani, 2006). Correspondingly, many business schools have introduced it in the business curriculum as a major and/or minor, with some requiring entrepreneurship as a core course for all business majors (Morris, Kuratko, and Cornwall, 2013). Generally, entrepreneurship studies focus on the tools needed for developing a new idea and launching a new business. Yet one widely established drawback of entrepreneurship is the low success rate for start-ups, with an estimated failure rate of over 90% (Patel, 2015). Further, Deibel (2018) argues that Acquisition Entrepreneurship (AE), which means becoming an entrepreneur by way of buying an already-established business with a proven cash flow rather than starting a business from scratch, can mitigate some of the inherent failure risk associated with starting a new business.

In recent years, AE has gained more attention in industry and academia. Although traditional entrepreneurship is more widely recognized, AE has existed across a range of industries and sizes. For instance, although most people know of Elon Musk as the inventor of Tesla, he is not an original founder of Tesla. He became a major investor in Tesla after his startup X.com failed (Kolodny and Black, 2021). Another less well-known example is Jennifer Braus, owner and CEO of Systems Design West. Her firm provides billing services to fire departments and municipalities offering emergency medical services in the Pacific Northwest region of the United States (Ruback and Yudkoff, 2017). Systems Design West has existed since 1989 and Jennifer Braus became the owner and CEO in 2015, just two years after earning her MBA.

AE has gained more traction in recent years most likely due to the evolving demographic landscape, shifts in the workforce, and the volatile economic climate. First, as Baby Boomers are nearing retirement, many are looking to sell their businesses. Baby boomers currently own over 2.3 million businesses in the United States. A majority of them have no succession plan, and in many cases, their children or grandchildren may not be interested in taking over the business. The COVID-19 pandemic has only exacerbated the surge of Baby Boomers' retirements (Schroeder, 2020). The large supply of businesses for sale creates a highly desirable market for AE deals (Sherman, 2019). Second, for those retiring business owners with heirs who *would* like to take over the business, many need assistance in succession planning. The future generation of soon-to-be acquirers can also benefit from formal education on the unique challenges and strategies that come with the transition. Not surprisingly, more business schools are offering some form of family business education (Sharma, Hoy, Astrachan, and Koiranen, 2007). Third, the United States is currently undergoing the biggest wealth transfer in history. It is estimated that millennials are inheriting over $68 trillion in wealth (Kelly, 2019). Therefore, many millennials are well-positioned to invest in these deals. All of these elements, along with the significant rise in investment vehicles targeted at Acquisition Entrepreneurship deals, creates a ripe environment for the rapidly growing AE market.

Coverage of AE would fit nicely within the finance curriculum given the heavy focus on firm valuation. While there is a dearth of scholarly writing on AE, Crabbs (1980) proposes a Small Business Management experiential learning course in response to students' increased interest in small business entrepreneurship. This article builds on that motivation as well as the growing industry trend. To my knowledge, there are no existing articles in the pedagogical literature that discuss the benefits of adding AE to the finance curriculum.

The remainder of this article provides a literature review of AE, an overview of the AE process, and a discussion on why it may be useful to introduce AE in the finance curriculum.

Literature Review

While AE is not new, the academic literature is very limited most likely due to data scarcity. However, a study on search funds by Stanford University's Center for Entrepreneurial Studies (CES) has provided useful insights since 1996 in its comprehensive study on search funds, which are the investment vehicle that is often used by aspiring acquisition entrepreneurs to fund AE deals. The term "search fund" was coined at Harvard Business School in 1984. Later, the term became more widespread at Stanford and is now part of the AE verbiage (Kelly and Heston, 2022). In the Stanford CES Search Fund Study, Kelly and Heston (2022) define search funds as "an entrepreneurial path undertaken by one or two individuals who form an investment vehicle with a small group of aligned investors to search for, acquire, and lead a privately held company for the medium to long term, typically six to ten years but at times shorter or longer." The study provides insights on emerging search funds trends, characteristics of first-time acquisition entrepreneurs using search funds, and the factors that impact deals for first-time acquisition entrepreneurs and investors in the United States and Canada.

From 1986-2021, over $2.3 billion has been invested in search funds to finance AE deals. Studies estimate an internal rate of return of 35-37% (Hunt and Fund, 2012; Kelly and Heston, 2022). In 2021 and 2022, the most common industries of acquired companies were services, software, tech-enabled services, and healthcare (Kelly and Heston, 2022).

While AE may help mitigate some of the risk associated with founding a company, there are also some unique risks associated AE. For example, undertaking an already existing business with an established operation and culture could actually be a challenge for the new owner. The due diligence process will likely reveal many aspects of the target company's operations and culture to help determine whether it will be a good fit; 75% of searches end with positive due diligence, yet there are still surprises (Ruback and Yudkoff, 2017). However, there may be some new unforeseen challenges for the acquirer. For instance, some of the company's prior success may have been partially due to the loyal employee or customer base. New ownership might disrupt this and the acquirer may experience an abrupt shift in operations or revenue that was not completely captured in the due diligence process. In addition, it is crucial to not overpay for the target company. Thus, valuation skills learned in finance, along with the guidance of skilled advisors, are essential to successfully evaluate and estimate past and future cash flows.

A natural question that may arise is who are the successful acquisition entrepreneurs? Diebel (2018) suggests that the strongest predictor of being a successful acquisition entrepreneur is having a growth mindset. A growth mindset could be summarized as embracing challenges, seeking opportunities for improvement, and persevering in times of adversity. On the other hand, someone with a fixed mindset accepts things just as they are without considering change or improvement. Fortunately, there are strategies on how to cultivate a growth mindset (Dweck, 2006).

Further, Deibel (2018) elaborates on the importance of having the right mix of attitude, aptitude, and action to be a successful acquisition entrepreneur. Understanding one's personal mix of the "three A's" should guide the search rather than first determining the target company's industry. Successful acquisition entrepreneurs should first assess their personal (or team's) attitude, aptitude, and action, and to align that with the target company opportunity, size, and industry. "Attitude" refers to adopting the growth mindset (Dweck, 2006). "Aptitude" refers to intelligence and competencies. Effective leaders understand their strengths and hire team members to whom they can delegate everything else (Deibel, 2018). "Action" refers to the willingness to take action with imperfect information. In this arena, perfectionism is a negative trait because successful entrepreneurs must be comfortable taking on calculated risks.

Further, acquisition entrepreneurs come from a diversity of professional backgrounds. The largest group (37%) have previous professional experience in private equity, investment banking, and finance, thereby aligning with this article's aim of incorporating AE into the finance curriculum, which provides the needed transferable skills. The next largest groups in terms of previous professional experience are Management Consulting (15%), and General Management (16%) (Kelly and Heston, 2022).

The Stanford CES study also provides demographic data on first-time acquisition entrepreneurs who used a search fund. The percentage of female acquisition entrepreneurs is steadily rising from 0% in the early 2000's now up to 13% in 2022. At the start of their search, 24% of the acquisition entrepreneurs were under 30 years old, 50% between 30-35 years old, and 22% between 36-40 years old. Fifty-five percent of acquisition entrepreneurs initiate their search for a company to acquire within three years of obtaining their MBA. While one might assume that previous experience is a strong predictor of successful AE, 74% of first-time searchers are 35 years old or younger and a majority commence the process soon after obtaining their MBA (Kelly and Heston, 2022). Thus, the limited experience of new graduates does not have to be a limiting factor to become a successful acquisition entrepreneur.

The Acquisition Entrepreneurship Process

While the AE process may not be familiar to many finance educators, many of the steps described are similar to traditional merger and acquisition (M&A) deals with some unique AE considerations. The search-and-acquisition process can be very lengthy and complex, involving many stakeholders such as brokers, accountants, and lawyers. Ruback and Yudkoff (2017) recommend budgeting for at least six months up to two years for a successful search.

One of the important early steps is to determine the source of financing. While search funds are a popular vehicle, many entrepreneurs self-finance or tap into their close networks for equity and/or debt. The source of financing will likely influence the size and type of business an acquisition entrepreneur can consider. For instance, debt financing may not be as available to acquire certain types of firms, such as firms in the technology industry, which usually have lower leverage. If working with a search fund, the investors may have certain parameters that guide the entrepreneur's search. In addition, entrepreneurs are strongly encouraged to engage legal counsel to ensure that the fundraising strategies and financing terms are appropriate and legal. During this stage, many search fund principals solicit investors with financial capital as well as intellectual and social capital, making them valuable advisors throughout the different phases on the process (Kelly and Heston, 2022).

Next, the search begins with the imperative step of developing the target statement to guide the search process. It is prudent for the acquisition entrepreneur to set a focus before beginning the search; otherwise, the process can be overwhelming and costly. In fact, the search is the lengthiest part of the process, with the median time for the search alone to be 23 months. Several factors influence the search such as the economic climate, the industry of the target firm, and any legal issues arising from the potential acquisition (Kelly and Heston, 2022). A well-developed target statement considers the opportunity/growth profile, size, and industry of the target company, thereby allowing the aspiring acquirer(s) to effectively communicate what they are looking for. The acquirer(s) should consider their own characteristics in developing the target statement to optimize how they will be working within the newly acquired firm. For instance, an aspiring acquisition entrepreneur with strong technology skills may consider a healthy firm that can greatly benefit from introducing up-to-date technology. One suggestion to formalize the self-evaluation process is to conduct a personal analysis of strengths, weaknesses, opportunities, and threats (SWOT) for the individual or team of searchers (Diebel, 2018).

Many acquisition entrepreneurs work with brokers to help identify potential target companies. As opportunities are presented, a thorough due diligence is conducted. Similar to M&A deals, the due diligence process requires learning as much as possible about the target company such as cash flows, legal issues, employees, physical equipment, patents, and more, typically with the assistance of an attorney and other professionals (Ruback and Yudkoff, 2017).

If all checks out in the due diligence stage and the acquirer(s) choose to move forward, one of the essential determinants of a successful deal is for the acquirer is to agree on a fair price for the target company. The required valuation skills for this step give finance majors a significant advantage. Valuing the deal requires determining potential synergies and the value-added. Even if finance students are not interested in becoming acquisition entrepreneurs, finance educators still have the opportunity to introduce one more area where these skills are put to use. Further, in my limited experience of introducing AE in an existing finance graduate course as a pilot, several students drew on their own experiences with a personal acquaintance who owns and/or manages a business, making it a more enriching learning experience.

Finally, if the acquirer and target agree to move forward with the deal, both parties will structure and close the deal. Depending on the deal arrangement, the former owner might even be retained with an incentive plan for a period of time. In AE, it's not uncommon for there to be a transitionary period after the deal closes that involves hands-on training from the former owner to the new owner.

A legitimate concern with AE is that the cost to purchase a business is prohibitive, but it is not as prohibitive as one might assume. Even if an interested student does not have all the financial, intellectual, and social capital necessary, various options exist to help them acquire a business. For example, Yale alumni Judd Lorson, who acquired a revenue management firm, did so by partnering with a search fund. Further, businesses can be financed similarly to a home. For simplicity's sake, say a business is valued at $1,000,000 or $500,000. It can be purchased with a $100,000 down payment (putting down 10% or 20%, respectively). The revenue that the business is already generating can be used to meet the loan obligations. Thus, there are different solutions to overcome the financial concerns of purchasing a business.

In addition, a lack of entrepreneurial and management experience may also be a deterrent for an inexperienced new graduate to pursue AE. Thus, one advantage of acquiring an already existing business is that, in many cases, the deal may be structured for the selling owner to offer hands-on training to the new owner as previously mentioned. In addition, taking on a new business with skilled and knowledgeable employees, as opposed to hiring all new employees, might alleviate some of the pressures that come with the transition period for the new owner.

Ruback and Yudkoff (2017) recommend that searchers focus on high-quality target companies with steady cash flows of $750,000 to $3 million. These companies are likely not exciting businesses in new, emerging industries, yet the authors consider them "enduringly profitable" (i.e. billing management, snow plowing, home health care, manufacturing). The competitive advantage of the acquisition entrepreneur is to grow the business by introducing the latest best management practices that the sellers are not willing and/or able implement.

Motivations to Incorporate AE in the Finance Curriculum

A few universities have already incorporated AE into the graduate business curriculum. For example, the Stanford University Center for Entrepreneurship collects and disseminates data on search funds, popular investment vehicle for acquisition entrepreneurs, and hosts conferences. The University of Chicago Center for Entrepreneurship and Innovation offers resources for students and alumni to access education, capital and mentorship in the form of clubs, funds, podcasts, conferences, and more. In addition, The Wharton School of the University of Pennsylvania, Massachusetts Institute of Technology Sloan School of Management, and Harvard Business School offer at least one MBA course and/or student clubs around AE. These programs offer hands-on, experiential, practical learning opportunities. For example, Yale alumni Judd Lorson knew that he was interested in acquiring a business upon graduation. With the guidance and support of Search Fund Accelerator, he acquired a South Florida revenue management firm with over 20 employees (Wasserstein, 2018).

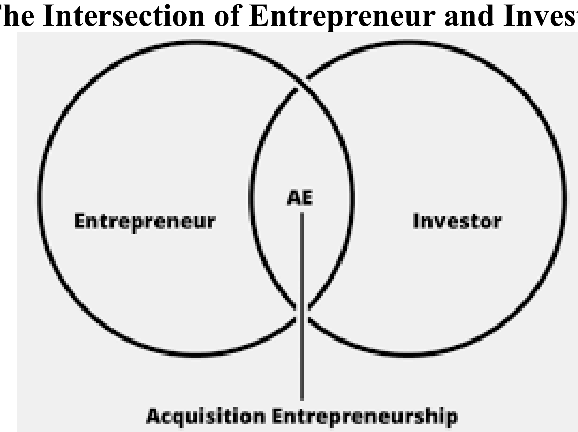
The examples provided are based on publicly available and accessible course and program offerings. There may be other institutions that also offer AE that are not captured in this article. While the motivation to launch these programs, clubs, and/or courses is not made available, it is fair to assume that similarly to the other common business disciplines offered, there is some student demand for this subject. For instance, Kerr (1973) documents that students criticized the lack of entrepreneurship coverage, as there used to be just a few institutions that offered entrepreneurship. Fast forward many years later, and it is a common discipline offered at many business schools. It may also be the case that some institutions have considered AE but then determined that offering it is not feasible or a good idea. However, anecdotal evidence seems to suggest that academia may not have caught on to this rapidly growing industry trend.

Understandably, most schools that offer courses in AE are through the Management and/or Entrepreneurship department. Entrepreneurship is an interdisciplinary area that touches on all areas of business. But a major focus of AE is cash flow valuation. Finding the right business and paying the right amount for that business is one of the most crucial steps of the AE process. In addition to understanding the different elements of successfully running and growing a business, the acquisition entrepreneur is also an investor that needs to understand the fair value of the business they are purchasing by estimating the present value of the expected future cash flows. AE is one more area in which finance professors can illustrate the relevancy of the fundamental valuation principles we teach. While there are several potential advantages, I will focus on three reasons that are important to educators, students, and business schools.

Leveraging the Curriculum

Leveraging the curriculum is an important advantage of including AE in the finance curriculum. As Figure 1 illustrates, successful AE heavily depends on two key components: (1) managing and growing the newly acquired business as an entrepreneur; and (2) purchasing the business at a fair price as an investor. Any entrepreneur needs at least some base level knowledge of the different business disciplines, including financial management, as well as collaboration with skilled professionals and advisors. Teaching this broad range of skills goes beyond the finance curriculum and could possibly lend itself to interdisciplinary curriculum opportunities.

Figure 1
The Intersection of Entrepreneur and Investor



Source: Deibel (2018)

For the second component, AE draws on the concepts that finance educators already teach but extends them to a new context. In most introductory finance courses, valuation is introduced to all students as the time value of money. At different points, students are taught several methods of valuation such as net present value (NPV), discounted cash flow (DCF), free cash flow analysis (FCFF), multiples, and more. As undergraduate or graduate finance students advance through their courses, valuation is a recurring theme in specialized, elective courses such as Advanced Corporate Finance or Investment Banking. Thus, the valuation pillar of AE is not new to the finance curriculum. With sound due diligence, training, and advisors, it seems reasonable that an acquisition entrepreneur with a strong finance background can increase their chances of success.

Not surprisingly, a majority of first-time acquisition entrepreneur searchers have some experience in finance (Kelly and Heston, 2022).

Interestingly, over one-third of first-time acquisition entrepreneurs started their entrepreneurial journey in the classroom from taking an AE course (Kelly and Heston, 2022). Exposure is vital for students. The interdisciplinary nature of AE draws on many business skills and requires some base level understanding of the core business subjects. Therefore, it may be more valuable to introduce the subject later in students' matriculation. A key component of AE is buying the business for the right price on the front end. This component directly relates to the valuation principles that finance students cover throughout their studies. Typically, in their first introductory-level finance course, students learn the concept of NPV. Valuation is continually covered throughout the curriculum in corporate finance and investments courses and then in special topics elective courses such as Mergers and Acquisitions, Investment Banking, Advanced Corporate Finance, and more. The same valuation principles already covered in other courses can be extended to introduce AE in the curriculum. Essentially, students will be learning the same concepts in a different context. For instance, the role of a search fund is similar to what finance educators already teach on private equity, but search funds specifically help acquisition entrepreneurs acquire and grow a business.

Business schools can choose the extent to which they are willing and able to incorporate AE, such as a center, extracurricular clubs, conferences, new courses, or as part of an already existing course. Even if the resources and interest exist to implement a new course or center, the governance process may delay initiation, so considering different options offers flexibility. While there are already models of AE courses from universities such as MIT, Wharton, Harvard, and Stanford, there are a variety of ways to implement this subject into an already existing course. Faculty who are interested in learning more about how to best implement this subject will likely find it useful to refer to the courses at these universities as a starting guide (some universities use the term "Entrepreneurship through Acquisition"). The purpose of this paper is to discuss the value of including AE in finance curricula.

Increasing Student Opportunities

Next, learning AE is aligned with many students' expectations, demands, and motivations for pursuing a business degree. According to Kim, Markham, and Cangelosi (2002), the top five reasons students choose to major in business (in order of ranking) are (1) interest to pursue a career associated with the major, (2) strong job opportunities, (3) required abilities match their strengths, (4) desire to run a business someday, and (5) projected earnings in the related career. In a more recent survey of the class of 2023 conducted by Handshake, an online platform that connects students and employers partnered with over 1,400 educational institutions, job stability and high salary are the equally top reasons why students consider a job (Handshake, 2022). As the job market becomes increasingly competitive, AE offers another career avenue for graduates to transition into a senior role with a salary at an already established company.

Further, the idea of "lifetime learners" is being encouraged much sooner in students' educational pursuits (Vanhonacker, 2021). Business schools can capitalize on this and create an environment in which students are encouraged to continually return to the university after graduation to further their education. A 2015 Bureau of Labor Statistics survey finds that, on average, "young baby boomers" (born 1958-1964) changed jobs 11.7 times in their lifetime (McKay, 2020). However, a recent Gallup report finds that millennials (born 1980-1996) have changed jobs at a rate three times *higher* than non-millennials (including all baby boomers and Gen X) in the past year alone. Relative to their non-millennial counterparts, millennials are less willing to stay at their current jobs, more open to other job opportunities, and less engaged in the workplace (Adkins, n.d.). This increase in job-hopping may be driven by factors such as changes in interests, the economic climate, and/or cultural expectations. Hence, even if a student were not interested in pursuing AE during their formal studies, exposing them early on may bring them back to their alma mater to tap into resources that can support their AE ventures.

Meeting Business School Goals

Finally, in addition to the pedagogical benefits, finance professors and business schools should consider incorporating AE into the finance curriculum for several reasons, many of which align with the values promoted by business schools and accrediting bodies such as the Association of Advance Collegiate Schools of Business (AACSB). AE creates another opportunity to garner the alumni base and to strengthen relationships, which is an ongoing challenge for many universities. Seventy-two percent of higher education institutions think they need to do more to attract and engage young alumni, and only 9% think that are "doing well" in this regard. Most alumni organizations admit that they struggle to engage the young alumni, and a majority say that their top short-term goal is to increase alumni engagement (Toyn, 2020). Successful AE graduates will likely be in a position to contribute back to the university sooner than the typical graduate, such as Judd Larson who is actively involved at Yale. In addition, AE offers an opportunity to get more industry stakeholders such as accountants, lenders, and lawyers involved, giving business schools more opportunities to engage with industry.

AE promotes the interdisciplinary approach that business schools continue to strive towards (Charroin, 2022). Based on the traditional structure of departmentalized business schools, it is often challenging to find opportunities for interdisciplinary collaboration. While the valuation component of AE is very important to selecting the right target firm to purchase, the aspiring entrepreneur must consider many other aspects, such as technological innovations, marketing, human resource, and management. The acquisition entrepreneur must combine their finance knowledge along with all of these areas for a successful outcome. Incorporating interdisciplinary work in the classroom as much as possible is important to equip students for how business decisions are actually made. The chief financial officer (CFO) of a corporation works alongside many stakeholders and considers different facets of the business to reach the optimal financial decisions. AE provides an avenue for students to get practice in interdisciplinary decision-making.

Pilot

Because of these opportunities, I was motivated to run a pilot at my university in which I incorporated an AE series into an existing Investment Banking seminar of upper-level finance graduate students. The series kicked off with an introduction of AE and discussions of the process. In addition, we reviewed a case study to understand the challenges that can arise and to brainstorm solutions. Finally, students formed into teams in which they identified their strengths, weaknesses, and wherewithal (using fictitious financial profiles), developed a focused target statement, and searched for potential target companies through online small-business marketplaces such as BizBuySell and LoopNet. The series concluded with a team pitch on their target statement, the selected target company, and their plans to grow the business.

While one course is an insufficient sample size to draw any conclusions, there is some promise that, with exposure to the formal AE process, more students (including those that were not initially interested in any form of entrepreneurship) will want to learn more about this field. In a short survey administered at the end of the AE series, an overwhelming majority of students said that they enjoyed this segment of the course and that this is the first time they were ever introduced to AE. Over 50% of the students responded that they would now consider dabbling into AE at some point in their lives.

Conclusion

As the business education landscape continues to evolve, it is imperative that business schools ensure that the curriculum offered remains relevant for students and encourages lifelong learning. Introducing Acquisition Entrepreneurship in the finance curriculum is one way to promote this. In the traditional finance curriculum, students already learn many of the tools required to become acquisition entrepreneurs. Given the changing demographics, workforce, and economic climate, there are probably more students well-positioned to engage in Acquisition Entrepreneurship than even they may realize. The best way I can capture the impact of early exposure to this area is by quoting a student's survey response to an open-ended question: "After seeing some of the businesses available for purchase[,] it let me know that [owning] such a business is within reach. I wanted to actually purchase the business [my team] chose…"

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On Multiple IRRs and Extending the Valid Interval to Include Post-Audits

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Discussions of the internal rate of return (IRR) and the possibility of multiple rates is usually confined to non-negative IRRs. This is justifiable ex ante since a firm must invest in projects that earn a return in excess of its cost of capital to stay in business. However, ex post, when all cash flows are realized and known, it is possible that a project has earned a negative return. To gauge the extent of loss in post audits, this paper advocates an extension of the valid interval for IRRs to (–1, ∞) and examines sufficiency conditions for IRRs to be in the interval.

Keywords: Multiple IRRs, Post-Audit, Valid Interval, Sufficiency Conditions

Introduction

Most research on the internal rate of return (IRR) and almost all textbook expositions of it focus on IRRs in the interval [0, ∞) and, for decision-making purposes, on whether IRR > k, the firm's cost of capital. For example, Norstrom (1972) establishes that for cash flows $CF_0 \ldots CF_n$, a sufficiency condition for IRR in [0, ∞), is that:

$$\boldsymbol{CF_0 < 0}, \sum_{t=1}^{t=\tau} \boldsymbol{CF_t < 0}, \sum_{t=\tau+1}^{t=n-1} \boldsymbol{CF_t > 0}, \text{ and } \boldsymbol{CF_n > 0}$$

Bernhard (1979) generalizes this condition further to include situations that the Norstrom condition fails to detect. The focus on the interval [0, ∞) is economically justifiable given that the firm's cost of capital is always a positive number and managers maximizing shareholder value should seek a return in excess of it. But such a perspective is based on estimates made *ex ante*, on expectations of future cash flows, and sidelines the fact that estimates can be wrong. Firms are cognizant of this view and many carry out audits *ex post* when all cash flows are realized and it can be determined whether or not a project was profitable. However, as Soares, Coutinho, and Martins (2007) point out, post-audit research is scarce primarily because firms are reluctant to release such data despite the obvious benefits to future forecasts of learning from past errors.

With both the *ex ante* and *ex post* perspectives in mind, and the possibility that in the post audit of realized cash flows the firm may discover that it has earned a negative rate of return – that is, in the interval (–1, 0) – it is justifiable that the range of economically meaningful IRRs be extended to (–1, ∞). The objective of this paper is to make such an extension. In proceeding, I discuss some of the mathematics behind the IRR and examine sufficiency conditions for uniqueness of the IRR in (–1, ∞). For clarification purposes, I use the term "IRR" only in reference to the real-valued root or roots.

Textbooks define conventional cash flows as those which have only one sign change. That is, cash flows begin negative, turn positive and stay positive until the project's end, or begin positive, turn negative and stay negative. Alternatively, projects with negative initial cash flows that then turn positive are defined as investing projects, while those with initial positive cash flows that later turn negative are defined as borrowing projects. Citing Descartes' rule of signs, textbooks then typically state that multiple IRRs – multiple real-valued roots – can arise when there are two or more sign changes in the cash flow stream. While this statement is correct in the sense that multiple IRRs can and do occur when there are multiple cash flow sign changes, the impression that it leaves on students – that there are indeed multiple IRRs whenever there is more than one sign change in the cash flow stream – is not. Descartes' rule, as typically stated, serves merely as an upper bound on positive roots. Panton (1999) illustrates this very well by showing that the polynomial stated in cash flow terms:

$$CF_0 + \frac{CF_1}{(1+r)^1} + \cdots + \frac{CF_{n-1}}{(1+r)^{n-1}} + \frac{CF_n}{(1+r)^n} = 0, \quad (1)$$

with roots 1/(1+r) does not generally have the same coefficients or number of sign changes as the polynomial restated with r as roots. That is, when the original cash flow equation is multiplied out and expanded to:

$$CF_0(r)^n + C_1(r)^{n-1} \ldots + C_{n-1}(r) + C_n = 0, \quad (2)$$

only CF_0 generally remains unchanged as a coefficient, and signs on the C_i are no longer necessarily the same as the signs on the CF_i. But Panton (1999) is focused on the presence and location of IRRs in [0, ∞). In order to extend the interval to (–1, ∞), I briefly address some of the math behind the IRR.

Math Background

According to the Fundamental Theorem of Algebra, the polynomial equation of degree n:

$$a_0 + a_1x + a_2x^2 + \ldots + a_{n-1}x^{n-1} + a_nx^n = 0,\ a_n \neq 0,$$

must have n roots. The roots need not all be unique, and could be real-valued or complex-valued. Further, if the coefficients a_i are real-valued, the complex roots, if they occur, always occur in conjugate pairs. In the case of the standard setup in finance, equation (1) above, or the transformed equation (2), the coefficients are always real-valued with the result that any complex-valued roots continue to occur in conjugate pairs.

If the initial and final cash flows have different signs, Noronha and Noronha (2021) show that this is a sufficient condition for at least one real-valued root (IRR) to exist in the interval (–1, ∞). Thus, with an initial cash outflow and final cash inflow, or initial inflow and final outflow, when n is odd – for example, a seven-period situation with eight cash flows – there will always be at least one real-valued root (IRR) in (–1, ∞), regardless of the number of sign changes in the original cash flow equation or in the transformed equation. When n is even, given the assumption of different signs on initial and final cash flows and, consequently, the impossibility of all complex-valued roots, there must be at least two IRRs. Thus, even in cases where cash flows are conventional, if n is even there will be two IRRs at a minimum. What usually happens in such cases is that one IRR falls in (–1, ∞), with other IRRs < -1 implying a loss of more than the total investment amount and thus not economically meaningful.

In situations where the initial and final cash flows have the same sign, the situation is less clear-cut. For n odd, at least one IRR will always exist, but it is not guaranteed to fall in (–1, ∞). For n even, it is possible that no IRR exists, or that an even number of IRRs does. In textbooks, such examples are those pertaining to a strip mining project or a nuclear plant project where there are initial outflows, intermediate inflows and final clean-up outflows required by environmental regulations. Also, when one encounters an example of cash flows where "no IRR exists," it should be clear that a) no real-valued roots exist and, b) the example must have an even number of periods.

But more than simple existence of the IRR in (–1, ∞), we are interested in conditions for the *uniqueness* of the IRR in (–1, ∞). Expanding on Soper's (1959) sufficiency condition for the IRR to be unique in (–1, ∞), Bernhard (1977) shows that the condition is equivalent to the unrecovered amount in the project never changing sign. That is, if CF_0 is the initial investment (or amount borrowed in case of a borrowing project), then the unrecovered amount (*UA*) in the project is defined as follows:

$$UA_0 = -CF_0,$$

$$UA_t = UAt_{-1}(1+IRR) - CF_t,\ t=1,2\ldots n$$

The sufficiency condition for a unique IRR in (–1, ∞) is then equivalent to having $UA_t > 0$ for $t=1,2\ldots n-1$, in the case of a lending (investing) project, or $UA_t < 0$ in case of a borrowing project. Note that $UA_n=0$.

Three Examples

Example 1

Consider the following six-period cash flow stream:

CF_0	CF_1	CF_2	CF_3	CF_4	CF_5	CF_6
–8	6	–1	7	–2	6	–2

There are six sign changes in the cash flows. Further, initial and final cash flows have the same sign, so upon first examination, neither an IRR in (–1, ∞) nor, indeed an IRR, is guaranteed. Following Panton (1999), expansion of the cash flow equation in terms of r gives:

$$8r^6 + 42r^5 + 91r^4 + 97r^3 + 47r^2 - r - 6 = 0$$

Under Descartes' rule of signs, with one sign change in the transformed equation, there can be at most one positive IRR. But, because n is even, if there are IRRs, there must be at least two. There are indeed two IRRs at 0.277 and –0.672, only one of which > 0. The IRR at 27.7% is easily located using Excel. Mathematica was used to locate the IRR at –0.672. It is easy to verify that, because the undiscounted cash flows sum to 6, the project return must be positive, so even though there are two economically meaningful IRRs, 27.7% must be the correct one.

Example 2

Now consider the effect of changing the time 6 cash flow from negative to positive:

CF_0	CF_1	CF_2	CF_3	CF_4	CF_5	CF_6
–8	6	–1	7	–2	6	2

There are five sign changes in cash flows, but now the initial and final cash flows have different signs so an IRR in (–1, ∞) is assured. Since the undiscounted cash flows sum to 10, the project return must be positive. Also, Descartes' rule of signs indicates one positive IRR at most. The expanded equation is:

$$8r^6 + 42r^5 + 91r^4 + 97r^3 + 47r^2 - r - 10 = 0$$

There are two IRRs at 0.336 (found using Excel) and –1.279 (found using Mathematica), with only one falling in the interval (–1, ∞).

Example 3

Soares, Coutinho, and Martins (2007) find that, in their sample, sales forecasts are overly optimistic, leading to negative forecast errors when compared to realized figures. Consider the following realized cash flows on a four-period investment project that has ended, was unprofitable, and is being post-audited.

CF_0	CF_1	CF_2	CF_3	CF_4
–12	8	–6	10	–1

The undiscounted cash flows sum to –1, so it is clear that the project both, failed to earn the cost of capital, and lost money. The expanded equation for the project is

$$12r^4 + 40r^3 + 54r^2 + 26r + 1 = 0$$

Since there is no sign change, Descartes' rule indicates no positive IRR, though this is also clear from the fact that with a positive IRR the sum to zero is impossible. However, negative IRRs are not ruled out and, if they exist, there must be at least two since n is even. There are two negative IRRs at –0.042 (using Excel) and –0.894 (using Mathematica), both in (–1, ∞). In such cases the least disputable conclusion is that the project lost at least 4.2% per period.

Even if one lacks programs like Mathematica to find multiple roots, on finding the first root using Excel one can use Bernhard's (1977) condition to verify uniqueness of the IRR in (–1, ∞). Below, I show that this condition is met in the case of *Example 2*, where all unrecovered amounts in periods 1…n-1 are positive, but not in *Examples 1* or *3*.

***Example 1* (IRR=0.277)**				***Example 2* (IRR=0.336)**				***Example 3* (IRR=–0.042)**		
Time	Cash Flow	Unrecovered Amount		Time	Cash Flow	Unrecovered Amount		Time	Cash Flow	Unrecovered Amount
0	–8	8		0	–8	8		0	–12	12
1	6	4.216		1	6	4.688		1	8	3.496
2	–1	6.384		2	–1	7.263		2	–6	9.349
3	7	1.152		3	7	2.704		3	10	–1.043
4	–2	3.471		4	–2	5.612		4	–1	0
5	6	–1.567		5	6	1.498				
6	–2	0		6	2	0				

Conclusion

A non-negative interval for IRRs is justified in *ex ante* decision-making. However, firms could have undertaken projects that are found to have earned negative returns *ex post*. Consequently, the possibility of finding negative IRRs in the post-audit necessitates an extension of the interval for IRRs to include such an eventuality. Sufficiency conditions for an IRR in (–1, ∞) require only that the initial and final cash flows have different signs. Sufficiency conditions for a *unique* IRR in (–1, ∞) require that the unrecovered amount in any period during the life of the project not change sign.

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