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The D'Artagnan Capital Fund: A Novel Approach to Moderating a Student Managed Fund

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Student-managed funds have become essential pedagogical instruments in many university finance departments in the United States, with each fund having a unique purpose and execution. In this paper, we share the details of the development, implementation and on-going management of a student-managed equity fund at a small Midwestern university. A complete perspective of the fund is presented which provides an A-to-Z guide to fund creation and execution. In addition to the general lessons learned over the last decade from experiences with the fund, the manuscript presents the current structure of our fund-management program, student responsibilities of managing the portfolio, fund performance, and the opportunity for engaging external constituencies.
Keywords: *portfolio management, student funds, attribution analysis, pedagogy.*

Introduction

The Department of Finance in the Williams College of Business is home to two student-managed investment funds with a current combined market value of over $3.7 million. The Xavier Student Bond Investment Fund is a $1.3 million fixed-income portfolio managed by MBA students and the D'Artagnan Capital Fund is a $2.4 million equity portfolio managed by undergraduate business students.

Student investing began at Xavier in 2004, when the Xavier University Board of Trustees transferred responsibility to manage $1 million dollars of the University endowment to eight undergraduate Finance majors. In November 2004, the Xavier Student Investment Fund was established as a $1 million portfolio of

high-grade bonds, managed by undergraduate students using a bond-indexing approach. In December 2007, the Board of Trustees approved the formation of a second student-managed fund, a $1 million, large-cap equity investment fund (later named the D'Artagnan Capital Fund) to be managed by undergraduate students. Management of the fixed-income fund was subsequently moved to the MBA level. Under student management, the value of the D'Artagnan Capital Fund (DCF) has grown to its current value of approximately $2.4 million.

As there are many different approaches to running a student-managed fund, the purpose of this paper is to share the details of the undergraduate student-investment program at Xavier as a capstone learning experience for the students, and to discuss the specific policies, procedures and mechanisms developed to run the program. Additionally, the performance results of student management of the DCF over the past five years are presented as well as student reviews of the experience.

Student Learning Outcomes

In addition to teaching equity valuation and portfolio management this program strives to develop:

- an understanding of stock valuation;
- an understanding of portfolio performance measurement;
- an understanding of the impact of economic events on stock prices;
- improved communication skills, both written and verbal; and
- the ability to work in small groups.

At Xavier, the DCF is viewed as a student-run business, with a significant fiduciary responsibility to the University. As discussed further below, second-semester students manage the operations of the Fund while first-semester students assume analyst roles. An emphasis is placed not only on the technical aspects of selecting stocks and managing the portfolio, but also on interpersonal skill development, assumption of responsibilities, and communication with outside constituencies.

Prospectus and Strategy Statement

The DCF is operated under the guidelines laid out in a prospectus approved by the Xavier University Board of Trustees authorizing the students to manage endowment funds. The DCF is a large capitalization equity fund which is benchmarked against the S&P500 index. The prospectus specifies:

- the types of securities that can be held (common and preferred stock);

- what securities are excluded (fixed income, commodities, derivatives, etc.);
- the portfolio be allocated across the ten SIC sectors in the S&P500;
- no more than ten (10) percent may be invested in cash; and
- weights in each sector not vary by more than plus-or-minus 50 percent of the industry sector weight in the S&P500 index.

The University pays the DCF an annual management fee of 50 basis points in the form of a budget transfer. These funds are used to cover some administrative costs and to fund other activities as described further below.

The prospectus is silent on the style of the fund, giving the finance department the flexibility to staff the fund in the best way possible. As the current instructor for the DCF has considerable experience with individual security valuation, the style during the professor's five-year-and-counting tenure has been to follow a bottom-up approach. However, if the department chooses to staff the classes with a different faculty member, the style could easily be shifted to another approach to best utilize the strengths of the person teaching the classes. A copy of the prospectus is available upon request.

DCF students developed the following statement to summarize the current emphasis of the Fund:

> *The D'Artagnan Capital Fund is an opportunities fund, which seeks to position itself in undervalued stocks in the marketplace utilizing a bottom-up approach. Our Analysts extensively research company financials, management, and industry competitors. They formulate financial valuation models, present their findings, and undergo peer review, all of which leads to investment decisions. Our goal as a Fund is to continuously outperform our benchmark—the S&P 500—on a risk-adjusted return basis while remaining in compliance with our prospectus.*

For more details, go to www.xavier.edu\equity-fund.

Investment Program Structure

The DCF is the underlying framework for the undergraduate student investment program at Xavier. This program is comprised of a three-course (nine-credit-hour) sequence consisting of a three-hour prerequisite course in business valuation followed by two classes in portfolio management, Portfolio Management I and II. In general, students enrolled in Portfolio Management II manage the Fund and provide guidance and leadership for the students enrolled in Portfolio Management I who take on roles as analysts. The specific duties and responsibilities of students in these two groups are further described below. The valuation class serves as a

general finance elective, but is a requirement for students wishing to participate in the management of the DCF.

In the valuation class, students learn discounted cash flow techniques and relative valuation methods for valuing individual stocks as well as portfolio performance metrics. They are introduced to the tools for stock valuation which will be used in the management of the DCF. During the semester, students in this class apply discounted cash flow and relative valuation approaches, working in groups, to value three to five publicly-traded companies. These valuations, submitted in formal reports, are presented to the class at the end of the semester. Textbooks over the last several semesters have included texts by Damodaran (2006) and by Stowe, et al (2007). Students typically take the valuation class in the first or second semester of their third year, usually following or taken concurrently with classes in investments, financial statement analysis, or financial modeling. It is strongly encouraged that finance majors take the valuation class later in their finance program so as to have a deeper understanding of finance and finance tools. Only two pre-requisites, financial accounting and introductory finance, are required for the valuation class so that students with different business majors are able to participate in the DCF experience if they so choose. Though DCF students are typically finance majors, participants in the DCF have included majors in accounting, economics, international business, and marketing. These non-finance majors bring a unique perspective to DCF discussions and can be useful for some of the non-investment related business functions in the DCF.

Upon successful completion of the valuation class with a minimum grade of B- (at least 80%), students are eligible for Portfolio Management I (PM1). In PM1, students are assigned the position of analyst for the DCF and begin to participate in the analysis, valuation and decision-making of the fund. As analysts, students in PM1 are placed into sectors and are managed by second-semester students enrolled in Portfolio Management II (PM2). First-semester students provide preferences on which sector they would like to analyze and the portfolio managers draft them into sector teams. As analysts, first-semester students, working with their managers, screen stocks using Bloomberg, Capital IQ, Yahoo Finance, and other tools. Analysts are assigned to monitor existing stocks in the portfolio and are required to create at least five new stock valuations for consideration by the Fund during the semester. Each of these valuations, representing a buy, sell, or hold recommendation, is presented to the fund for consideration. PM1 students take a prospectus quiz early in the semester to emphasize the importance of following the guidelines provided by the Board of Trustees. Additionally, first-semester students are graded on their stock valuations and presentations, participation (both peer review and instructor review), an exam on portfolio performance metrics and a final exam covering valuation, performance metrics, and knowledge of their sector and sector holdings. (Examples of the valuation reports, presentations, and exams are available upon request.) Some students choose to participate in

the management of the DCF for only a single semester. A recently-introduced Minor in Finance allows students to participate for one semester in the DCF in partial fulfillment of the requirements for the minor. After completing PM1 with a minimum grade of B (at least 84%), students are eligible to take PM2. During this class, second-semester students hold positions of Officer or Sector Manager of the DCF. They are responsible for supervising the Analysts (first-semester students), developing agendas for class, inviting guest speakers, and scheduling additional activities. The typical officer and manager roles are defined in Table 1. Students apply for these roles, providing evidence that shows why they should be chosen. The current professor's view on officer roles is that students who worked the hardest and provided the best analysis in PM1 be given first choice of officer roles. Sometimes a shy, reserved student requests to be the Chief Executive Officer (CEO). If the professor deems that the student contributed significantly as a first-semester student, he/she could be chosen for the role as a way of helping the student develop interpersonal and leadership skills. The professor believes that it is important that students be given the support and learning experiences necessary to develop skills beyond technical skills used in analysis. When promoting the program, finance professors and advisors describe the DCF courses as challenging courses combined with a 15-plus hour per week internship. We find that many students list their DCF positions under the experience section of their resume.

Table 1. Example of Officer/Manager Roles in the D'Artagnan Capital Fund

Chief Executive Officer	The CEO of the D'Artagnan Capital Fund leads the Fund. The CEO manages the Fund, ensuring deadlines are met and sets the strategic objectives for the Fund.
Chief Financial Officer	The CFO's primary responsibility is to work with the Controller in calculating the monthly, annual, and semi-annual performance reports to guarantee their accuracy. Also, the CFO creates and balances the Fund's budget for the year and tracks the invoices and fees to send to Xavier University.
Chief Investment Officer	The CIO's primary responsibility is to manage the analysts and lead the morning meetings. In doing so, he or she creates a stock presentation calendar, ensures that the analysts are ready to present their pitches as well as contacting professionals in the Greater Cincinnati Area to come in and speak to the Fund or to observe how we operate. Additionally, the CIO records the trades made and sends them to the faculty advisor for execution.
Chief Operating Officer	The COO is responsible for overseeing the managers and leading the manager meetings. In addition, he or she coordinates the Fund's events and communicates the Fund's activities to external contacts through various outlets such as the Fund's website and social media accounts.

Table 1. (*Continued*)

Chief Compliance Officer	The Chief Compliance Officer is responsible for ensuring that the Fund's holdings remain consistent with the prospectus at all times. He or she will also confirm that the trades are within the bounds that the Fund can operate.
Chief Economist	The Chief Economist is responsible for monitoring the macro-economic environment and communicating the research to the Fund in order to help achieve its strategic objectives set by the CEO.
Controller	The Controller's primary responsibility is to assist the CFO in preparing the monthly, annual, and semi-annual performance reports. The Controller also documents the trades executed and monitors the performance calculating engines to assure accuracy.
Director of Financial Literacy	The Director of Financial Literacy's key responsibility is organizing and promoting the Fund's youth mentoring program with the Alliance Academy, a local grade school, which teaches personal finance, professional skills, and encourages students to begin planning for higher education. He or she is also responsible for assisting analysts in developing models and ensuring their calculations are accurate.
Sector Managers	Sector managers work with analysts to value companies and make strategic allocation and investment decisions to be presented to the entire group. Typically we have 8 or fewer sector managers. We typically combine Industrials and Materials due to the small size of the Materials sector and combine Information Technology and Telecommunications due to the small number of firms in the Telecommunications sector. Depending on the semester if we have fewer than 16 managers some of the officers will also have manager titles as well.

Classes for analysts from PM1 and Managers from PM2 are scheduled to meet separately each Monday during the semester. Early in the semester the analysts work with the professor to present models and stocks in a more forgiving environment. Later in the semester the managers are invited to the presentations to create more critical conversations. On Mondays, in their separate meeting, managers meet to discuss sector recommendations, operational duties and responsibilities and personnel issues. Both classes meet together on Wednesdays during the semester wherein stock pitches are presented. Following a pitch, an up or down vote is conducted at the conclusion of the pitch by the combined-group to ascertain whether to include the stock in the portfolio. We have found that secret balloting is useful to avoid popularity and band wagon based voting. Survey Monkey is used to collect votes, provide analytical support and counter evidence and critique the presentation. As a result of monitoring stocks in the fund, analysts will also make

recommendations of which positions should be liquidated. Additionally monthly, semi-annual, and annual performance metrics are presented and discussed.

For continuity of management of the DCF, both classes are scheduled every semester. This results in an overlap of first- and second-semester students each semester. Students typically become analysts either during the spring of their junior year or fall of their senior year. Students that are analysts in their junior year become officers and managers at the end of the spring semester and manage the portfolio over the summer. Management over the summer is typically in the form of monitoring the stocks in the portfolio and having discussions via conference calls and email. The DCF does not experience a high turnover of the portfolio in the summer, which is consistent with a long run strategy of having the best-valued stocks in the portfolio.

Student interest and participation in the DCF has grown considerably over the years. Currently there are approximately forty students assuming the analyst rolls in PM1 and 17 fulfilling management and officer rolls in PM2. Until now, no restrictions have been placed on the size of the classes other than the aforementioned grade requirements. To account for the growing number of candidates, a process had recently been put into place to restrict the number of students in PM2 to keep the numbers of managers and officers to a manageable size.

Evolution of Fund Structure

The first student investment fund available to Xavier students was a $1 million fixed income fund. A fixed-income fund was chosen because of the perception that there were a greater number of fixed-income analyst jobs in the Midwest relative to equity analysts. However, given the liquidity of bond markets and taking into consideration that a $10 million trade in such markets is considered small, from the outset the small size of the student fund presented challenges. To minimize the cost of trading, trades were piggy-backed on trades executed by the University endowment's fixed income manager and occasionally student-recommended trades were delayed or over-ruled. Even still, it was a significant learning experience for students to participate in the Fund. After a couple of years of experience with the fixed-income fund, department faculty decided that such a fund would be better suited for our MBA students and an equity fund was created for the undergraduates. The faculty found that is easier to prepare the MBA students to manage a fixed-income portfolio than the undergraduates, given the technical nature of bond-portfolio analytics.

Initially the two-semester sequence of classes associated with the D'Artagnan Capital Fund was treated as an introductory class followed by an advanced class. As the students became adept at valuing equities using discounted-cash-flow techniques and relative valuation methods relatively quickly, the two-class

structure left second semester students wanting more of an educational challenge. Hence, the two-class structure currently employed was introduced, giving the second-semester students opportunities to develop leadership and interpersonal skills.

The Trading Process

After the students vote for a change in the portfolio, the officers meet and begin the process of trade creation. The students create a trade form (see Table 2) and forward it to the professor along with the rationale for the trade and voting results. A Xavier University alum who works at UBS serves as custodian/broker for the account. The professor reviews the trade, sends it to the local UBS trading desk, and follows up with a confirmation call. The alum supports the process and provides help with performance-reporting questions and research reports if the students are interested. Additionally he provides trades at a low cent-per-share basis.

Table 2. Example of trade order. Students send a PDF file of the trades they would like to make along with a justification and results of voting. The professor forwards the PDF file to the trading desk at our local UBS office and calls to confirm the trade.

Account			#XXXX		
Account Name			Xavier University		
Date			4/26/2016		
Buy/Sell	**Quantity**	**Shares/Dollars**	**Symbols**	**Name**	**Market Limit**
Sell	297	Shares	**MON**	Monsanto Company	Market
Sell	492	Shares	**LH**	LabCorp	Market
Buy	810	Shares	**CF**	CF Industries Holdings	Market
Buy	270	Shares	**AGN**	Allergan	Market

Data Sources and Investment Tools

The Department of Finance at Xavier is home to the Fifth Third Trading Center, a trading room sponsored by the Fifth Third Bank Foundation. The two main resources housed in the trading room used in the student investment program are the Bloomberg Professional Service and the Standard & Poor's Capital IQ Platform. Students are introduced to Bloomberg and Capital IQ in the valuation class and capabilities of each are used extensively in the management of the DCF. The Bloomberg Professional Service is very useful for determining cost of debt estimates, for use in discounted cash flow models such as free cash flow to the

firm valuation and for researching economic data. Capital IQ is very simple to use and useful for downloading historical financials to serve as a basis for pro forma cash flow forecasts and for downloading trading multiples for performing a comparable-companies analysis.

In addition to Bloomberg and Capital IQ, StatPro is used by the DCF for analyzing and reporting portfolio performance. (For a discussion of using Bloomberg in Finance Classes see Coe (2007)) The Fund experimented with other portfolio analysis software tools such as Bloomberg but found that most of these tools use holding-based analysis to track performance. This requires daily closing methods to ensure that portfolio tracking is synchronized with the portfolio's actual performance. Daily closing is not practical in a program where students are not available each day the market is open. During the summer and winter breaks, for example, the students manage the portfolio remotely and daily closings are not practical. Statpro is transaction-based which is useful in this situation because it allows a student to enter all the transactions and trades that occur since the last time the software was updated.

Portfolio performance tracking is one of the more difficult aspects of running the student-managed portfolio. In talking with portfolio managers and operations professionals at local investment firms, DCF managers found this is a challenge frequently faced in "real-world" portfolio management. However, in the "real world" there is a daily closing process used by firms to keep their portfolio tracking in sync with the actual performance of their funds. Student managers of DCF have developed methods to adjust the Statpro results and keep the portfolio tracking in sync with the actual performance of the Fund.

At the conclusion of each semester, management of the fund is handed off to a new group of students. In addition to the portfolio tracking and analytic tools, methods to aid in the transition of knowledge and procedures from one set of students to the next are vital. Student managers use an online Wiki page in Canvas, a learning management system, to maintain an operations manual which students can access. This allows new students to learn how a particular activity was performed in the past. For instance, if a password is changed for a shared system, students update the operations manual so that it will be available for other students. One of the challenges fund managers face is the effort required for them to update the operations manual, keeping it brief enough to be useful while providing enough information to keep continuity across students and time—a problem not uncommon in the "real world."

Presentations and Fiscal Year

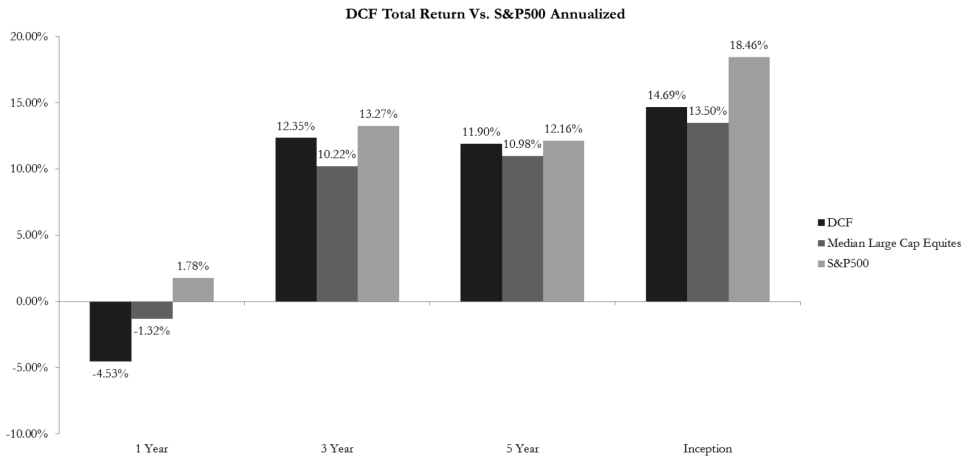
Although Xavier University has a June 30 fiscal year-end, the DCF has chosen a March 31 fiscal year-end to better reflect the academic school year. This allows for a mid-year performance report and presentations to be done in October and

November. Year-end performance reports and presentations are done in April and May. In addition to individual stock pitches and reports which the students make to the Fund team during the semester, the students also make formal presentations to outside constituent groups each semester. These include:

- Presentation to the Finance Club (FMA)—Each semester, DCF students present their fiscal-year results to undergraduate business students who are interested in joining the fund experience. In addition, the program preparation and requirements are discussed.
- Presentation to the Faculty—Each semester, DCF students invite professors from all departments in the College to a presentation of the year's results and to ask questions about the class. Finance and business faculty typically ask challenging questions about the portfolio. This experience makes for a great learning environment.
- Presentation to the Board of Executive Advisors—The Department of Finance has an external advisory board consisting of area finance professionals. Presentation by students to the BEA is a great opportunity to get the Department's Board involved with the students. The BEA members typically ask challenging questions and give guidance based on their work experiences. A subset of the Board also provides oversight to the DCF.
- Presentation to Former Students—For this presentation, former DCF students are invited to come back to campus each semester and listen to the year's results. Less formal than the other presentations, these presentations result in the most challenging questions for DCF managers. Former students provide great insight as they have all sat in the student's seats at some point in the past and can share empathy and insights. This presentation is also fun for our professors who have the opportunity to engage with former students.
- Presentation to the Board of Trustees—In the spring of each year, the DCF officers present the results of the Fund operations to the Investment Advisory Committee of the University Board of Trustees. The Board evaluates the performance of the DCF group against that of the other money managers of the University's endowment. The Board typically hosts a luncheon where students have a chance to interact with them and discuss the experience.
- Presentation to Family Members—The day before graduation, the seniors bring their families to the trading room and give one last presentation before they graduate. This is always a fun event for the students to show off the great work they have done throughout the year. It also seems to excite the parents about the knowledge and passion their students have obtained during their degree.

Portfolio Performance

The return performance of the DCF since inception for the period ending March 31, 2016 is presented in Figure 1. As shown on the table accompanying Figure 1, the DCF has performed in the top 46 percent of large cap, actively-managed funds for the last 3 years and in the top 43 percent for the last 5 years.



	One-year	Three-year	Five-year	Since Inception
DCF Return	-4.53%	12.35%	11.90%	14.69%
Percentile	67th	46th	43rd	NA
Median Active Mgr	-1.32%	10.22%	10.98%	13.50%
S&P 500 Return	1.78%	13.27%	12.16%	18.46%
Percentile	33rd	26th	19th	NA

Figure 1. The Performance of the D'Artagnan Capital Fund since inception for the period ending March 31, 2016 (Source for percentile rankings: Morningstar)

Performance metrics for the DCF are shown in Tables 3 and 4. Table 3 presents standard performance metrics calculated for the year ending March 31, 2016. Table 4 presents an attribution analysis for the last five years.

Table 3. Annual DCF performance metrics for the period ending March 31, 2016

Performance Metric	DCF	S&P 500
Total Return	-4.53%	1.78%
Excess Return	-6.31%	NA
12 Month Beta	1.06	1.00
Sharpe Ratio	-0.417	0.060
Treynor Ratio	-0.0449	0.0154
Jensen's Alpha	-6.41%	
M^2	-12.28%	

It is interesting to note that the area in which the students perform best is in security selection for information technology. Is this because they are particularly informed about technology based on their age? Or did they just get lucky and hold Apple at the right time? Perhaps this is an area for future study.

Table 4. Five Year Attribution Analysis

	Asset Allocation	Security Selection	Excess Return
Energy	-2.11%	1.26%	-0.85%
HealthCare	-1.20%	-4.63%	-5.83%
Financials	0.49%	-2.08%	-1.58%
Discretionary	-1.13%	-3.95%	-5.08%
Information Technology	0.01%	4.68%	4.69%
Industrials	-0.19%	-1.92%	-2.11%
Utilities	0.08%	0.21%	0.28%
Staples	0.00%	-0.06%	-0.06%
Materials	-0.32%	-0.55%	-0.87%
Telecommunications	0.18%	0.53%	0.71%
Other	-0.41%	-1.62%	-2.03%
Cash	-0.74%	0.01%	-0.73%
Five Year Total	-5.34%	-8.12%	-13.46%
Annualized	-1.07%	-1.62%	-2.69%

Program Performance

The feedback regarding the overall program from the department's Advisory Board members and former students is favorable. Students in the program also evaluate the program favorably. Table 5 shows two questions from the course teaching evaluations over the last five years. Over the last five years the average teaching evaluation for the relative worth of the second semester DCF class was 4.99, on a scale of one (low) to five (high), and the relative worth of the first semester DCF class was 4.82. Students also evaluated the courses as making them work harder than a typical class.

Table 5. Selected responses from course teaching evaluations (scale 1:5)

Course	Relative Worth of Course	Makes Me Work Harder
Fall 2010		
FINC 490 Portfolio Management I	4.9	4.9
Spring 2011		
FINC 490 Portfolio Management I	5	5
FINC 492 Portfolio Management II	5	5
Fall 2011		
FINC 490 Portfolio Management I	4.9	4.7
FINC 492 Portfolio Management II	5	5
Spring 2012		
FINC 490 Portfolio Management I	4.7	4.6
FINC 492 Portfolio Management II	5	4.9
Fall 2012		
FINC 490 Portfolio Management I	4.9	4.6
FINC 492 Portfolio Management II	5	4.7
Fall 2013		
FINC 490 Portfolio Management I	4.7	4.8
FINC 492 Portfolio Management II	5	5
Spring 2014		
FINC 490 Portfolio Management I	5	4.8
FINC 492 Portfolio Management II	4.9	4.9
Fall 2014		
FINC 490 Portfolio Management I	4.73	5

Table 5. (Continued)

Course	Relative Worth of Course	Makes Me Work Harder
FINC 492 Portfolio Management II	5	5
Spring 2015		
FINC 490 Portfolio Management I	4.82	4.75
FINC 492 Portfolio Management II	5	5
Fall 2015		
FINC 490 Portfolio Management I	4.56	4.94
FINC 492 Portfolio Management II	4.75	5
Average of Finc 490 Port Mgmt I	4.82	4.81
Average of Finc 490 Port Mgmt I	4.99	4.94

Additional Activities

In addition to managing a portfolio of over $2 million, students in the D'Artagnan Capital Fund also participate in other activities that are not part of a typical classroom experience. In addition to creating an experiential opportunity of managing real money and a fund that is run like a business, students are encouraged to participate in activities with the greater Cincinnati community.

Perhaps, the most important outside activity is the Fund's financial literacy and mentoring program. This program invites seventh and eighth grade students from a neighborhood school to come to campus every other week to participate in activities that teach business, finance, and careers. The neighborhood school is an inner-city school in a low-income area. In addition to coming to campus several times a semester, students are taken on field trips each fall and spring. Approximately 30 seventh and eighth graders attend sessions on a regular basis. In addition to the classes and activities that introduce and teach business and finance concepts, the program is important as it introduces these students to a college environment and college students who are ready to teach them. This relationship with a local grade school is a good learning experience for our portfolio managers and analysts as well because it exposes them to students that are typically from a different background from their own. The financial literacy and mentoring program is funded in part by grants from U.S. Bank and Financial Executives International.

In addition to the financial literacy program the DCF has hosted and moderated several investment symposiums. Below is a partial list of events DCF students have held:

Mutual Fund Workshop with the Investment Company Institute (Inaugural Cincinnati Investment Symposium)—Invited four Investment Company Institute professionals from Washington, D.C. to discuss mutual fund issues.

So You Want to Start a Mutual Fund—Invited mutual fund service providers to campus to discuss how to start a mutual fund and to discuss what is involved in the day-to-day operations of a fund.

Second Annual Cincinnati Investment Symposium—Portfolio Analysts and Managers invited for panel discussions on various investment topics. Attended by area investment managers and finance faculty.

Third Annual Cincinnati Investment Symposium—Portfolio Analysts and Managers invited for panel discussions on various investment topics with Keynote Speaker from Cleveland Fed. The Symposium was attended by area investment managers and finance faculty as well as the students.

In addition to hosting a local investment conference, interested DCF students travel to annual investment conferences such as R.I.S.E., G.A.M.E., and ENGAGE. The DCF has supported contingencies of over 20 students at these conferences which provide a great opportunity to learn from investment professionals and to engage with students from other universities to discuss best practices of managing student funds.

Over the last five years the DCF had six teams participate in the CFA Challenge. The CFA challenge is a case study competition in which students perform an in-depth valuation of a publicly-traded company. The students meet the Company's management team, or investor relations personnel, and work with a local CFA and professor. Given the extensive valuation modeling the students do as part of the DCF, the CFA Challenge is a natural fit to have their skills evaluated by CFA professionals and compete with other schools.

In addition, several students interested in exploring law school have worked with one of our Business Law instructors, and a 40 Act attorney, to re-write the fund prospectus in a format which would be used by funds wishing to register with the SEC. This has been an opportunity for these students to explore a business area that is tangential to the finance field that they have been studying as majors.

Summary

In this paper, the operation and management of the D'Artagnan Capital Fund, a novel approach to running a student investment fund, is described. A roadmap is presented which could be used by institutions and professors who are running, or are considering setting up, a student investment fund. In addition to the basic duties of managing a portfolio, we describe external and internal educational opportunities that tie in with the program. Finally, we demonstrate the five-year performance which indicates the approach we employed outperformed 57 percent of the professional large cap active managers as covered by Morningstar.

The experiences at Xavier University could easily be replicated by any finance department but would require significant buy-in from the faculty and Board of Trustees. When executed well, the experience of running a fund has shown to be one of the best academic experiences for both students and faculty. This real-world in-school experience gives participating students a leg-up in the job market and their future careers.

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Linking the Capital Budgeting Decision to the Security Market Line

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In managerial finance textbooks capital budgeting techniques usually appear after a chapter covering the security market line. However, little is done to connect these two important topics. This paper introduces a capital budgeting technique, which links the capital budgeting decision to the security market line. Teaching this technique should help students develop a bridge between project selection and the return on equity.
Keywords: *Stock return, net present value, capital budgeting, security market line, pedagogy*

Introduction

Capital budgeting is a very important area of managerial finance; an understanding of how a firm creates value for shareholders is crucial. Most textbooks cover capital budgeting after introducing the security market line equation in a chapter on risk and return. Typically, four project selection techniques are presented: internal rate of return, net present value, payback period, and the profitability index. Net present value is usually identified as the most important of these techniques because it quantifies the wealth created for stockholders, and this fits with the goal of the firm to maximize shareholder wealth.

Net present value, if positive, is the wealth created for common stockholders over and above paying them a fair return on a project. For instance, a project's net present value may be estimated at $42,910 when using 10.8% as a fair return on retained earnings. The interpretation would be that stockholders are expected to receive more than a fair return for the use of their funds. That is, they are expected to receive their 10.8% return plus $42,910 in present value terms. Mixing a percentage return with a dollar amount is not very clear.

Estimating the project's return on retained earnings and plotting it on the security market line should help students better understand why a project is acceptable, or not. It will enable them to connect a capital budgeting project to something they recently learned, the security market line.

History of Discounted Cash Flow Techniques

Offering an historical perspective on a topic can help students appreciate its importance especially when the topic dates back centuries.

Simon Stevin, mathematician and engineer, introduced a discounted cash flow technique for comparing financial investments in 1582; see Jones and Smith, (1982). He assessed the value of loans using present value analysis and included compound interest tables. Three centuries later, Arthur Wellington (1887) published his book, *The Economic Theory on the Location of Railways*. Wellington, an American civil engineer, was one of the first writers to use present value analysis to determine the worthiness of nonfinancial investments. His work was meant to help railroad managers determine whether a new line should be built. Choosing the wrong project in the railroad industry would be tragic in light of the magnitude of a fixed asset investment.

Irving Fisher's (1907) *The Rate of Interest: Its Nature, Determination, and Relation to Economic Phenomena* is the first reference to net present value in American economic literature. Twenty-three years later, a Stanford engineering professor, Eugene L. Grant (1930), published his textbook, *Principles of Engineering Economy*. He is credited with having presented present worth and equivalent annual cost methods for assessing capital budgeting projects in a single textbook. In 1960, Bierman and Smidt published, *The Capital Budgeting Decision*, which emphasizes the net present value method for assessing capital budgeting projects. Finally, Charles Horngren's (2000) *Cost Accounting: A Managerial Emphasis* also became a leading textbook in managerial accounting; it too is a strong advocate for the net present value method.

Net Present Value

The firm obtains capital from bondholders and shareholders (i.e. preferred and common stockholders) in order to invest in fixed assets, which are expected to generate cash inflow. In order to determine a project's net present value, the first step is to estimate the cash inflows available for bondholders and shareholders. Then, using a risk-adjusted discount rate, often the firm's cost of capital, the present value of the expected cash inflow is found. If it is higher than the investment into assets, the project's net present value is positive; its cash inflows are expected to be high enough to adequately compensate bondholders and preferred stockholders for the use of their capital. Common stockholders, on the other hand, are expected to receive remuneration over and above the amount necessary to compensate them for risk. For example, if a project's net present value is fifty-thousand dollars, the firm's common stockholders will share this wealth in addition to receiving a fair return for the use of their funds. This assumes the firm is financially sound. The formula for net present value is:

$$Net\ Presenter\ Value = \sum_{Yr=1}^{N} \frac{CIF_{Yr}}{(1 + RADR)^{Yr}} - II_0 \quad (1)$$

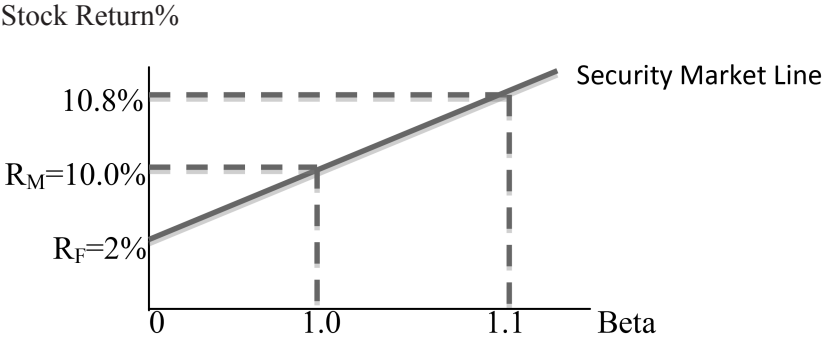
Where:

N = Project's life in years,
Yr = Year,
CIF = Expected cash inflow,
RADR = Risk-adjusted discount rate,
II_0 = Initial investment into assets in year zero.

Estimating the Fair Return on Investor Capital

A firm may be thought of as a collection of previously accepted capital budgeting projects. It follows that if a project under consideration is average in terms of its risk the firm's cost of capital may be used as the risk adjusted discount rate. This rate can be estimated by examining the returns on the firm's outstanding securities or by examining bond and stock yields at similar companies; adjustments need to be made in order to accommodate flotation costs. For instance, the cost of debt can be approximated by assessing the yields on similar bonds issued by the firm itself or by firms in its industry. The same approach can be used to approximate the costs of preferred and common stock financing. Managers may use the security market line equation to estimate a fair return for the use of retained earnings. They need the forgoing estimates in order to determine whether a project under consideration will provide at least a fair return on investor-supplied capital.

Below is a graph of the security market line equation assuming the risk-free rate is estimated to be 2%, the market return is estimated to be 10%, and the stock's beta is estimated to be 1.10:



The fair return for the use of retained earnings is 10.8%.

Below is the formula for the weighted average cost of capital given the contribution by common stockholders is retained earnings:

$$WACC = RADR = (W_d)(C_d)(1\text{-}T) + (W_{ps})(C_{ps}) + (W_{Ce})(C_{re}) \quad (2)$$

Where:

WACC = Weighted average cost of capital,
RADR = Risk adjusted discount rate,
W_d, W_{ps}, W_{ce} = Weights of debt, preferred stock and common equity,
Cd, Cps, Cre = Costs of debt, preferred stock and retained earnings,
T = Corporate tax rate.

Given the cost of debt is 5%, the cost of preferred stock is 6%, the cost of retained earnings is 10.8%, a 30% tax rate, and the following capital structure weights, the weighted average cost of capital is:

$$WACC = (0.4)(5\%)(1\text{-}30\%) + (0.1)(6\%) + (0.5)(10.8\%) = 7.4\% \quad (3)$$

The Zero Net Present Value Project

A project with a net present value equal to zero is fair. Its cash inflow provides bondholders, preferred stockholders and common stockholders with sufficient returns. In a world with perfect competition, one may expect all projects to have net present values equal to zero. All stakeholders (e.g. employees), including bond and shareholders, are expected to be compensated fairly when a project's net present value equals zero.

For the project below (Table 1) the internal rate of return (IRR) is 7.4%, the firm's weighted average cost of capital. In this instance, bondholders and shareholders are expected to be sufficiently compensated for the use of their capital.

Table 1. Zero Net Present Value Project (IRR = 7.4%)

Year	Cash Flow	$PVIF_{7.4\%}$	Present Value
0	−$1,000,000.00	1.0000	−$1,000,000.00
1	246,000.00	0.9311	229,050.28
2	246,000.00	0.8669	213,268.42
3	246,000.00	0.8072	198,573.94
4	246,000.00	0.7516	184,891.94
5	248,947.63	0.6998	174,215.42
			Net Present Value = $0.00

Common stockholders, the firm's owners, are compensated with the funds remaining after bondholders and preferred stockholders have been paid. The return on retained earnings (i.e. common stockholder capital) can be estimated as follows:

$$IRR = (Wd)(Cd)(1-T) + (Wps)(Cps) + (Wce)(Cre) \quad (4)$$

$$Cre = [IRR - (Wd)(Cd)(1-T) - (Wps)(Cps)]/Wce \quad (5)$$

Where:

- IRR = project's internal rate of return,
- C_{re} = estimated return on retained earnings (i.e. common stockholder capital),
- W_d, W_{ps}, W_{ce} = Weights of debt, preferred stock and common equity,
- C_d, C_{ps} = Costs of debt and preferred stock,
- T = Corporate tax rate.

Substituting the numbers from the example above into equation five, the return on retained earnings is:

$$C_{re} = [7.4\%_{IRR} - (0.4)(5\%)(1-0.3) - (0.1)(6\%)]/0.5 = 10.8\% \qquad (6)$$

If the firm accepts this zero net present value project, bondholders and shareholders will be sufficiently compensated if all goes according to plan. That is, bondholders will earn a 5% yield, preferred stockholders will earn a 6% yield, and common stockholders will earn a 10.8% return on their $500,000 of retained earnings.

If we assume the project's cash inflows are invested at the firm's weighted average cost of capital instead of at the project's internal rate of return we would use the modified internal rate of return developed by McDaniel, McCarty and Jessell (1988) in Equation 4 and 5 instead of the internal rate of return.

Positive Net Present Value Projects and the Return on Common Stockholder Capital

The following project (Table 2) has higher cash inflows. Its internal rate of return is 9%, which is higher than the firm's weighted average cost of capital (i.e. 7.4%). The net present value of this project is $42,910.34, which is an estimate of the wealth created for common stockholders over and above their required return of 10.8%. When this project is announced to the public, the value of the firm's shares (i.e. capitalization) should rise by this amount, unless the market's estimated value of the announced project differs from the firm's estimate.

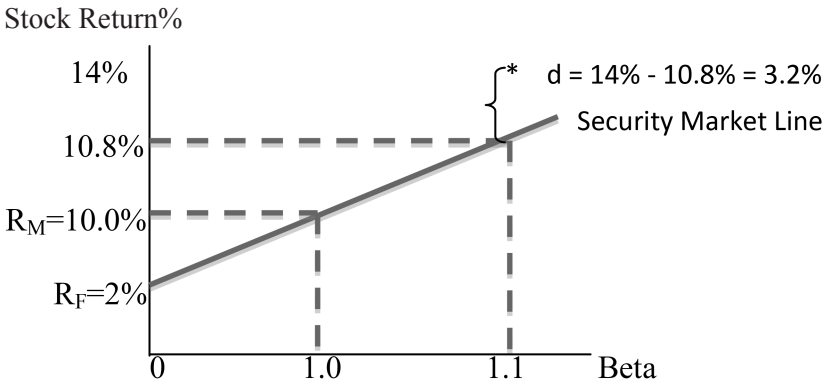
Table 2. Positive Net Present Value Project (IRR = 9.0%)

Year	Cash Flow	$PVIF_{7.4\%}$	Present Value
0	−$1,000,000.00	1.0000	−$1,000,000.00
1	257,000.00	0.9311	239,292.36
2	257,000.00	0.8669	222,804.81
3	257,000.00	0.8072	207,453.27
4	257,000.00	0.7516	193,159.47
5	257,500.00	0.6998	180,200.43
		Net Present Value =	$42,910.34

The return on common stockholder capital (i.e. retained earnings) can be estimated as follows:

$$C_{re} = [9\%_{IRR} - (0.4)(5\%)(1-0.3) - (0.1)(6\%)]/0.5 = 14\% \quad (7)$$

If the firm accepts this project, the estimated return on retained earnings is 14%. This is a much clearer metric than stating the common stockholders earn their required 10.8% return plus extra wealth in the amount of $42,910.34. We can say this project is expected to return 5% to bondholders, 6% to preferred stockholders and 14% to common stockholders. The expected return for common stockholders plots above the security market line.



The acceptance of this project is expected to pay a 14% return to common stockholders; this is 3.2% higher than the required return of 10.8%.

Negative Net Present Value Projects and the Return on Common Stockholder Capital

The following project's (Table 3) cash flows are lower such that its internal rate of return is only 6%. Since the firm needs to pay an average return of 7.4% to bond and shareholders its net present value is negative (i.e. −$37,103.09).

Table 3. Negative Net Present Value Project (IRR = 6.0%)

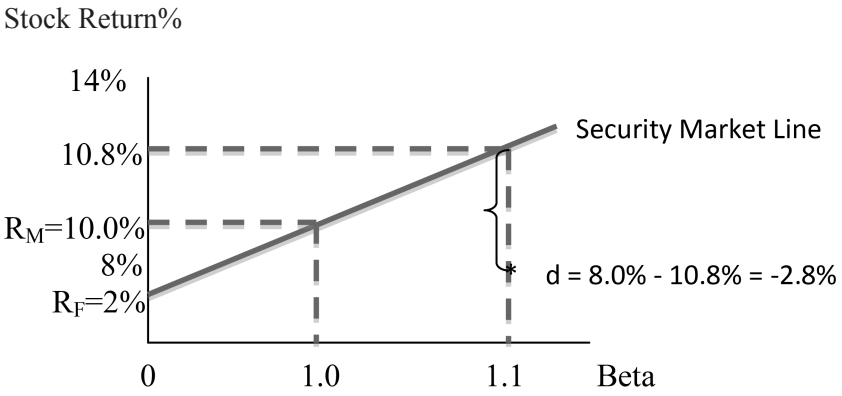
Year	Cash Flow	$PVIF_{7.4\%}$	Present Value
0	−$1,000,000.00	1.0000	−$1,000,000.00
1	237,000.00	0.9311	220,670.39
2	237,000.00	0.8669	205,465.91
3	237,000.00	0.8072	191,309.04
4	237,000.00	0.7516	178,127.60
5	239,100.00	0.6998	167,323.97
		Net Present Value =	−$37,103.09

The return on common stockholder capital (i.e. retained earnings) can be estimated as follows:

$$C_{re} = [6\%_{IRR} - (0.4)(5\%)(1-0.3) - (0.1)(6\%)]/0.5 = 8\% \quad (8)$$

If the firm accepts this project, the estimated return on to common stockholders only 8%.

We can say the project is expected to return 5% to bondholders, 6% to preferred stockholders, and 8% to common stockholders. The expected return on common stockholder capital plots below the security market line.



The acceptance of this project is expected to pay an 8% return to common stockholders; this is 2.8% lower than the required return of 10.8%.

Conclusion

Managerial finance textbooks typically cover the security market line before covering capital budgeting techniques, but little is done to connect these two topics. This paper introduces a capital budgeting technique, which links the capital budgeting decision to the security market line. This should help students form a bridge between these two very important topics.

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Using Political Event Derivatives to Illustrate the Binomial Option Pricing Model

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When students first learn about the binomial option pricing model (BOPM), the idea of modeling a stock price distribution with a binomial tree can appear abstract. In this paper, we present a simple way to illustrate the BOPM to a scenario that is a true binomial event, an election result. Using the quoted prices on the Iowa Electronic Market (IEM) for election futures contracts, synthetic options can be created and priced using the BOPM. This exercise provides the opportunity to illustrate to students how an option on a binomial event is the same as a futures contract on the same event.

I. Introduction

Understanding and valuing options is a difficult topic for many students. Most students are intrigued with the unlimited upside and limited downside for a long call option. And, when option valuation is discussed in class, students seem to gravitate toward the Black-Scholes Option Pricing Model, feeling that the Binomial Option Pricing Model is too simplistic to be applied to option valuation. However, it is important that students understand that every model has limitations, and understanding those limitations can be critically important. In this paper, we discuss how we value a binary option on a futures contract, the relationship between the futures contract and a call option on that futures contract, and the limitations of the Black-Scholes Option Pricing Model.

Teaching option valuation can be difficult because many concepts seem esoteric to students at first. Several authors have created different techniques to teach options. For example Johnson and Stretcher [2009] use an Excel spreadsheet with spinner buttons to allow students to vary inputs and immediately see the resulting change in option value. Trading games are another method that is used to introduce options valuation and payoffs to students. For example, Pavlik and

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Neinhaus [2004] create a multi-round options trading game and Cooper and Grinder [1997] create an options game in which the students use Black-Scholes and put-call parity to value options. Smolira and Travis [2011] allow students to purchase options on their final exam grade using points on their grade.

II. Iowa Electronic Market

The Iowa Electronic Market (IEM) is an actual derivatives trading market. It is operated by the faculty of the University of Iowa Henry B. Tippie College of Business. The first IEM presidential election derivatives were traded on the 1988 U.S. Presidential election. Trading accounts can be opened in amounts between $5 and $500. Contracts have been traded on a variety of U.S. political events such as the outcomes of Congressional and Presidential elections and monetary policy actions of the Federal Reserve. Each contract is a futures contract with two potential outcomes.

As an example, on April 17, 2013, one of the contracts available for trading on the IEM was a futures contract on whether Democrats would control both houses of the U.S. Congress after the 2014 election. The closing price from the previous day was quoted at 0.225.[1] This means that for $0.225, one could buy a contract that pays $1 if the Democrats take control of both houses of Congress; the contract pays $0 if this event does not occur. One aspect of the IEM that becomes interesting is that the price of these contracts can be interpreted as the market participants' expected probability of the event. In other words, the average IEM market participant at that time expected that there is a 22.5 percent probability the Democrats would take control of both houses of Congress in 2014. Over time, if information in the markets makes the likelihood of the event increase, then the price of the contract will increase. The tradability of the exchange allows for the contract to be bought and sold at any time up to the expiration date.

In order to set up trading for a class with real money on the IEM, the professor must email IEM and set up a course. Once the course is set up, students register online, print out a confirmation page, and mail the funds for trading, plus a $5 set-up fee, to the University of Iowa. Accounts are limited to $500 and all funds are kept in an account at the University of Iowa. It is also possible to set up an account to try out the IEM, which will permit the participant to do everything a trader can do except trade in real-money contracts. The process of implementing a trade on the IEM is straight-forward once an account is established. No short sales are

[1] One feature that distinguishes the IEM futures contracts is that they require an upfront investment. Traditional futures contracts on other exchanges normally only require full payment at contract expiration.

permitted in the IEM, but "bundles" are sold that replicate a short sale without the need for the IEM from guaranteeing the other side of the trade. A bundle consists of one unit of each contract listed in the market at a total cost of $1. If you sell one or more of the contracts in the bundle, it acts much as a short sale and the $1 paid for the bundle equates to the margin deposit.

Previous research has shown that the market price data from the IEM is, on average, more accurate than polls at predicting U.S. Presidential elections [Berg, Nelson, and Rietz, 2008]. While there are various "play-money" simulations available, Rosenbloom and Notz [2006] find that real-money markets, such as the IEM, are more accurate for non-sports events.

Other research finds that the IEM are accurate at predicting IPO valuations. In Google's 2004 IPO, the IEM traded contracts on the market capitalization of Google at the end of the first day of trading. Berg, Neumann, and Rietz [2009] find that the IEM market was more accurate than preliminary prices ranges from the prospectus. Further, the IEM valuation was accurate before much of the information about the IPO, such as number of shares, was released.

Further extensions of the implications of trading on the IEM document how prices on the IEM are related to real-world events. For example, Goodell and Vahamaa [2013] find that market uncertainty, as measured by the VIX, is positively related to the election probability of the eventual presidential winner and overall changes in market uncertainty.

Simkins and Maier [2004] discuss how the IEM can be used to discuss efficient markets in classroom setting. Gruca [2000] discusses how the now defunct motion picture box office futures can be used to integrate finance and marketing, while most other research, such as Fowler [2008] focuses on the political science implications of IEM presidential and congressional futures.

An extension of these traded futures contracts is the creation of synthetic options contracts. The pricing and analysis of these options are the focus of this paper. After going through the process of applying the Binominal Option Pricing Model (BOPM) and Black-Scholes Option Pricing Model (BSOPM) to value options on these futures contracts, interesting contrasts can be made between the futures and options markets.

III. Illustration

On August 17, 2012, futures contracts were being actively traded on the upcoming 2012 U.S. Presidential election. For a futures contract on whether the Democrats will win the White House, the closing price for the previous day was 0.600. That meant that for the purchase price of $0.60, this futures contract would pay $1 if the Democrats win the 2012 U.S. Presidential election. This same contract pays nothing, if the Democrats do not win. Since the futures price can be

calculated as the discounted expected value of the outcome, the price of 0.600 can be interpreted as a probability.[2] More formally, it is the probability given to the event by the average IEM market participant.

As a classroom exercise, students could be asked to price an in-the-money call option on this futures contract. This can be a simple way to illustrate the BOPM. Using the risk-neutral BOPM method, the price of a call option can be calculated by the following:

$$C = [\, q\, C_u + (1 - q)\, C_d \,] / (1 + r)^t$$

$$C_u = \text{Max}(uF - X, 0)$$

$$C_d = \text{Max}(dF - X, 0)$$

Where u is the up-step multiplier for the futures price, d is the down-step multiplier for the futures price, C_u is the option value in the upward state, C_d is the option value in the downward state, C is the option price today, r is the risk-free rate, t is the time until option expiration, F is the current futures price, and X is the strike price of the option. From this scenario, u can be calculated by the up-step futures price divided by the current futures price:

$$u = 1 / F = 1 / 0.600 = 1.66666667$$

The value of d can be calculated by the down-step futures price divided by the current futures price:

$$d = 0 / F = 0 / 0.600 = 0$$

With a risk-free rate of 0.09 percent, which was the 3-month U.S. Treasury bill rate at the time, the value of the risk-neutral probability q can be calculated as follows:

$$q = (1 + r - d) / (u - d) = (1 + 0.0009 - 0) / (1.666666667 - 0) = 0.60054$$

It is worth noting, that in absence of discounting (i.e. if r = 0), the q value is the same as the current futures price. The BOPM call option value is as follows:

$$C_u = \text{Max}(1 - 0.6, 0) = 0.4$$

$$C_d = \text{Max}(0 - 0.6, 0) = 0$$

$$t = (\text{“}11/6/2012 - \text{“}8/17/2012\text{”}) / 365 = 0.2219 \text{ years}$$

[2] On that date, the 3-month U.S. Treasury bill rate, an approximation of the risk-free rate, was 0.09 percent. The discounting effect on the price is thus negligible over such a short time horizon and with such a low interest rate.

$$C = [\ 0.60054 * 0.4 + (1 - 0.60054) * 0\ ] / (1 + 0.0009)^{0.2219} = 0.2402.$$

Thus, the price of an at-the-money call option on that date was \$0.2402. From this point, some interesting comparisons can be made between the futures contracts and the options on those futures contracts. One way to compare these contracts is by their potential payoffs relative to the initial investment. For an investment of \$0.24, the option pays \$0.40 if you win and \$0 if you lose. Measured by investment returns, the option pays a return of [(0.4 − 0.24) / 0.24] = 66.67 percent if you win and 0 percent if you lose. In comparison, for an investment of \$0.60 in the underlying futures contract, the futures contract pays \$1 (66.67 percent return) if you win and \$0 (0 percent) if you lose. The option contract and the futures contract pay the same returns with the same probabilities. The option is simply a smaller contract.

We can also calculate the delta of the call option as the change in option value divided by the change in the futures value. The delta of this option is:

$$\Delta = (.40 - 0) / (1 - 0) = .40$$

If one were to buy either .4 futures contract or one option, the same payoffs will be produced with the same probabilities. Thus, the futures contract and the option contract are essentially the same security.

This exercise also allows the instructor to relate pricing of an option using the BOPM to the BSOPM, as well as limitations to the BSOPM. Specifically, the BSOPM values a call option as an asset-or-nothing call option minus a cash-or-nothing call option. Because students are often interested in the valuation of exotic options, we show that the BSOPM can be used to value a binary call option as:

$$C = Qe^{-Rt}N(d_2)$$

where Q represents the fixed payout of the option, in this case, \$0.40. Since the call option allows us to buy the futures at \$0.60, the payout from the call is \$0.40 or \$0.

We can calculate the return of the underlying futures contract. The futures contract has a return of 66.67 percent with a probability of 60 percent and a return of −100 percent with a 40 percent probability. The resulting standard deviation is 81.63 for 81 days, or 173.28 percent annualized. With a strike price of \$0.50, a stock price of \$0.60, and the annualized futures standard deviation of 69.31 percent, the call option price using the BSOPM is:

$$C = \$0.40e^{-0.0009(81/365)}N(.4268) = \$0.1707$$

Note, the price of the option under the assumptions of the BSOPM is significantly different than the price using the BOPM. The major factor that results in a different option price is the BSOPM assumptions of a normal continuous distribution. The standard deviation of 173.28 percent is obviously non-normal as there are only two possible outcomes, each with different probabilities. Other

limitations of the BSOPM specifically, and financial models in general, can also be examined for these options.

For example, in general, increasing the volatility of the underlying asset increases the value of the options. Therefore, we would expect that the implied volatility of the option in the BSOPM would be greater than 173.28 percent. However, calculating the implied volatility required to yield an option price of $0.2402 with the BSOPM is an annualized 85.09 percent. The reason for this seeming contradiction is because the option is a binary option that is relatively deep in-the-money. This option pays $0.40 if the Democrats gain 50.1 percent of the vote and nothing otherwise. In this case, a zero volatility is optimal since that would imply no chance that the option finishes out-of-the-money. A higher volatility increases the probability that the option finishes out of the money.

We can also relate this seeming contradiction to option strategies, as well as several Greeks. For example, gamma, which measures the delta sensitivity of an option, and the vega, or dollar impact on the option price for a one percent change in volatility, are not as expected in this case. For out-of-the money call options, both gamma and vega are positive. For a bull spread and binary option, both the gamma and vega can be negative.

This result provides an excellent classroom opportunity to begin comparing options and futures in markets where the outcomes are not as simple as either winning or losing such as derivatives on financial assets or commodities. In a binomial world, futures and options are the same, but that is not the case for derivatives on continuous price distributions. Futures and options are known to have very different potential payoffs in a continuous price distribution, such as a stock price index. The valuation of the call option utilizing the BSOPM also provides an opportunity to discuss the limitations of financial models and the importance of understanding the underlying assumptions of any model.

IV. Classroom Experience

This illustration was given in a senior level undergraduate course on derivatives in the Fall 2012 semester. Since most of the semester occurred during the run up to the U.S. Presidential election, the IEM futures on this election were of interest. After providing the above illustration and analysis, the class was given the opportunity to trade these contracts synthetically with play money at the beginning of each class period. It was a useful experience for the students to see how the prices of these contracts changed over time as events unfolded during the election.

As stated before, the IEM exchange trades derivatives on a variety of political events. It is most likely that during any time in the future, there would be a contract of interest for illustration and analysis for an investments or derivatives course.

Evaluation of student comprehension of the relation between IEM futures and option prices was done by exam question, including quantitative and conceptual

problems. For example, students were asked to calculate the value of a cash-or-nothing call option, as well as the delta of the option. Discussion of the valuation of political event derivatives also gave the students a practical application of the limitations of the BSOPM, especially as it relates to the normality assumption. This led directly to conceptual questions about the normality assumption of the BSOPM and how violation of this assumption can affect the value of an option.

V. Conclusions

This paper provides an example of how derivatives on political events can be used to illustrate the application of the Binomial Option Pricing Model in an actual binomial environment. This illustration provides a unique classroom opportunity to compare futures and options contracts on binomial versus continuous distributions and to expose the weaknesses of the Black-Scholes Option Pricing Model on a non-normal underlying distribution.

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A Trading Game for an Introductory Finance Course with Assessable Learning Outcomes

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This paper describes a simple stock trading game that may be incorporated into an introductory finance course with a small amount of time devoted to instructor administration. The game is designed to help students meet course learning objectives through experiential education. Student learning is assessed using results from a post-game quiz for direct evidence. Student reflection on the trading experience may serve as indirect evidence of learning. Quiz results suggest that students significantly improve their understanding of two course learning objectives and increase their perceived knowledge of the stock market. While improved learning outcomes may not be exclusively associated with the trading game, the majority of participating students have expressed that the game enhances their understanding of course material.

INTRODUCTION

This paper describes a simple trading game that may be used in an introductory finance course to enhance learning about common stock and the stock market. The game is designed to help students meet learning objectives, adds an experiential learning component to the course, and requires a relatively small amount of instructor effort in administration and grading. The game produces assessable learning outcomes that provide feedback for improving the course.

The game is motivated by the desire to implement a simple yet meaningful learning activity for all students and particularly for non-finance business majors who will likely take only one finance course in their undergraduate curriculum (Bianco and Bosco, 2011). One author has experimented with a variety of trading games in introductory finance courses and has found the time in administration to be burdensome, a high degree of student burn-out if the game endures for a long period of time, and infrequent or frivolous trades that do not appear to help the student gain knowledge. Shortening the duration of the game, tying trades to

learning objectives, and placing the burden of proof on the students has greatly improved the experience for both the students and the instructor.

Research supports the notion that practical experience enhances learning in finance. King and Jennings (2004) find higher learning outcomes in a course that includes a technology-based experiential component compared to outcomes in a traditional lecture setting without an experiential component. Moffit, Stull and McKinney (2010) assess learning that occurs in a stock trading simulation without any related content coverage by the instructor and suggest that simulations are effective teaching tools without instructor intervention in the learning process. Additionally, market simulations are associated with increased student enjoyment in learning (Ascioglu and Kugele, 2005).

The trading game is described and presented as it would be assigned to students. Since the game is based on course learning objectives, the paper also demonstrates how to assess learning by using a pre-test, post-test, and the last trading assignment. The game begins with a short-answer pre-test to establish baseline knowledge prior to classroom discussion of equity markets and valuation. Four weekly trading assignments are designed to give students some practical experience with topics related to two course learning objectives. A post-test that is essentially the same as the pre-test records student knowledge at the game's conclusion. Student scores on the post-test directly measure learning outcomes for assessment purposes. A comparison of pre- and post-test average scores as well as student reflection on the trading experience provide indirect evidence of learning.

To generate the assessment results presented herein, two instructors at the same university administered the game over two semesters in a total of seven sections of an introductory finance course required for all business majors. All sections of the introductory finance course have common learning objectives. One instructor required the game as part of the course, and the other instructor offered the game as an extra credit assignment. Results presented in this study are with the permission of participating students, where inclusion of a student's work for this research is voluntary and not rewarded with grade credit. The authors have obtained approval from the university's Institutional Review Board for this research prior to commencing the investigation.

Students participating in this research completed four assignments over a four-week period that included required trades and a brief reflection on each trading experience. Students also completed the pre- and post-tests given as pop quizzes at the beginning and conclusion of the game. Post-test results indicate that the majority of students either meet or exceed expectations for each learning objective. Matched sample results from the pre- and post-tests indicate an increase in knowledge related to each learning objective. Further, student perception of stock market knowledge significantly improved at the conclusion of the game. Finally, student reflections on their trading experiences indicate an improved understanding of the stock market and the relationship between risk and expected

return. Collectively, this indirect evidence suggests that the trading game enhances students' ability to meet course learning objectives.

This paper's contribution is to share an experiential learning activity that works for both students and instructors in an introductory finance course. Market simulations are a routine component of investments courses to augment student learning through experience (Dicle and Levendis, 2011 provide a good list of articles on stock market games). Less common (and undocumented to the authors' knowledge) is the inclusion of a trading game in an introductory finance course as a pedagogical tool. Like all trading games, this game enhances student learning through practical experience researching and trading stocks, helping students connect material presented in classroom lectures with marketplace outcomes. What makes this game unique is that trading and reporting criteria are founded in the course learning objectives. Further, the game is designed to hold students accountable for their participation, where students provide easily graded proof of fulfilling the game's requirements. The post-test provides feedback to the student on learning that is internalized and not memorized. The scores on the post-test may be used by the instructor for assessing learning outcomes and to generate "closing the loop" ideas such as areas for improved topic coverage or game modification to better aid student learning.

The remainder of the paper is organized as follows: Section 2 reviews the literature related to the benefits of experiential learning in finance in the broader context of financial literacy and the costs of including a trading game in an introductory finance course. The pre-test, trading simulation and post-test are described in Section 3. Section 4 demonstrates how the game's components provide both direct and indirect evidence of learning, and Section 5 concludes.

LITERATURE ON THE BENEFITS AND COSTS OF INCLUDING A TRADING GAME

A trading game is a form of financial education through experiential learning. Financial education is in principle associated with financial literacy. Hastings, Madrian and Skimmyhorn (2013) identify a variety of financial literacy definitions based on a meta-analysis of research on financial education, financial literacy, and their associations with financial outcomes. Such definitions include knowledge of financial products, knowledge of financial concepts, mathematical skills necessary for effective decision making, and engaging in activities that build financial literacy (e.g., trial and error with financial products). They conclude that the evidence is mixed regarding the efficacy of financial education on financial literacy and financial outcomes. Further, financial outcomes (e.g., experience with financial activities and products) may cause financial literacy. Hilgert, Hogarth and Beverly (2003) find that practical experience with financial activities improves financial outcomes in a broad cross-section of households where households with good

financial outcomes are most likely to gain financial literacy through experience, family, friends, and the media.

Lusardi, Mitchell and Curto (2010) find that relatively low financial literacy among the youth is partially offset by higher education and parental transfer of financial knowledge. Among college students, Cude, Lawrence, Lyons, Metzger, LeJeune, Marks and Machtmes (2006) also find that parents are important to their children's attainment of financial literacy.

While a solid connection between financial education and financial literacy remains an open question, students desire to attain more financial knowledge (Cude et al., 2006). Ford, Devoto, Kent and Harrison (2007) find that the feeling of intimidation by financial markets is negatively related to knowledge of financial markets. The authors suggest that engaging students in market simulations may reduce intimidation. A trading game may improve student learning (e.g., King and Jennings, 2004; and Moffit et al., 2010) and increase student enjoyment of the course (e.g., Ascioglu and Kugele, 2005). Enhancing financial literacy may be viewed as the broader goal of financial education. A trading game provides experience with financial concepts, products and markets, where such experiences may enhance financial literacy (e.g., Hilgert et al., 2003).

Three costs are associated with including a trading game in an introductory finance course: content coverage, resource cost and assessment of student learning. Most of the content of introductory finance courses is driven by designated learning objectives. Bianco and Bosco (2011) find that over 90% of AASCB accredited schools surveyed cover valuation and capital budgeting in the required finance course for all business majors. Although equity valuation content coverage is fairly standard in an introductory finance course, including a trading game requires that some class time be devoted to explaining the game to students and answering related questions. Running a trading game requires a trading platform. Many free sites are available, but require the instructor to learn the system prior to the game. Finally, the game requires time in monitoring students, grading student submissions and assessing learning value. McClatchey and Kuhlemeyer (2000) discuss the results of their survey regarding ease with which faculty can incorporate a stock market game into a course. Their survey results suggest that, on average, instructors find stock market games easy to implement, easy for students to understand and play and require little class time to explain. Also, using third-party vendor sites makes the game incorporation relatively easy for the instructor compared to faculty-designed and managed games.

GAME DESIGN AND DESCRIPTION

The game is designed to help students meet course learning objectives that map to the College of Business mission of "preparing students to succeed in the

global marketplace through educational excellence." The College has defined undergraduate program learning goals and objectives to guide the fulfillment of its mission. The game relates to the undergraduate program learning goal of business literacy, stating that students will comprehend business concepts across functional areas. This program goal is associated with the learning objective that students will demonstrate an understanding of the basic concepts of finance. The introductory finance course supports this undergraduate program learning objective with a set of standardized course learning objectives and related "traits." The two course learning objectives and related traits that form the basis of the game are:

1. Explain the role and functioning of bond and stock markets
    a. Differentiate between primary and secondary markets and explain the roles of each one
    b. Describe how stocks are quoted and traded on the NYSE and the NASDAQ
    c. Identify and describe the most commonly used stock market indexes

2. Explain the risk-return tradeoff in financial markets including the use of the Capital Asset Pricing Model (CAPM)
    a. Explain the difference between standard deviation and beta as measures of risk
    b. Explain the risk-return tradeoff and be able to explain what kind of risk is priced by the market, systematic (beta) risk with the CAPM or total (standard deviation) risk for less than fully diversified investors.

The trading game commences as students begin to learn about equity valuation in the introductory finance course and runs for a four-week period. In the authors' courses, the game constitutes 5% or 50 of the 1,000 possible course points. This includes six graded items: a pre-test (5 points), four trading assignments (35 points total) and a post-test (10 points). The pre-test is administered at the start of the game to establish baseline knowledge related to the above learning objectives. The four trading assignments (Parts 1-4) are administered in sequence with weekly deadlines, where the Part 2 is assigned on the due date for Part 1 and so on. The trading assignments are posted in the course management system (e.g., Blackboard) and students submit their reports electronically to the course management system. The post-test contains the same questions as the pre-test (except for questions related to prior trading experience) and is administered as a pop quiz shortly after the due date for the last trading assignment. The pre-test is discussed next. This is followed by a description of the simulated trading experience, a presentation of the four trading assignments as they are assigned to students and a discussion of the post-test and measurement of learning outcomes.

Pre-Test

At the start of the game, students complete a pre-test as a pop quiz that measures baseline knowledge. The pre-test is titled as a "survey" to reduce any student anxiety related to test taking. The pre-test (Appendix) contains thirteen questions, two "ice-breaker" questions related to the student's prior experience with the stock market ("Do you own stocks" and "Have you ever traded stocks" with Yes or No responses), one question about the student's perception of stock market knowledge (Likert scale from 1 to 5, 1="very low" and 5="very high"), and ten short answer questions related to the learning objectives (two questions for each of the five traits). The pre-test is worth 5 points in the authors' courses, or 0.5% of the possible course points. Students are told that they will receive full credit for making an honest effort to complete each question and are encouraged to write "don't know" instead of guessing at an answer. Although students receive full credit for completing the pre-test, responses to the learning objective questions are graded for accuracy and the score is shared with students for feedback. Graded pre-tests are not returned to students and answers are not formally reviewed during the game. However, material related to each learning objective is discussed in lectures and students complete homework and quizzes on this material during the game.

The pre-test serves as an orientation to the learning objectives for this part of the course and provides feedback to students regarding their current knowledge related to the stock market. Mean responses to the pre-test may also be used to create indirect measures of learning for assessment purposes. The same question on the perception of stock market knowledge and the same ten short-answer questions related to the learning objectives are administered as a post-test at the end of the trading game. An Excel function may be used to calculate the differences in the pre- and post-test mean responses for perceptions of stock market knowledge and the two learning objectives.

The Trading Simulation

The authors set up a private, password-protected contest called a "league" on Wall Street Survivor (WSS, wallstreetsurvivor.com) to conduct the simulated trading game. WSS is a free web-based platform for stock, ETF and option trading. The league is set to expire after five weeks (four weeks are needed for the trading game, one additional week to allow the instructor to grade the last trading assignment). Each student is endowed with $100,000. There is no portfolio reset or position limits. Short selling is allowed, but options and margin trading are disallowed since these topics are not covered in the course. The private WSS league allows for competition across multiple sections of students and reduces the time cost of verifying student participation and trades.

Each assignment requires trades linked to the course objectives and contains a small reflective thinking component. As a brief overview of the four assignments, Part 1 helps students become familiar with the secondary market and trading stock. Part 2 focuses on building an understanding of market indices, their constituents, and discovering previously unfamiliar issuers. Part 3 is designed to increase student understanding of the CAPM, focusing on beta and continuing the theme of discovering previously unfamiliar issuers. Part 4 emphasizes the distinction between market and firm-specific risk and includes a brief questionnaire for reflection on the learning experience. Part 2 is assigned on the due date for Part 1, Part 3 is assigned on the due date for Part 2 and so on.

Each trading assignment is easily graded by holding students accountable for their participation. The burden of proof of assignment completion is entirely on the student and eliminates "detective" work on the part of the instructor. Students must provide at least two screenshots from their WSS accounts related to each assignment. The first screenshot verifies that the endowment is fully invested (no more than $100 cash at the time of the screenshot) and must show the league name, cash balance, and market value of the portfolio. The second screenshot is of the transactions history in WSS, displaying the date and time of the required trades, ticker symbol, quantity traded and other trading data. Students are free to trade as often as they desire and to hold more than $100 cash each week, but must show that they are fully invested and have made the required trades at the time of the screenshot. Full credit is awarded for providing the required screenshots, completing all of the required trades, displaying a cash balance of $100 or less and completing the reflective thinking component. Points are deducted for not meeting the assignment criteria (e.g., screenshots do not show all required trades, missing trade related information, etc.) or for late submission. Students are not penalized for capital losses that occur from trading. Grading involves accessing submitted assignments through the course management system, checking screen shots for the required trades, and verifying that students have provided related information. Time devoted to grading the submissions for a class of 25 students is no more than 30 minutes.

The four weekly trading assignments are presented below as they would be assigned to students. Each assignment contains a preamble that requires trades to clear by 4:00 p.m. Eastern Time on the due date (the preamble is omitted for brevity). The first three parts are each worth 8 points, and Part 4 is worth 11 points.

Part 1

Details about your account and trading rules

a. You have an initial $100,000 to invest in stocks or ETFs of your choice

b. You must fully invest your funds (don't hold cash)
c. Stocks must have a minimum price of $0.25 to purchase, and $3.00 to short
d. Each trade costs $10 in commission, regardless of the number of shares bought or sold
e. The contest ends on Friday, May 16, 2014, end of the trading session

Tasks to Complete

1. Join the FIN310SP2014 "league" on wallstreetsurvivor.com by linking to the following URL (select link or paste into a browser): http://www.wallstreetsurvivor.com/league/FIN310SP2014

Select "Join Private League," enter the password "saints," no quotes, all lower case.

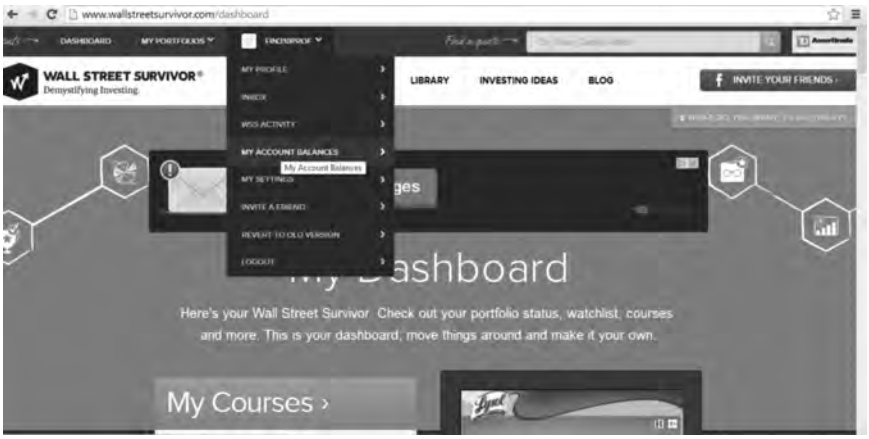
2. Create an account. Click "Sign Up." Fair warning: if you create an account but do not link to the above contest, then you are ineligible for credit.

After selecting "Sign Up," you will be asked if you want to sign up with Facebook or with WSS (Wall Street Survivor). Choose WSS. After signing up, join the class league if you are not already in. You are in the class league if FIN310SP2014 appears under "My Portfolios."

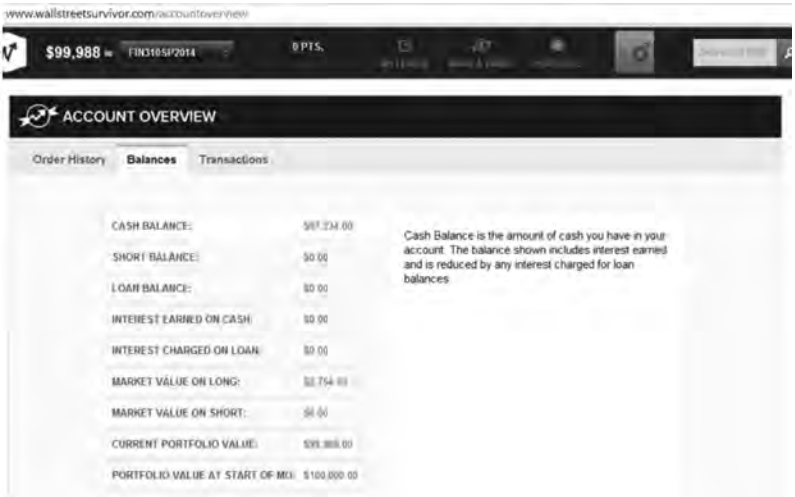
3. Spend $100,000 in either purchases or sales (shorts) of stocks or exchange traded funds (ETFs) by trading a minimum of 5 different stocks/ETFs by Friday, April 11, 4:00 p.m.
   a. You may trade more than 5 stocks, but invest all of your money.

b. This is a large amount of money, so you need to plan the number of shares that you want to trade in each stock to meet the minimum requirement.

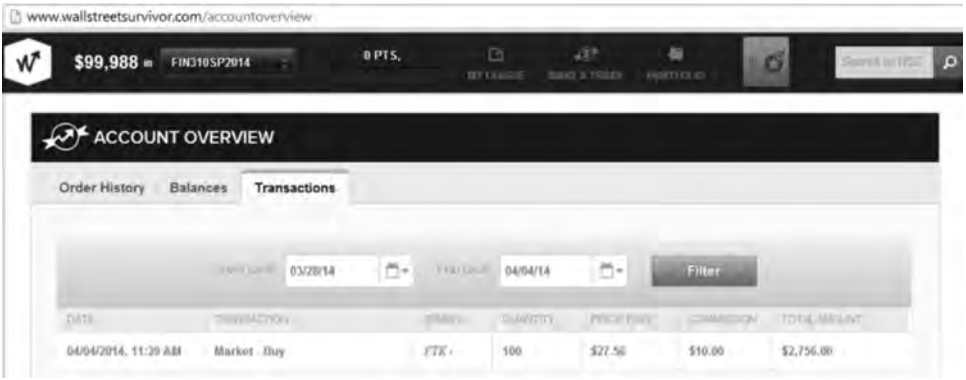
4. Create a Word document, save your file using the file name TP-1-firstname-lastname.
   a. Type your name and Wall Street Survivor name at the top of the Word doc
   b. Select "My Account Balances" from the pull-down menu under your survivor name. This will take you to "Account Overview."



   c. Select the "Balances" tab in Account Overview. Take a screenshot of your account balances, specifically showing your cash balance. Your cash balance should be $100 or less at the time of the screenshot. Paste the screenshot into your Word document.



d. Select the "Transactions" tab in Account Overview. Take a screenshot of your transactions required for this part of the project. You may filter the transactions by date to show the required trades. Make sure that your screenshot shows the required trades. You may need multiple screenshots. Paste the screenshot into your Word document.



e. Below your last screenshot, type the following information for each of the 5 securities traded to meet the requirements for this part of the project:
    i. Ticker symbol, issuer name, number of shares

For example,

Ticker	Issuer	Shares
FTK	Flotek Industries, Inc.	100

1. Upload your Word document by the deadline.

Trading Basics

- To trade securities in this contest, you must place "buy" and "sell" orders through a broker. Wallstreetsurvivor is your broker.
- To execute a trade, the broker must receive an order specification which includes
    - The name of the security
    - Whether to buy or sell
    - Order size
    - Order type
    - Time limit of the order
- Name of the security: enter the stock's ticker symbol
- Buy or sell: hmmmmm…
- Order size: how many shares do you want to buy or sell?

- Order type
    - Market order: an immediate transaction at the best available price
    - Limit order: you name the price and the time limit for the transaction
- Trading hours are M-F, 9:30-4:00, Eastern time
- The Internet has lots of free advice, data, and other information available. Use any and all information to make rational investment decisions.

Part 2

Tasks to Complete

1. Make the following required trades by Friday, April 18, 4:00 p.m. (trades must clear by 4:00 p.m.) while continuing to keep your $100,000 fully invested in either purchases or sales (shorts) of stocks (it's ok to have a small amount of cash, but no more than $100).
    a. Buy a minimum of 100 shares of a stock that is a member of the Dow Jones Industrial Average (Dow or DJIA)
    b. Buy a minimum of 100 shares of a stock with a price that is less than $15 per share
    c. Buy a minimum of 100 shares of a stock containing the name of an animal (e.g., lion, tiger, bull, bear, etc.)
    d. Buy a minimum of 100 shares of a stock issued by a restaurant business

Note: Make four separate trades, don't double count one trade as meeting two or more of the above criteria. Make new trades for this part of the project. Don't count trades that you made for Part 1.

2. Create a Word document, save your file using the file name TP-2-firstname-lastname.
    a. Type your name and Wall Street Survivor name at the top of the Word doc
    b. Select "My Account Balances" from the pull-down menu under your survivor name. This will take you to "Account Overview."
    c. Select the "Balances" tab in Account Overview. Take a screenshot of your account balances, specifically showing your cash balance. Your cash balance should be $100 or less at the time of the screenshot. Paste the screenshot into your Word document.
    d. Select the "Transactions" tab in Account Overview. Take a screenshot of your transactions required for this part of the project. You may filter the transactions by date to show the required trades. Make sure that

your screenshot shows the required trades. You may need multiple screenshots. Paste the screenshot into your Word document.

e. In the space below the screenshot, type the following information related to each required trade (a table is a nice way to summarize this information):
    i. Ticker symbol and issuer names
    ii. Exchange (NYSE, Nasdaq, etc.)
    iii. Share price
    iv. Number of shares purchased
    v. Market capitalization (market cap)
    vi. Beta

3. Upload your Word document by the deadline.

Part 3

Tasks to Complete

1. Make the following required trades by Friday, Apr. 25, 4:00 p.m. (trades must clear by 4:00 p.m.) while continuing to keep your $100,000 fully invested in either purchases or sales (shorts) of stocks (it's ok to have a small amount of cash, but no more than $100).
    a. Buy a minimum of 100 shares of a stock with a beta greater than 1.5
    b. Buy a minimum of 100 shares of a stock with a beta of 1.0 or less
    c. Buy a minimum of 100 shares of a stock containing the name of a color (e.g., green, blue, yellow, etc.)
    d. Buy a minimum of 100 shares of a stock issued by a bank or some other firm in the financial services industry (e.g., credit card issuer, insurance co., brokerage firm, etc.)

Note: Make four separate trades, don't double count one trade as meeting two or more of the above criteria. Make new trades for this part of the project. Previous trades don't count.

2. Create a Word document, save your file using the file name TP-3-firstname-lastname.
    a. Type your name and Wall Street Survivor name at the top of the Word doc
    b. Select "My Account Balances" from the pull-down menu under your survivor name. This will take you to "Account Overview."
    c. Select the "Balances" tab in Account Overview. Take a screenshot of your account balances, specifically showing your cash balance. Your

cash balance should be $100 or less at the time of the screenshot. Paste the screenshot into your Word document.

d. Select the "Transactions" tab in Account Overview. Take a screenshot of your transactions required for this part of the project. You may filter the transactions by date to show the required trades. Make sure that your screenshot shows the required trades. You may need multiple screenshots. Paste the screenshot into your Word document. In the space below the screenshot, type the following information related to each required trade (a table is a nice way to summarize this information):
    i. Ticker symbol and issuer names
    ii. Number of shares purchased
    iii. Market cap and the exchange name (NYSE, Nasdaq, etc.)
    iv. Beta

3. Upload your Word document by the deadline.

Part 4

Tasks to Complete

1. Make the following required trades by Friday, May 2, 4:00 p.m. (trades must clear by 4:00 p.m.) while continuing to keep your $100,000 fully invested in either purchases or sales (shorts) of stocks (it's ok to have a small amount of cash, but no more than $100). [2 points per trade]
    a. Buy a minimum of 100 shares of a stock that has experienced a firm-specific event immediately preceding the time of your purchase of the stock (within the week).
        i. A firm-specific event may be positive or negative news, such as the introduction of a new product, a legal issue, a product recall, the purchase of another firm, earnings announcement, etc. Find firms with firm-specific events by scanning the news on Yahoo! Finance, MarketWatch, Bloomberg, the Wall Street Journal, or any other business news site.
        ii. Take a screenshot of the news item, showing the title of the article and the date. You will paste this screenshot in your report. Make sure the event that you choose is reported within the same week.
    b. Buy a minimum of 100 shares of a stock in the energy sector (e.g., gas, oil, solar, wind, etc.)
    c. Buy a minimum of 100 shares of a stock in the transportation sector (e.g., rail, shipping, air travel, etc.)

Note: Make three new separate trades, don't double count one trade as meeting two or more of the above criteria.

2. Create a Word document, save your file using the file name TP-4-firstname-lastname.
    a. Type your name and Wall Street Survivor name at the top of the Word doc
    b. Select "My Account Balances" from the pull-down menu under your survivor name. This will take you to "Account Overview."
    c. Select the "Balances" tab in Account Overview. Take a screenshot of your account balances, specifically showing your cash balance. Your cash balance should be $100 or less at the time of the screenshot. Paste the screenshot into your Word document.
    d. Select the "Transactions" tab in Account Overview. Take a screenshot of your transactions required for this part of the project. You may filter the transactions by date to show the required trades. Make sure that your screenshot shows the required trades. You may need multiple screenshots. Paste the screenshot into your Word document.
    e. Paste the screenshot of your news item reporting the firm-specific event, showing the title, date, and part of the text of the article.
    f. Summarize the following information related to each required trade in a table such as:

Ticker	Issuer	Industry/Sector	Shares	Market Cap	Beta

        i. Ticker symbol and issuer name
        ix. Industry or sector
        ix. Number of shares purchased
        ix. Market cap and beta

[-2 points if missing, 0 otherwise]

3. Take a screen shot of "Your Rank." Go to "My Portfolios," use the pull-down menu to find FIN310SP2014, select "My League" tab. Your Rank is found on the middle of the page. Paste the screenshot into your Word document. [-2 points if missing, 0 otherwise]

4. Answer the following questions below the last screen shot:
    a. State the overall rate of return on your portfolio (from Your Rank screenshot) [-1 point if missing, 0 otherwise]
    b. State the overall rate of return for the S&P 500 index (from Your Rank screenshot) [-1 point if missing, 0 otherwise]
    c. Briefly compare your performance to that for the S&P 500 [1 point]

d. Did the required trades lead you to discover companies that were previously unknown to you? Briefly elaborate using specific examples. [1 point]
e. What do you consider to be your biggest trading success during this trading game? Briefly elaborate with a specific example. [1 point]
f. What is your biggest trading failure in this short game? Briefly elaborate with a specific example. [1 point]
g. Did this trading game help you better understand the stock market, beta, and the risk-expected return tradeoff in financial markets? Briefly address all three items (it's okay to be honest, honesty is appreciated here). [1 point]

5. Upload your Word document by the deadline.

Post-Test and Measurement of Learning Outcomes

Students complete a post-test as a pop quiz shortly after completing the last trading assignment. The post-test contains the same questions as the pre-test, excluding the first two questions that ask about the student's prior experience with stocks ("Do you own stocks" and "Have you ever traded stocks") and is worth 11 points (1 point per question). Students are told that their scores will be based on the number of correct responses to motivate effort in answering the questions.

Responses to the learning objective related post-test questions provide direct evidence of learning. Individual student scores are categorized as: does not meet, meets, or exceeds learning expectations related to each of the two learning objectives. The cutoff points for these categories are 69% or below, 70-89%, and 90% or above, respectively. As discussed above, indirect evidence of learning may be measured by comparing mean pre- and post-test responses for the perception of stock market knowledge and learning objective questions. The reflection component of Part 4 also provides indirect evidence of learning. Students are credited with points for providing reflection on their learning experience, but the answers are not graded for accuracy provided each item is answered in sufficient detail. Assessment results from the game are described next.

SAMPLE DESCRIPTION AND RESULTS

The trading game is assigned to students enrolled in the introductory finance course at the authors' university. This course is required for all students pursuing an undergraduate degree in business or economics. Enrolled students are typically in their junior or senior years and traditional college age (early to mid-twenties). The data for this study are collected from a total of seven sections over two semesters

during Fall 2012 and Spring 2013. Although the game is offered to all enrolled students, the sample includes only those students who completed both the pre- and post-game pop quizzes. Students who participated in only one of the quizzes are dropped from this study. A total of 95 students participated in both the pre- and post-tests over two semesters, 51 students in Fall 2012 and 44 students in Spring 2013. All 95 students have granted permission to use their results in this study.

Students are asked about their experience with the stock market prior to the start of the game, including stock ownership and experience trading stocks on the student's own account or through a simulation. Twenty eight percent of the participants state that they own stocks, and 37% of students state that they have traded stocks through a simulation exercise or on their own account. Students who own stocks are not necessarily the same students who report experience trading stocks. Only 15% of participating students both own and have traded stocks prior to the game.

Table 1: Descriptive Statistics

	Maximum score	Pre-game mean score (standard deviation)	Post-game mean score (standard deviation)
Explain the role and functioning of bond and stock markets	6	2.59 (1.36)	3.80 (1.55)
Explain the risk-return tradeoff in financial markets including the use of the Capital Asset Pricing Model	4	1.36 (0.89)	2.81 (1.16)
Student perception of knowledge of the stock market	1-5	2.23 (0.81)	3.05 (0.84)
Sample size		95	95

Descriptive statistics for the pre- and post-test results are displayed in Table 1. The pre-game average score related to the first learning objective, "stock market roles and functions," is 2.59 out of 6 possible points, with the average post-game score increasing to 3.80. The pre-game average score for the second learning objective, or "risk-return," is 1.36 out of a possible 4 points, and the average post-game score is 2.81. The pre- and post-tests ask students to rate their knowledge of the stock market using a Likert scale from 1 to 5, where a score of 1 corresponds to a rating of "very low" and a score of 5 indicating that perceived knowledge is "very high." The pre-game average perception of stock market knowledge is 2.23, corresponding to a rating between "low" and "medium." The post-game average is 3.05, a rating around "medium."

Table 2: Measurement of Post-Game Learning Outcomes

Learning objective		Total number of students assessed	Number of students not meeting expectations (percent)	Number of students meeting expectations (percent)	Number of students exceeding expectations (percent)
Explain the role and functioning of bond and stock markets	Fall	51	17 (33%)	28 (55%)	6 (12%)
	Spring	44	17 (39%)	21 (48%)	6 (14%)
Explain the risk-return tradeoff in financial markets including the use of the Capital Asset Pricing Model	Fall	51	22 (43%)	15 (29%)	14 (27%)
	Spring	44	16 (36%)	6 (14%)	22 (50%)

Although the descriptive statistics discussed above combine the results from two semesters, assurance of learning typically involves measuring individual learning outcomes for a given semester. Table 2 displays direct evidence of learning collected from the post-test for each semester in which the game is offered. Since the post-test is administered as a pop quiz, student responses are more likely to reflect leaning and not memorization. Learning outcomes are for the combined sections, three sections in fall and four sections in spring semesters. For the stock market roles and functions learning objective, 67% of fall and 61% of spring semester students either meet or exceed expectations. Regarding the risk-return learning objective, 57% and 64% of students meet or exceed expectations for fall and spring semesters, respectively. These learning outcomes help to identify areas where the trading game may be augmented to better aid students in meeting learning objectives.

Table 3: Paired Sample Means Tests

	Change (post-game mean score minus pre-game mean score)	t statistic (p-value)
Explain the role and functioning of bond and stock markets	1.21	8.39 (0.0000)
Explain the risk-return tradeoff in financial markets including the use of the Capital Asset Pricing Model	1.45	10.08 (0.0000)
Student perception of knowledge of the stock market	0.82	8.70 (0.0000)

Paired sample means tests of to pre- and post-game quiz responses provide indirect evidence of learning. Table 3 shows the change in means scores for questions related to the two learning objectives and the perception of stock market knowledge. The results of paired sample means tests provide evidence that students improved their ability to meet each learning objective at the conclusion of the trading game. The t statistic for the stock market roles and functions learning objective is 8.39 (p-value<0.0001), and the t statistic for the risk-return learning objective is 10.08 (p-value<0.0001). The paired sample means test also indicate that students' perception of stock market knowledge has increased at the conclusion of the trading game. The t statistic for the differences in scores is 8.70 (p-value<0.0001). These results are also supported by a Wilcoxon signed-rank test (not tabulated) that rejects the null hypotheses of no difference in pre- and post-game perception of stock market knowledge.

Part 4 of the trading game also provides indirect evidence of learning. Student reflections on their experience with the game reveal discovery of previously unknown issuers and more importantly, a better understanding of the stock market, beta, and the relationship between risk and expected return. While a few students did not think the game helped with understanding the relationship between risk and expected return, the majority have expressed that the game helped to better understand the stock market. The quotes below are representative of student reflection on the game:

> "Without the required trades, I would have most likely never have known or heard about these companies, and thus feel that the required trades are important as it forces the student to conduct research and find companies that are unknown to them, and forces them to become educated on the company."

> "I now know the difference of the beta within companies and I know that companies that have a higher beta are riskier, but have a higher return in the long run. In the stock market you can play safe and buy stocks that are not as risky, but the return will not be as high, it's a matter of how well does one handle risk."

> "I actually learned a lot from this finance project because I had no idea what people did with stocks or what they meant that the market price dropped and they lost money. I knew that it was an investment but I never knew that every second counted. That things could change within a second and either be positive or negative news to stockowners. I enjoyed this project because now I know what it means to actually be part of the finance world."

Overall, the indirect evidence suggests that learning about stock market roles and functions and the risk-expected return relationship occurs during the game. Learning may not be the direct result of the trading game because students engage in other learning activities during the game. However, the indirect evidence supports the notion that the game enhances learning.

CONCLUSION

This paper describes a simple stock trading game for introductory finance courses that is tied to learning objectives and produces assessable learning outcomes. The game is conducted over a short window to avoid student burn out, gives students an authentic experience with lecture topics, and holds students accountable for their participation in the game. Placing accountability on the students greatly reduces the time spent in grading student reports. The majority of students participating in this study meet or exceed expectations related to learning objectives. Compared the baseline scores on the pre-game quiz, students demonstrate a significant improvement in their understanding of two course learning objectives. Indirect evidence indicates that the game itself enhances learning about the stock market, beta, and the relationship between risk and expected return.

This game may be modified to suit the learning objectives in other introductory finance courses, a money and banking course, or an introductory economics course. The important contribution is to share a meaningful experiential component that may be an easily implemented into a course that reaches a broad constituency of students with the larger aim of improving financial literacy.

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APPENDIX

Pre-Game Survey

Answer each question to the best of your ability. If you don't know an answer, then state "Don't Know" where appropriate. The survey will be graded based on the number of completed answers. Incorrect answers will not be penalized. Total points=5.

1. Do you own stocks? (Circle your response) Yes No

2. Have you ever traded stocks either using your own money or through a simulated trading experience? (Circle your response) Yes No

3. How do you rate your knowledge of the stock market? (Circle your response)

Very Low	Low	Medium	High	Very High
1	2	3	4	5

4. Why do corporations issue stocks? Write a brief answer in the space below.

5. "If you buy 10 shares of Apple stock that has been trading for 5 years, then Apple receives your cash." Is this statement true or false? Briefly explain.

6. Briefly describe how you would buy a stock that trades on the New York Stock Exchange. Write a brief answer in the space below.

7. You want to buy a stock that is listed on the NASDAQ. The bid price is $20.25, and the ask price is $20.50. Which price will you pay, the bid or the ask? Briefly explain.

8. The Dow Jones Industrial Average is a stock index. How many stocks are included in this index? Write a brief answer in the space below.

9. "The Standard and Poor's 500 is a stock index that measures the performance of 500 firms that have relatively small market capitalizations." Is this statement true or false? Briefly explain.

10. Apple's stock has a beta of 0.86 and a standard deviation of returns equal to 10.4%. Microsoft's stock has a beta of 1.12 and a standard deviation of returns equal to 7.4%. Which stock has relatively more total risk? Briefly explain.

11. Apple's stock has a beta of 0.86 and a standard deviation of returns equal to 10.4%. Microsoft's stock has a beta of 1.12 and a standard deviation of returns equal to 7.4%. Which stock has relatively more market risk? Briefly explain.

12. Apple's stock has a beta of 0.86. Microsoft's stock has a beta of 1.12. Which stock has a higher expected return? Briefly explain.

13. What is the relationship between a stock's risk and its expected return? Write a brief answer in the space below.

A Classroom Game for Teaching Asset Allocation

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Trading games are frequently used to teach students about investing in stocks, but a game that simulates the asset allocation process is more difficult to find. This paper introduces a single-period asset allocation game for students of all disciplines that is particularly suitable for an undergraduate Personal Finance course. It also provides alternate versions and game variations for extended or advanced play.
Keywords: stock market game, finance education, asset allocation, personal finance, pedagogy

INTRODUCTION

Trading games are often used in Investments courses to simulate the process of investing in stocks. Depending on the structure of the game, students may have the opportunity to consider macroeconomic factors, develop a trading strategy, and place trades. A simulation allows the student to apply the theory learned in class in a controlled environment without putting real dollars at risk.

Multiple articles have been written on the effectiveness of trading games. Some have focused on the pedagogical approach utilized by instructors, while others have summarized potential advantages and disadvantages to student learning. Overall, the use of simulations appears to be a positive development that enhances student engagement and learning.

However, stock selection is only one part of the investment process. Brinson, Hood, and Beebower (1986) started a debate over the importance of asset allocation with their research on the determinants of portfolio performance. In their initial study, they found that over 90% of the variability of portfolio returns is determined by asset allocation rather than security selection and market timing. Stock selection certainly still plays an important role, and stock trading games can therefore add value. However, if Brinson, et al are correct, and asset allocation is the primary driver of portfolio risk and return, then surely a simulation that focuses on asset allocation would also be worthwhile.

New software that mimics the asset allocation process is now available, and is especially worthy of consideration for use with students in a portfolio management

course. But what about the non-business student who will never take an investments course? Regardless of whether they invest in individual securities, most students will someday need to make allocation decisions in a retirement plan. How can asset allocation be taught in a personal finance course that attracts non-business majors from across campus? Furthermore, can it be taught in a concise manner that does not require significant class time?

This paper introduces an asset allocation game that can be played in a single class period. While some prior understanding of basic asset classes is required, the game is suitable for all majors and all levels of previous investment experience. The game stresses the volatility of returns in the short-term, the growth potential for returns in the long-term, and the uncertainty inherent in making allocation decisions.

Literature Review

While few if any articles on asset allocation games exist, the literature on stock market trading games is more developed. Early articles introduced the basic concept. Tessema (1989) provided an overview of using a stock market game in the classroom, especially as it relates to portfolio planning and monitoring. Williams and Walker (1993) commented on a stock market experiment in their broader discussion of the use of games and simulations in teaching economics. Bell (1993) introduced a game for upper-division students that focuses on valuation.

Subsequent articles revealed the development of new methods and trading systems. Downing (2012) described a unique system used at Seattle Pacific University in which students post trade offers that are visible by all other students in microeconomics classes. Dicle and Levendis (2011) offered an open-source trading game that is interwoven with the capital asset pricing model and generates automated feedback for students.

All of the games and simulations are built upon the belief that hands-on involvement enhances student learning. The literature supports this idea and identifies multiple benefits for both students and instructors. Wood, O'Hare, and Andrews (1992) discussed the costs and benefits of using the Stock Market Game in the classroom. While they concluded that additional research is needed, they also suggested that the benefits may outweigh the costs. Cebula and Toma (2000) concluded that gaming exercises improve student learning and instructor evaluations. Marriott, Tan, and Marriott (2015) surveyed the use of simulations in finance programs in the UK and offered evidence of an enhanced learning experience for students. Dolvin and Pyles (2011) showed that performance in a game does not impact student knowledge and interest level. Importantly, Huagn and Hsu (2011) found that incorporating online games significantly enhances the attainment of student learning outcomes. Specifically, they discussed the use of online games to teach topics such as budgeting, risk and return, consumer credit,

and investments. Finally, Harter and Harter (2010) compared students who participated in a stock market game as a course supplement to those who relied solely on the primary course content. They found that students who participated in the game outperformed those who did not on a financial literacy assessment.

To summarize, since their introduction, stock market trading games have taken on a variety of forms and structures. The evidence suggests that, as a whole, simulations lead to improved student interest and learning. It therefore seems worthwhile to consider a single-class asset allocation game designed to aid students in making future allocation decisions.

Playing the Game

The objective of the simulation is to accumulate the highest ending portfolio value. Before introducing the game, the instructor will need to prepare 1) a student portfolio tracking form, 2) the results for each round of the game, and 3) and the full results table. Ideally, the results for each round and the full results table will be combined into a single powerpoint file that can be used to display results throughout the game. For a copy of a sample file, please contact the author.

	Large Stocks			International Stocks			Bonds			Cash			Total
	Qty	*Price*	*Value*	*Qty*	*Price*	*Value*	*Qty*	*Price*	*Value*	*Qty*	*Price*	*Value*	Value
0	5	$40	$200	10	$20	$200	2	$100	$200	400	$1	$400	$1,000
1													
2													

The portfolio tracking form is used by each student to make allocation decisions and track results. At the beginning of the game, each student starts with a portfolio value of $1,000 that is divided among large stocks, international stocks, bonds, and cash. At the conclusion of each round, the student may "reallocate" funds from one asset class to another.

For example, before round one, a student may elect to reallocate from bonds to international stocks. If the student decides to sell one bond, the resulting $100 (one bond times the round zero price of $100) can be used to purchase five shares of international stock ($100 divided by the $20 round zero price). The student would then record the new quantities on row one, which would now reflect 5 large stocks, 15 international stocks, 1 bond, and 400 units of cash. The student would then await pricing updates for round 1 before calculating the new value of each holding. After all students have made reallocation decisions for round one, the instructor reveals the price changes for round one. The student should write the asset prices on the portfolio tracking form.

ROUND 1							
Large Stocks		Intl Stocks		Bonds		Cash	
Price	Change	Price	Change	Price	Change	Price	Change
$38	-6.00%	$18	-9.20%	$102	1.68%	$1	0.00%

After recording the new prices, the student calculates a new total portfolio value, and makes any reallocation decisions. To calculate the portfolio value after each round, the student should multiply the quantity of each asset times the adjusted price for the same period. To continue the example given above, the student would now have $190 in large stocks (5 shares times $38 per share); $270 in international stocks (15 shares times $18 per share); $102 in bonds (1 bond times $102); and $400 in cash (400 units times $1). The total portfolio value is thus the sum of the assets, or $962. After calculating the new portfolio value, the student makes reallocation decisions for round two, the instructor reveals the prices for round two, and so on. The game ends when all rounds have been completed.

Game Versions

One of the advantages of the simulation is that it can accommodate a variety of historical trading periods and instructional purposes. While the primary intent of the simulation is to provide a broad overview of asset allocation in a single class period, the instructor may elect to play multiple versions over several class periods to more fully develop student understanding of the portfolio management process. This study will highlight four sample games and the supporting rationale for each.

Game 1, Financial Crisis. The base version of the game simulates actual monthly returns during 2008. As the market begins to fall in the second half of the year, students are confronted with a decision: are stocks due to rebound or will they continue to go lower? Participants often become invested in their decisions, and are surprised as large and international stock values continue to fall. The game is also helpful in instructing students about momentum, and is especially effective when paired with Game 4.

	Large Stocks		Intl Stocks		Bonds		Cash	
	Price	*Change*	*Price*	*Change*	*Price*	*Change*	*Price*	*Change*
0	$40	NA	$20	NA	$100	NA	$1	NA
1	38	-6.00%	18	-9.20%	102	1.68%	1	0.00%
2	36	-3.24%	18	1.43%	102	0.14%	1	0.00%
3	36	-0.44%	18	-1.09%	102	0.34%	1	0.00%
4	38	4.86%	19	5.38%	102	-0.22%	1	0.00%
5	38	1.32%	19	1.04%	101	-0.73%	1	0.00%
6	35	-8.42%	18	-8.25%	101	-0.08%	1	0.00%
7	35	-0.85%	17	-3.15%	101	-0.08%	1	0.00%
8	35	1.43%	17	-4.06%	102	0.95%	1	0.00%
9	32	-8.92%	14	-14.51%	101	-1.34%	1	0.00%
10	27	-16.79%	11	-20.08%	98	-2.38%	1	0.00%
11	25	-7.15%	11	-5.49%	101	3.26%	1	0.00%
12	25	1.04%	11	5.99%	105	3.75%	1	0.00%

Game 2, Security Selection. While asset allocation is the primary determinant of portfolio variability, security selection and market timing are also contributing factors. Game 2 is for the instructor who wants to explore an additional component of portfolio management. In this version, three stocks are compared with the S&P 500: stock A (Apple), which is more volatile than the index; stock B (Coca-Cola), which is less volatile than the index; and stock C (Walmart), which exhibits a negative correlation with the market during the time period considered. The instructor can use the game to explain that some large cap stocks are much more risky than others, even if all share a common asset classification. The importance of diversifying within an asset class is therefore necessary to account for variations in firm-specific risk. A discussion of firm-specific risk naturally leads to a discussion of systematic risk, which naturally leads to a discussion of beta, which can be identified for each of the three stocks.

	Large Stocks		Stock A		Stock B		Stock C	
	Price	*Change*	*Price*	*Change*	*Price*	*Change*	*Price*	*Change*
0	$40	NA	$198	NA	$61	NA	$48	NA
1	38	-6.00%	135	-31.66%	59	-3.86%	51	6.75%
2	36	-3.24%	125	-7.64%	59	-0.92%	50	-2.23%
3	36	-0.44%	144	14.78%	61	4.12%	53	6.23%
4	38	4.86%	174	21.22%	59	-3.29%	58	10.06%
5	38	1.32%	189	8.51%	57	-2.74%	58	-0.04%
6	35	-8.42%	167	-11.29%	52	-9.22%	56	-2.67%
7	35	-0.85%	159	-5.07%	52	-0.92%	59	4.31%
8	35	1.43%	170	6.65%	52	1.11%	59	0.08%
9	32	-8.92%	114	-32.96%	53	1.56%	60	1.39%
10	27	-16.79%	108	-5.34%	44	-16.68%	56	-6.81%
11	25	-7.15%	93	-13.87%	47	6.38%	56	0.13%
12	25	1.04%	85	-7.90%	45	-3.41%	56	0.32%

Game 3, Market Timing. The selection of monthly returns from 2008 is effective in playing Game 1 because of the shock value in seeing the steady decline in asset values. However, it is also misleading in that it represents an extreme scenario. An investor who invested during any year since that time would experience a much different result. Consequently, Game 3 is based on monthly returns from 2009, and if used in conjunction with Game 1, allows the instructor to discuss the role of market timing in portfolio performance. A student who employs the same strategy in both games will experience very different results between the two.

	Large Stocks		Intl Stocks		Bonds		Cash	
	Price	*Change*	*Price*	*Change*	*Price*	*Change*	*Price*	*Change*
0	$40	NA	$20	NA	$100	NA	$1	NA
1	37	-8.43%	18	-9.81%	99	-0.88%	1	0.00%
2	33	-10.65%	16	-10.26%	99	-0.38%	1	0.00%
3	36	8.76%	17	6.34%	100	1.39%	1	0.00%
4	39	9.57%	19	12.80%	101	0.48%	1	0.00%
5	41	5.59%	22	11.83%	101	0.73%	1	0.00%
6	41	0.20%	22	-0.57%	102	0.57%	1	0.00%
7	44	7.56%	24	9.13%	104	1.61%	1	0.00%
8	46	3.61%	25	5.44%	105	1.04%	1	0.00%
9	48	3.73%	26	3.83%	106	1.05%	1	0.00%
10	47	-1.86%	25	-1.25%	106	0.49%	1	0.00%
11	50	6.00%	26	2.00%	108	1.29%	1	0.00%
12	51	1.93%	26	1.44%	106	-1.56%	1	0.00%

Game 4, Time Horizon. The first three games are all based upon monthly returns for a period of one year. However, because most investors adopt a longer time horizon, Game 4 incorporates annual returns over a ten-year period. Modeling an extended time period can help students understand the tension between stock volatility in the short term and the benefit of long-term market returns. The focus of Game 4 is 2004-2013, or the ten years surrounding the 2008-2009 crisis.

One effective technique is to play Game 4 in conjunction with Game 1. As students complete the 2008 monthly game, they are made aware of the risk inherent in investing. When they see large cap stocks down 37% and international stocks down 43%, they gain a better idea of what it may have felt like to experience the greatest market downturn since the depression. Some may even exclaim that they'll never invest in stocks.

However, when Game 1 is coupled with Game 4, students see a much bigger picture. Even with the downturn in 2008, investors in large U.S. stocks during the ten-year period earned a cumulative return of 104%, and investors in international stocks earned 95%. As bad as 2008 was, it was more than absorbed by the returns in surrounding years.

	Large Stocks		Small Stocks		Intl Stocks		Bonds		Cash	
	Price	*Change*	*Price*	*Change*	*Price*	*Change*	*Price*	*Change*	*Price*	*Change*
0	$40	NA	$10	NA	$20	NA	$100	NA	$1	NA
1	44	10.90%	12	18.40%	24	20.30%	104	4.34%	1	0.00%
2	47	4.87%	12	4.56%	27	13.47%	107	2.43%	1	0.00%
3	54	15.82%	15	18.26%	35	26.37%	112	4.32%	1	0.00%
4	57	5.49%	14	-1.50%	38	11.19%	119	6.97%	1	0.00%
5	36	-36.99%	10	-33.84%	22	-43.39%	126	5.24%	1	0.00%
6	45	26.48%	12	27.25%	29	31.77%	133	5.94%	1	0.00%
7	52	15.06%	15	26.85%	31	7.76%	142	6.54%	1	0.00%
8	53	2.11%	15	-4.16%	27	-12.13%	153	7.84%	1	0.00%
9	62	16.02%	17	16.26%	32	17.27%	159	4.21%	1	0.00%
10	82	32.37%	24	38.81%	39	22.78%	156	-2.02%	1	0.00%

Variations in Play

Regardless of the version selected, the instructor may further enhance the learning experience by introducing additional complexities into the simulation. For example, prior to the beginning of play, the instructor may elect to modify the objective. Students might be asked to agree on a particular goal that corresponds to the time period represented in the game. If the game is intended to represent twelve months, the class may elect to think in terms of saving for a spring break trip. Or if the game is intended to represent ten years, the objective may become saving for a down payment on a home. In each case, allocation decisions can be interpreted relative to the indifference between reaching the goal and falling short, and a shift in the stated objective from maximizing portfolio value to reaching a target goal leads to a discussion of risk aversion and utility.

The concept of risk aversion can be further explored in a game with a stated objective of saving for retirement. Assuming retirement at age 67, the instructor assigns alternate beginning ages for groups of students in the class (e.g. 25, 35, 45, 55) and initiates Game 4. One would certainly expect differences in risk capacity to be reflected in different allocations for younger and older savers. Inconsistencies

in student allocations provide an opportunity for the instructor to discuss the influence of risk aversion on personal risk tolerance.

In addition to risk, numerous other considerations can easily be incorporated into the simulation. Transactions costs are easily modeled as a commission on every trade (reallocation decision). Taxes on gains can be calculated at the end of each round to stress the importance of tax strategy (assuming a non-qualified account). Systematic withdrawals can be added to simulate distribution planning from an institutional account. All three can contribute to a discussion of the appropriate timing of reallocation and rebalancing decisions.

CONCLUSION

Countless variations can be considered, especially if utilized in an upper-level finance course. However, in each game, and with each alternate scenario, the focus remains on the asset allocation decision. Just as stock trading games have become a popular way to simulate hands-on experience through trading decisions, an effective asset allocation simulation can help provide a better understanding of the asset allocation process. The single-period game introduced in this paper can help students understand volatility, returns, and the components of successful portfolio management. While the base version is intended for non-business majors in a personal finance course, variations in play provide the instructor the flexibility to extend the simulation to multiple periods, to consider additional complexities, and to use the game for more advanced learning.

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Using PHP and Other Web-Based Technologies to Teach Finance

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Web-based technology has changed nearly every aspect of our lives, including teaching and learning. This paper explores the potential of web-based technologies, such as PHP and HTML5, to make teaching easier and to help students learn difficult financial concepts. To begin, a systematic approach to the technology adoption process is described. The paper then moves on to examine ways to use technology that encourage student participation and interaction through the use of a dart portfolio that can be easily tracked online by students and through an online risk assessment test that offers students immediate feedback regarding their own level of risk aversion. Next, web-based applications are discussed that can enhance cases and generate multiple versions of exams that are accurate and relatively easy to grade. Finally, this study explores the use of web-based technologies to help assess student learning and close loops as required by business accreditation.

INTRODUCTION

Web-based technologies offer many opportunities to enhance the teaching and learning experience for students and to help instructors create fast, efficient, and accurate teaching tools. Whether it is an older technology such as PHP or a newer offering such as HTML5 combined with JavaScript and Cascading Style Sheets (CSS3), there are unlimited opportunities to develop interesting and useful tools for students and faculty. Some might argue that this task is best left to textbook publishers or to the creators of learning management systems (LMS), but since these programmers and publishers are not teachers, they must often simply guess at the technologies that would be most useful in the classroom. Publishers, for instance, tie themselves to a particular technology by making huge investments into platforms such as LearnSmart®, myFinanceLab™, and CourseMate. After making these investments, publishers are compelled to spend a great deal of time and effort convincing instructors that these particular platforms offer the best solutions to their teaching needs. Oftentimes, the publishers' technologies fill a crucial educational need and serve students and faculty well. At other times, the attempt to use publisher-

produced technologies seems a bit like trying to fix a mechanical problem on a car with a sledgehammer. The good news about using web-based technologies to develop teaching and learning tools is that almost all of them are free, they are easy to use, and there is help available on the web. The only limits are time and imagination. It is in our best interests as finance teachers to make use of these web-based technologies to develop online tools that improve student learning, encourage active learning, and foster an interest in the discipline of finance.

Over the past few years, the author has developed a number of applications using PHP and other web-based technologies. All of these developments began with a problem or a need. Then a search for a technological solution, if appropriate, began. This is an important principle: first define the problem, and then look for a solution to the problem. Too often, someone presents a technology to instructors and urges them to find a problem or need that the technology can solve or fulfill. More often than not, this approach leads to unsatisfactory results.

The following technology adoption process ensures the effective use of technology in the development of teaching and learning tools:

1. Identify the problem.
2. Determine whether technology should be used to address the problem or not. Perhaps the problem can best be addressed with a non-technological solution such as a change in teaching or testing methods.
3. If technology is a viable approach, identify the technology or technologies that will best address the issue. This might mean using third party technologies that already exist or creating the technology in-house.
4. Implement the technology.
5. Gather feedback.
6. Refine the technology.

In the first section, several learning tools are described that have been developed over the years. The technologies used in each application are discussed, and some examples of what can be achieved with web-based technologies are shown. The next section explains how these technologies can be used to assess teaching and learning, and the last section discusses future research and uses of such technologies.

APPLICATIONS

The Dart Portfolio

The Dart Portfolio was made famous by Burton Malkiel in his book *A Random Walk Down Wall Street* [Malkiel, 1975] and popularized in the *Wall*

Street Journal's experts vs. the darts contest. [Metcalf and Malkiel, 1994] It is often used as an icebreaker in introductory finance classes. All that is required is a bulletin board with names of publicly traded stocks attached and some darts. Students volunteer to throw the darts at the bulletin board and select a portfolio of stocks that the class can then follow over the term. In pre-internet days, students could best follow the stocks selected by subscribing to the *Wall Street Journal* and tracking the portfolio's stock prices every day in the *Money & Investing* section of the paper.

The advent of the internet resulted in a plethora of portfolio trackers that can track a Dart Portfolio's stock much more efficiently. However, there are two problems with these trackers: (1) Students must have access to the portfolio tracker's username and password if an instructor wants them to access the Dart Portfolio outside of class, and (2) Students with access to the portfolio tracker are often sorely tempted to make unauthorized trades in the Dart Portfolio. One way to alleviate these two problems is to use a portfolio tracker that does not need a username or password to access the portfolio, but no financial web site currently offers such a tracker. Fortunately, there are several web sites, such as http://www.codingforums.com/archive/index.php/t-146819.html, that explain how to pull stock prices from Yahoo! Finance to a web page using a technology known as PHP. This means that any professor who is willing to make a small investment of time can develop his or her own customized portfolio tracker that can be accessed directly by students without the need for a username or password.

PHP is a scripting language that was developed in the 1990s in an effort to transform static web pages into dynamic, interactive web pages. Once the ability to interact with databases was added, PHP became a powerful web development tool. One of the greatest advantages of PHP, according to author David Powers, is that, "You can start writing useful scripts without the need to learn lots of theory, yet be confident in the knowledge that you're using a technology with the capability to develop industrial-strength applications." [Powers, 2010] This makes PHP especially useful to educators with little programing experience. If one knows a little bit about programming languages, it is easy to learn PHP. Many colleges and universities already have web servers that are set up to handle PHP scripted web pages, but if not, it is a fairly straightforward process to enable PHP on a server or to set up a test server on a computer for development purposes. Best of all, the PHP software is free and can be downloaded at http://www.php.net.

With PHP, it is possible to set up a Dart Portfolio during class while students are throwing the darts. Exhibit 1 shows the web page where the instructor enters the names, tickers, and closing prices of selected stocks. Students never actually see this input page because it is running on a computer that is not connected to the overhead projector. The stock prices for dart-selected stocks can be projected from the lectern computer for the class to see. Once the date of creation for the portfolio has been set and the names, tickers, and closing prices of all ten stocks have been

recorded, the only thing that remains is to input the closing value of the S&P 500 index and click the submit button.

Exhibit 1: Data Entry Form for the Dart Portfolio

The input page submits all of the data to a MySQL database, and the portfolio can then be shown to the class as an html page on the overhead projector. Exhibit 2 shows the sample portfolio of stocks display page submitted from the Exhibit 1 input page. All students need to access this page is its URL. (The example portfolio can be viewed at http://finance.ewu.edu/finc335/portfolio3.php.)

Exhibit 2: Dart Portfolio Display Page

The Dart Portfolio
Spring 2015

Ticker	Name	Number of shares	Current Price Per Share	Today's Change	Market Value	Gain/Loss Since Inception
BAC	Bank of America Corporation	1,000	$15.41	+0.02	$15,410.00	0.00
CSCO	Cisco Systems, Inc	1,000	$27.25	-0.27	$27,250.00	0.00
CTRX	Catamaran Corporation	1,000	$59.49	-0.05	$59,490.00	0.00
EXC	Exelon Corporation	1,000	$32.85	-0.76	$32,850.00	0.00
GPRO	GoPro, Inc	1,000	$40.89	-2.52	$40,890.00	0.00
DRYS	Dry Ships, Inc	1,000	$0.7371	-0.0229	$737.10	0.00
REXX	Rex Energy Corporation	1,000	$3.70	-0.02	$3,700.00	0.00
MAC	Macerich Corporation	1,000	$78.73	-5.60	$78,730.00	0.00
MON	Monsanto Company	1,000	$116.96	+4.42	$116,960.00	0.00
AA	Alcoa	1,000	$13.00	+0.08	$13,000.00	0.00
				Total Value of the Portfolio	$389,017.10	0.00

This portfolio was created on April 1, 2015. Its original value was $389,017.10
The closing value for the S&P 500 Index on that date was 2,059.69.
The S&P 500 Index currently stands at 2,059.69.
To date, the return for this portfolio is 0.00 percent,
and the return on the S&P 500 index is 0.00 percent.

Once the portfolio is available for students to view, a few moments can be spent discussing some of the features of the portfolio such as the fact that although there is an equal number of shares of each stock, the market value of each stock in the portfolio varies dramatically. Students can also be asked to point out any of the companies in the portfolio with which they are familiar. In this example portfolio, students would probably be familiar with Bank of America, Cisco Systems, and GoPro. Later in the term, the class can explore the unfamiliar companies.

In fact, the portfolio plays an essential role throughout the term. In addition to familiarizing students with the different types of companies that issue securities, the portfolio stocks can be used to teach about financial ratios, stock valuation, dividend policy, risk and return, diversification, benchmarking, market indexes, stock splits and so forth. It offers students a practical real world look at the tools and techniques they are learning in the course. Of course, the portfolio is also a great resource for exam questions. It is no longer necessary to rely on the tired, old hypothetical Company X; instead, the instructor can use real world information that is familiar to her students. In fact, students often comment after a test that they noticed questions where one of the stocks in the Dart Portfolio was the basis for the problem.

Unlike other portfolio trackers on the web, the PHP-generated portfolio developed here cannot automatically account for stock splits or stock dividends.

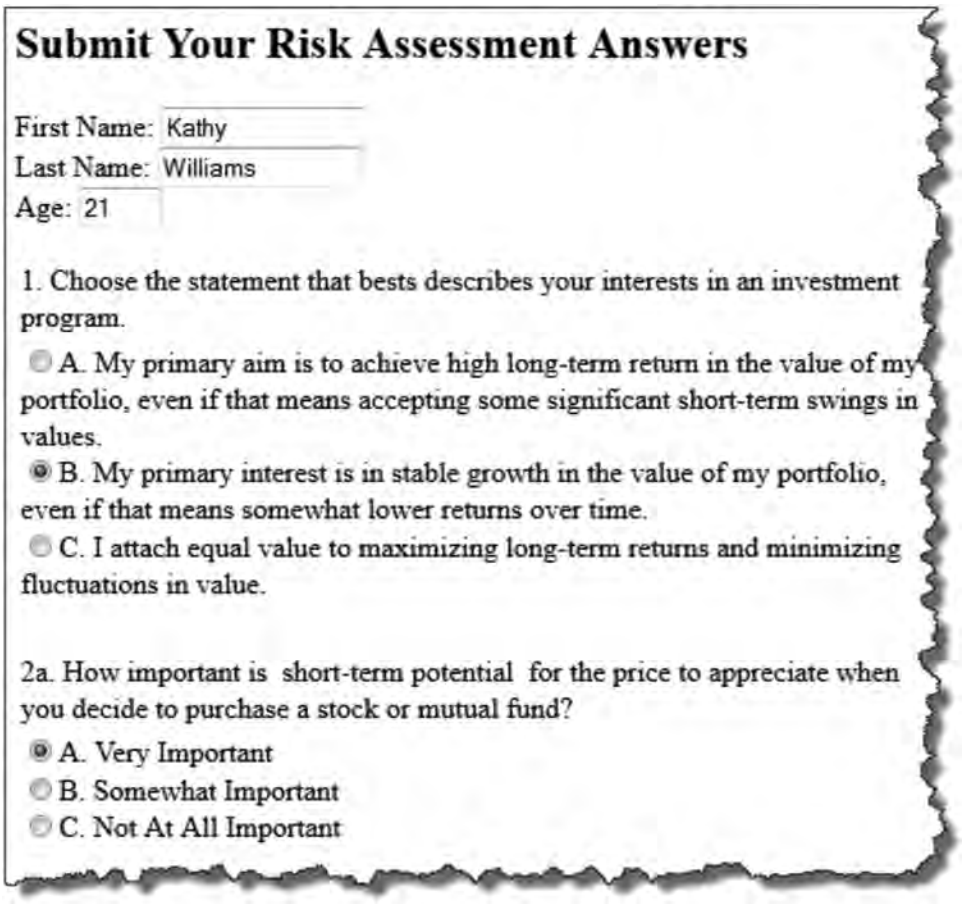
There are two approaches to dealing with a stock split should one occur during the term. The first approach is to tell students on the first day of class about stock splits and their possible effect on the portfolio. Students can then be directed to a stock splits calendar, such as the one at Stock Splits.net (http://www.stocksplits.net/splits.htm), where they can search the coming months to see if any of the Dart Portfolio stocks have set a date for a stock split. If any are found, the class can then discuss how this should be handled in the portfolio. The second approach is to do nothing and wait. If a split happens, it gives the instructor an opportunity to ask the class why the price of the affected stock fell so dramatically. Either way, it is necessary to adjust both the price and the number of shares manually in the PHP code to reflect the split.

The Dart Portfolio is a wonderful tool that is used throughout the class. It gives students concrete, real world examples to work with and makes learning finance relevant.

An Interactive Risk Test from *Fundamentals of Investments: Valuation and Management*

The success of the Dart Portfolio led to an investigation of other educational uses for web technologies. For instance, there is a quiz in *Fundamentals of Investments: Valuation and Management* [Jordan, Miller and Dolvan, 2015, pp. 43-45] that is a useful tool for helping students understand the concept of risk tolerance. Unfortunately, the process of calculating an individual's risk score by hand is cumbersome and often leads to a miscalculated score. Developing a PHP page that allows students to take the risk tolerance quiz online and receive an immediate and accurate score upon completion was the answer to this problem. (See Exhibit 3)

Exhibit 3: Online Questions for the Risk Tolerance Quiz



Submit Your Risk Assessment Answers

First Name: Kathy
Last Name: Williams
Age: 21

1. Choose the statement that bests describes your interests in an investment program.

○ A. My primary aim is to achieve high long-term return in the value of my portfolio, even if that means accepting some significant short-term swings in values.

◉ B. My primary interest is in stable growth in the value of my portfolio, even if that means somewhat lower returns over time.

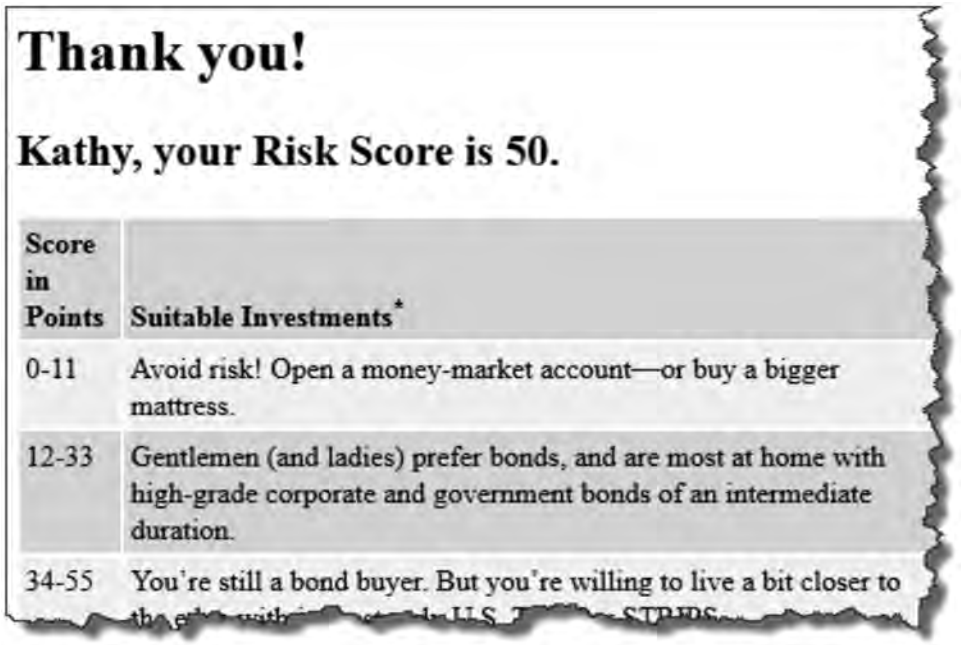
○ C. I attach equal value to maximizing long-term returns and minimizing fluctuations in value.

2a. How important is short-term potential for the price to appreciate when you decide to purchase a stock or mutual fund?

◉ A. Very Important
○ B. Somewhat Important
○ C. Not At All Important

Once a student has answered all of the questions and submits them, he or she immediately receives their risk score along with the **Suitable Investments** table, found in Jordan, Miller, and Dolvan, that describes suitable types of investments for persons within given risk score ranges.

Exhibit 4: Risk Score and Explanation



Thank you!

Kathy, your Risk Score is 50.

Score in Points	Suitable Investments*
0-11	Avoid risk! Open a money-market account—or buy a bigger mattress.
12-33	Gentlemen (and ladies) prefer bonds, and are most at home with high-grade corporate and government bonds of an intermediate duration.
34-55	You're still a bond buyer. But you're willing to live a bit closer to

All of the answers submitted by the students along with their risk scores are posted to a database, which can be checked periodically in order to see if students have completed the assignment. The online risk tolerance quiz is suitable for investments classes as well as personal finance classes. It is very easy for students to complete and gives them instant feedback. In this era of wireless internet access, many students complete the quiz in class immediately after it is assigned.

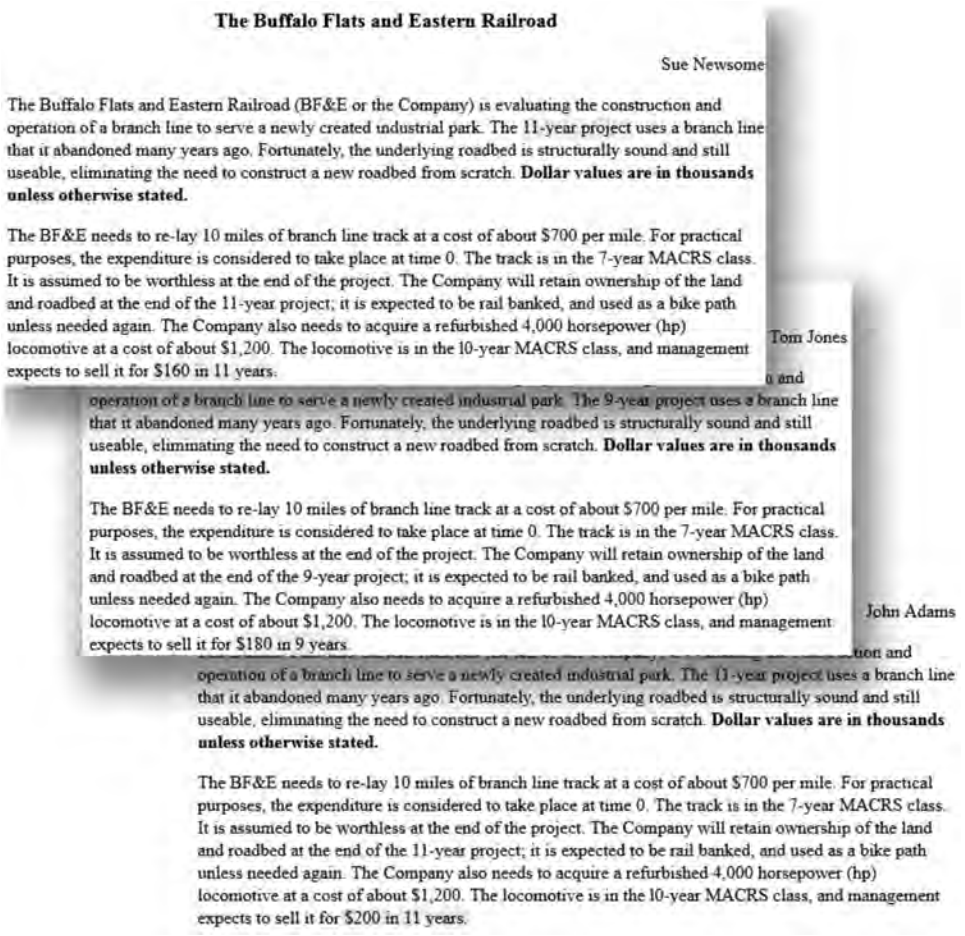
Cases

Cases are a powerful tool for learning and applying financial concepts. However, two issues make cases problematic. First, cases tend to be either too simplistic or too complicated. There is no technological solution to address this issue; it can only be overcome by carefully reading scores of cases and selecting the best ones for classroom use. The second issue is that if a single static case is assigned to the class, there is a great temptation to cheat when the numeric answers to the case are the same for every student. Algorithmically generated homework problems and test questions are common nowadays but not algorithmically generated cases.

The Journal of Financial Education has published many excellent cases. For instance, "Buffalo Flats and Eastern Railroad (BF&E)" [Rozycki, 2011] is an interesting case that describes how to use an Excel spreadsheet to perform

Monte Carlo simulations without special add-ins. The BF&E case was modified using a PHP program that created several randomly generated variables for the case making each student's solution to the case unique. The PHP program also calculated the solutions for each student's case. The case was completed in two stages; the first stage required students to develop their unique version of the static capital budgeting worksheet described in Rozycki [pp. 103-104]. After students successfully completed the static worksheet, they were given a copy of Rozycki's article, which they then used to convert their static worksheets into Monte Carlo simulations. Exhibit 5 shows part of the unique cases that were generated by the PHP program for three hypothetical students. Differences in each case are highlighted.

Exhibit 5: Algorithmically Generated BF&E Cases



The Buffalo Flats and Eastern Railroad

Sue Newsome

The Buffalo Flats and Eastern Railroad (BF&E or the Company) is evaluating the construction and operation of a branch line to serve a newly created industrial park. The 11-year project uses a branch line that it abandoned many years ago. Fortunately, the underlying roadbed is structurally sound and still useable, eliminating the need to construct a new roadbed from scratch. **Dollar values are in thousands unless otherwise stated.**

The BF&E needs to re-lay 10 miles of branch line track at a cost of about $700 per mile. For practical purposes, the expenditure is considered to take place at time 0. The track is in the 7-year MACRS class. It is assumed to be worthless at the end of the project. The Company will retain ownership of the land and roadbed at the end of the 11-year project; it is expected to be rail banked, and used as a bike path unless needed again. The Company also needs to acquire a refurbished 4,000 horsepower (hp) locomotive at a cost of about $1,200. The locomotive is in the 10-year MACRS class, and management expects to sell it for $160 in 11 years.

Tom Jones

and operation of a branch line to serve a newly created industrial park. The 9-year project uses a branch line that it abandoned many years ago. Fortunately, the underlying roadbed is structurally sound and still useable, eliminating the need to construct a new roadbed from scratch. **Dollar values are in thousands unless otherwise stated.**

The BF&E needs to re-lay 10 miles of branch line track at a cost of about $700 per mile. For practical purposes, the expenditure is considered to take place at time 0. The track is in the 7-year MACRS class. It is assumed to be worthless at the end of the project. The Company will retain ownership of the land and roadbed at the end of the 9-year project; it is expected to be rail banked, and used as a bike path unless needed again. The Company also needs to acquire a refurbished 4,000 horsepower (hp) locomotive at a cost of about $1,200. The locomotive is in the 10-year MACRS class, and management expects to sell it for $180 in 9 years.

John Adams

tion and operation of a branch line to serve a newly created industrial park. The 11-year project uses a branch line that it abandoned many years ago. Fortunately, the underlying roadbed is structurally sound and still useable, eliminating the need to construct a new roadbed from scratch. **Dollar values are in thousands unless otherwise stated.**

The BF&E needs to re-lay 10 miles of branch line track at a cost of about $700 per mile. For practical purposes, the expenditure is considered to take place at time 0. The track is in the 7-year MACRS class. It is assumed to be worthless at the end of the project. The Company will retain ownership of the land and roadbed at the end of the 11-year project; it is expected to be rail banked, and used as a bike path unless needed again. The Company also needs to acquire a refurbished 4,000 horsepower (hp) locomotive at a cost of about $1,200. The locomotive is in the 10-year MACRS class, and management expects to sell it for $200 in 11 years.

Since this was a first attempt at creating algorithmically generated cases, only a few variables were selected for random generation. The life of the project ranged

from 8 to 11 years, the sale of the locomotive at the end of the projected ranged from $160,000 to $240,000, the number of carloads generated per year ranged from 3,600 to 5,200, and transportation expenses as a percentage of revenue ranged from 27 to 37 percent. Exhibit 6 shows the solutions for the case assigned to hypothetical student Sue Newsome.

Exhibit 6: Solutions for Sue Newsome

1.

Year	0	1	2	3	4	5	6	7	8	9	10	11
Revenues		$4,160.00	$4,284.80	$4,413.34	$4,545.74	$4,682.12	$4,822.58	$4,967.26	$5,116.28	$5,269.76	$5,427.86	$5,590.69
Maintenance		832.8	857.78	883.52	910.02	937.32	965.44	994.41	1,024.24	1,054.97	1,086.62	1,119.21
Transportation expenses		1,248.00	1,285.44	1,324.00	1,363.72	1,404.63	1,447	1,490.18	1,534.88	1,580.93	1,628.36	1,677.21
Depreciation		1,120.30	1,930.30	1,397.10	1,012.54	735.74	712.84	703.70	390.80	78.72	78.60	39.36
Earnings before interest and taxes		958.90	211.28	808.72	1,259.46	1,604.42	1,697.52	1,778.97	2,166.35	2,555.15	2,634.28	2,754.91
Taxes		335.62	73.95	283.05	440.81	561.55	594.13	622.64	758.22	894.30	922.00	964.22
Earnings after taxes		623.29	137.33	525.67	818.65	1,042.87	1,103.39	1,156.33	1,408.13	1,660.85	1,712.28	1,790.69
Add depreciation		1,120.30	1,930.30	1,397.10	1,012.54	735.74	712.84	703.70	390.80	78.72	78.60	39.36
Operating cash flows		1,743.59	2,067.63	1,922.77	1,831.19	1,778.61	1,816	1,860.03	1,798.93	1,739.57	1,790.88	1,830.05
Net working capital	100	416.00	428.48	441.33	454.57	468.21	482.26	496.73	511.63	526.98	542.79	
Δ net working capital	-100	-316.00	-12.48	-12.85	-13.24	-13.64	-14.05	-14.47	-14.90	-15.35	-15.81	542.79
Cost of tracks	-7,000.00											
Cost of locomotive	-1,200.00											
Sale of Locomotive												160
Tax on Sale of Locomotive												-56.00
Net cash flows	-8,300.00	1,427.59	2,055.15	1,909.92	1,817.95	1,764.97	1,802.18	1,845.57	1,784.03	1,724.22	1,775.08	2,476.84

2. The NPV at 10% is $3,549.15
   The NPV at 20% is -$504.15
3. The IRR is 18.3349%

The algorithmically generated cases worked very well. Although each case had a unique solution, the cases were similar enough that students could benefit from working together on them with less danger of one person simply copying another person's spreadsheet. The solutions generated by the PHP program allowed for a quick grading of the students' spreadsheets and for timely suggestions for improving their spreadsheets before they attempted Rozycki's Monte Carlo procedure.

Algorithmically Generated Exams

The following question was actually on an introductory finance course's final exam:

> Adam wants to buy a new $28,000 Toyota Prius. However, her [sic] previous bankruptcies make it impossible for her [sic] to borrow any money. As a result, he has to pay cash for the car. He doesn't have any cash now. However, he has set a goal to save some money at the end of each month for six years to save up the $28,000 he would need. Assuming that he could earn 7.2% interest, compounded monthly, on his savings, how

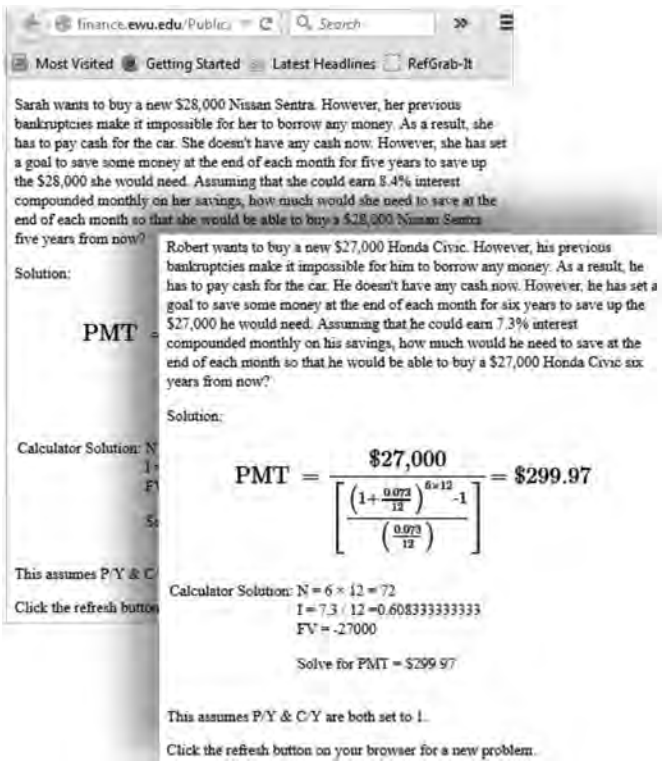
much would he need to save at the end of each month so that he would be able to buy a $28,000 Prius five [sic] years from now?

The author of this question made four versions of each test in order to reduce cheating but, unfortunately, failed to take the time to proofread each version carefully. This question was supposed to be a revision of the following question:

> Jennifer wants to buy a new $25,000 VW Bug. However, her previous bankruptcies make it impossible for her to borrow any money. As a result, she has to pay cash for the car. She doesn't have any cash now. However, she has set a goal to save some money at the end of each month for five years to save up the $25,000 she would need. Assuming that she could earn 6.6% interest, compounded monthly, on her savings, how much would she need to save at the end of each month so that she would be able to buy a $25,000 VW Bug five years from now?

Stylistic issues aside, a carefully constructed problem written in either PHP or JavaScript can mitigate the problems encountered when writing several versions of a test. Exhibit 7 shows the advantages of writing this problem in PHP:

Exhibit 7: Algorithmically Generated Test Problems



The PHP page that generated different versions of the above problem can be viewed at http://finance.ewu.edu/Public/gender_problem.php

For each problem in Exhibit 7, the pronouns match the gender of the name and the years of saving are consistent throughout. With a click of the browser refresh button, a new problem can be created and then copied and pasted into a test. Furthermore, the solution for this new problem can be quickly transferred to the test key. Using such methods gives an instructor access to a huge number of accurate revisions for this particular problem.

This has been such a useful tool that the author now uses either PHP or JavaScript to generate all exams—printed or online. HTML5 used along with JavaScript and Cascading Style Sheets (CSS) offers some very useful features for developing new teaching and learning tools, but before addressing HTML5, we will examine the process of test generation using PHP.

There is no denying it, it takes a great deal of time to write an exam in PHP, but the benefits are well worth the effort. Once a test has been written, it can be opened in a web browser and printed. Of course, an instructor can avoid many problems by taking the test themselves and noting any errors on the printed version of the exam. Corrections made to both the PHP code and the static html version of the exam will ensure that the students' copies of the exam are unlikely to contain errors. If an instructor needs more than one version of the exam, simply generate a new version of the exam and repeat the process above.

A static version of an exam can be created by viewing the page source of an exam in the web browser, copying the html generated by the PHP page, and pasting it into an html file. Exhibit 8 shows how to view the page source in the Firefox web browser while Exhibit 9 shows how to use the Firefox menu to "select all" of the html to be copied and pasted into a new html document.

Exhibit 8: Creating a Static Exam Document

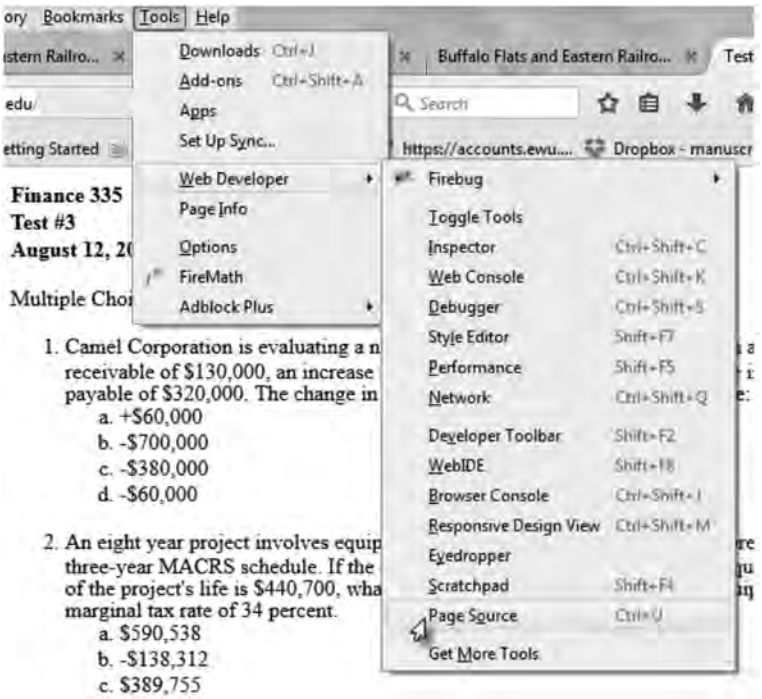
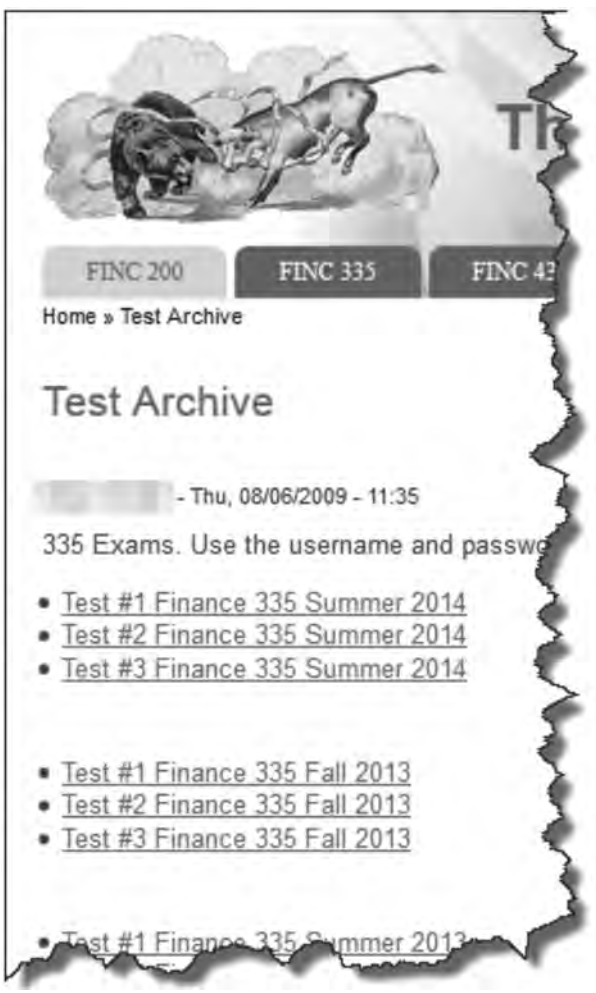


Exhibit 9: Cutting and Pasting PHP Generated HTML into a New Document

Each PHP generated exam creates two versions of the exam; the first is a clean copy that can be printed and distributed to students on the day of the test. The second is an exam key, which serves two purposes: (1) to use to grade exams and (2) to make available in an online test archive. An online test archives offers several benefits. First, students can get a good feel for an instructor's tests, his writing style, and his exam expectations. Instructors benefit by having a well-organized structure for old exams that serve as a reference when writing new exams. The availability of the archive to students also incentivizes an instructor to be constantly looking for new and different ways to write questions.

Exhibit 10: The Test Archive

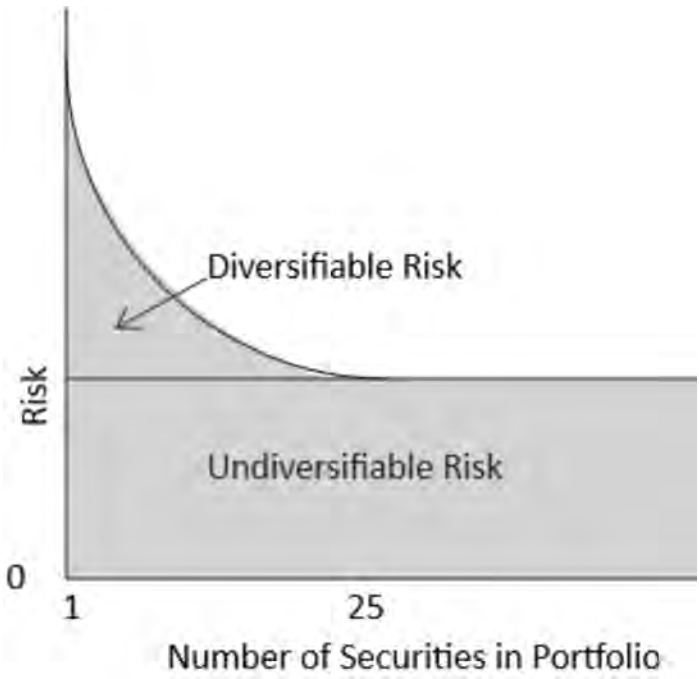


The URL below leads to an example of a static PHP generated exam: http://finance.ewu.edu/Public/PHP_Static_Exam_Example.html

Useful Features in HTML5

One of the most exciting newer features of HTML5 is the Canvas tool (not to be confused with the LMS also known as Canvas.) According to Fulton and Fulton [2013], "HTML5 Canvas is an immediate mode bitmapped area of the screen that can be manipulated with JavaScript. Immediate mode refers to the way the canvas renders pixels on the screen. HTML5 Canvas completely redraws the bitmapped screen on every frame by using Canvas API calls from JavaScript." This means that Canvas can be used for everything from drawing a static graph to creating interactive video games such as Angry Birds™. Exhibit 11 depicts a standard graph often used to show the risk-reducing benefits of diversification. This graph was created using the HTML5 Canvas tool.

Exhibit 11: Portfolio Risk Graph Created with HTML5 Canvas

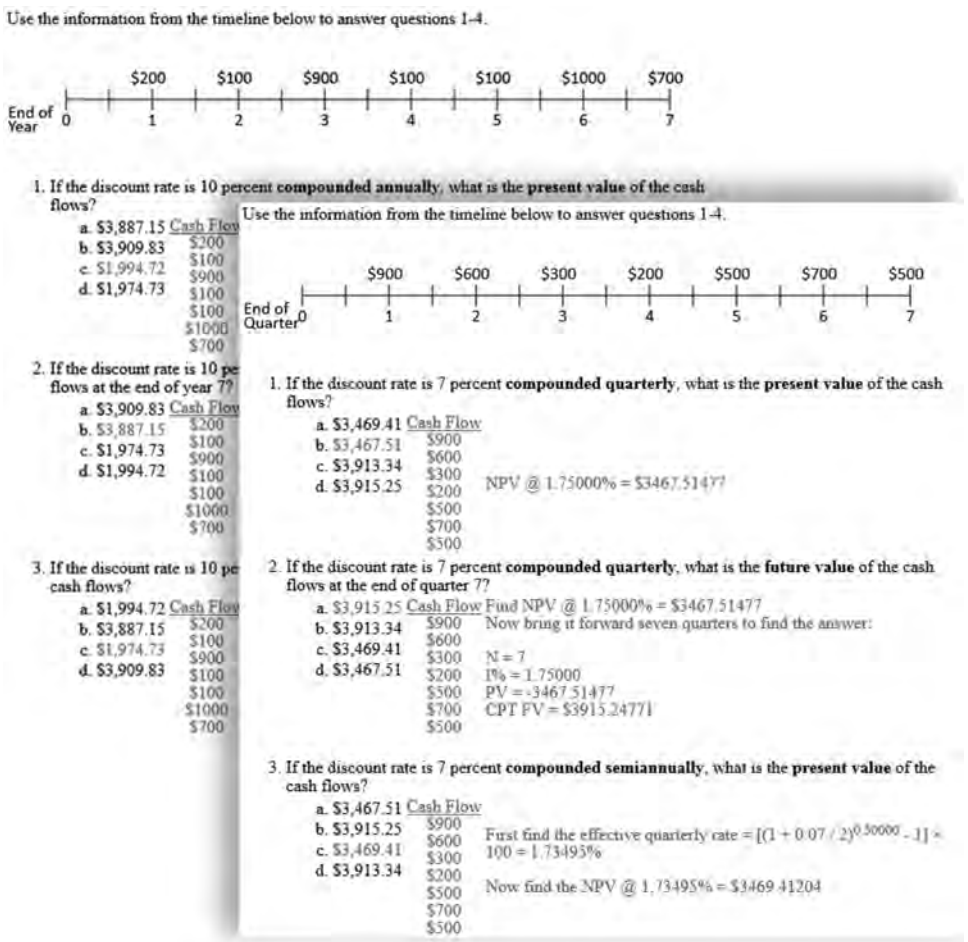


There are several advantages to using Canvas to create images. First, unlike a jpg or gif file, these images can be magnified with less distortion or pixilation. This makes them very useful for enlarging and displaying in a classroom. Second, although the image in Exhibit 11 is static, the Canvas tool can be used to easily make it interactive so that students can use a device such as a slider to increase the number of assets in the portfolio and then watch as the risk of the portfolio falls.

Third, jpg or gif images can be created from a Canvas display simply by right-clicking on the display and choosing the "Save Image as" option.

HTML5 Canvas along with JavaScript and CSS3 can also be used to generate exams. Exhibit 12 is a screenshot of several exam questions that were written using HTML5. The timelines were created using Canvas and generated dynamically. Note that cash flows in the exam question solutions, which are also dynamically generated, correspond to the cash flows on the timeline.

Exhibit 12: Exam Timeline Generated with HTML5 Canvas



Finally, it is possible to use PHP and HTML5 generated questions in learning management systems for online testing purposes. For instance, the Canvas LMS has a quizzing feature that generates online exams. The grouping feature of Canvas allows several questions to be added to a group. Canvas then allows the instructor

to choose the number of randomly selected questions from each group for the exam. One method that works well is to create a group for each test question and to populate that group with five to ten questions. Then set up Canvas to select one question at random from each group for each unique student exam. For short answer questions, creating an html document with solutions for each version of the problem is a straightforward cut-and-paste process that makes grading unique short answer questions almost effortless. Exhibit 13 shows a group of short answer questions in a Canvas LMS quiz.

Exhibit 13: Canvas LMS Grouped Short-Answer Questions

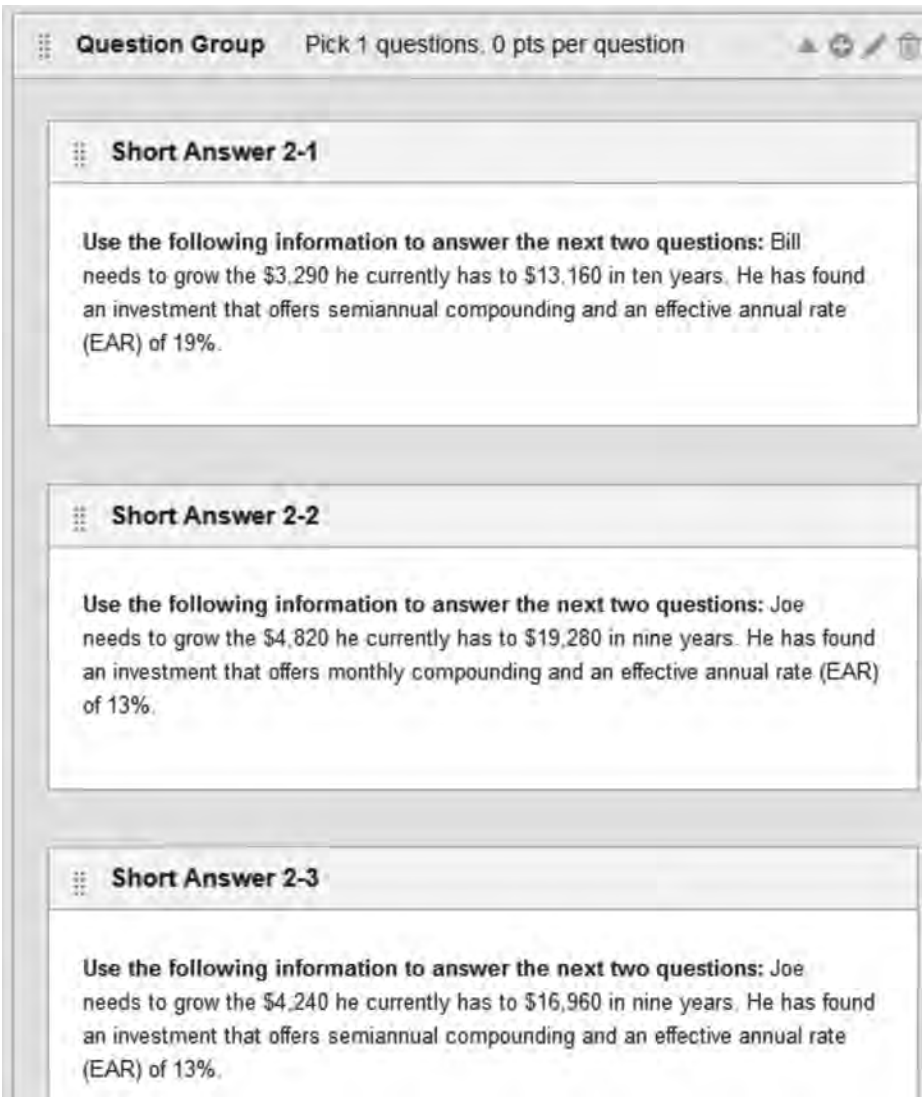


Exhibit 14 shows a portion of the html solutions document used for grading this short-answer question. Using dynamically generated test questions in Canvas LMS groups allows for the creation of online exams that are very similar to the exams given in the traditional classroom. Each online exam is unique because it randomly selects different versions of each question from a group of five to ten variations. The multiple choice versions are automatically graded in the Canvas LMS, but the short-answer questions must still be graded by hand. All online exams are proctored and time limited in an effort to reduce cheating.

Exhibit 14: Solutions to Short-Answer Grouped Questions in the Canvas LMS

2. (2-1) Bill needs to grow the $3,290 he currently has to $13,160 in ten years. He has found an investment that offers semiannual compounding and an effective annual rate (EAR) of 19%.
   a. What is the semiannual interest rate?

      semiannual rate = $(1.19^{1/2} - 1) \times 100 = 9.09\%$

   b. How long will it take him to achieve his desired future value amount if he makes this investment? Will Bill have the money he needs in ten years if he makes this investment?

      I% = 9.087121
      PV = -3290
      FV = 13160
      CPT N = 15.9386950906 ÷ 2 = 7.97 years.

      Yes, it will only take 7.97 years to grow his money to $13,160. So he will easily meet his goal.

   (2-2) Joe needs to grow the $4,820 he currently has to $19,280 in nine years. He has found an investment that offers monthly compounding and an effective annual rate (EAR) of 13%.
   a. What is the monthly interest rate?

      monthly rate = $(1.13^{1/12} - 1) \times 100 = 1.02\%$

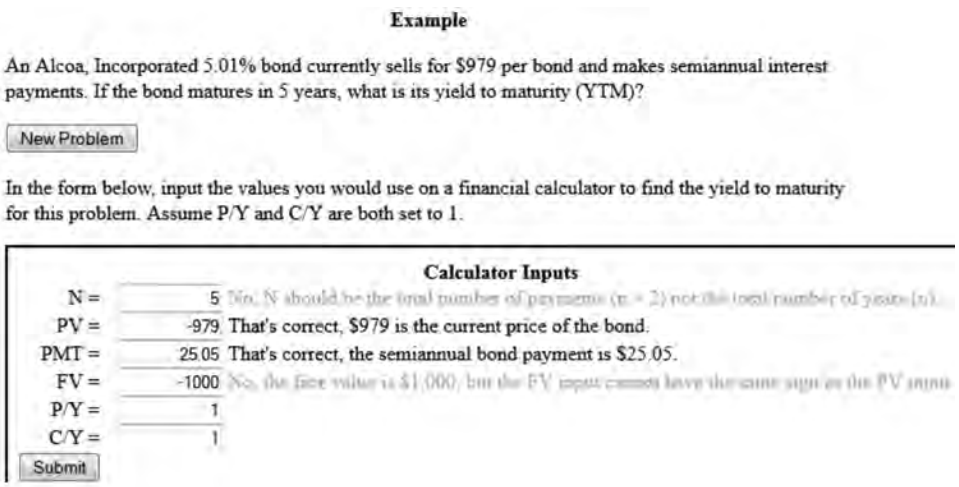
AACSB ASSESSMENT

Any accredited business school understands the need for effective assessment processes that "close the loop" in an effort to improve student learning. Technology, of course, offers many opportunities to aid in the assessment process. For example, test results in introductory finance courses indicated that students were having problems with bond yield to maturities (YTM) and with finding the after-tax salvage value of equipment under consideration for a capital budgeting project. The former issue was mostly a matter of understanding how to use a financial

calculator to calculate YTM while the latter problem arose primarily from an inability to calculate the equipment's book value.

The YTM issue was addressed by placing more emphasis on the process in the lecture. Furthermore, a number of technologies were employed to help students learn how to find a bond's YTM. It is common practice to recommend that students keep the P/Y and C/Y inputs on their financial calculators set to one. Adjustments can then be made to the number or periods input (N) and the interest rate input (I% or I/Y) when dealing with compounding that occurs more frequently than once a year. This means that when a student calculates a typical bond's YTM, the result produced by the calculator is a semi-annual yield. To find the annual YTM, students must then multiply the result their calculator gives them by two. Many students often omit this last step. To remedy this omission, the author developed an HTML5 webpage that employs JavaScript and videos to teach students how to calculate YTM. Exhibit 15a is a display of the aforementioned web page, which includes some common input errors.

Exhibit 15a: Online YTM Tutorial—Incorrect Inputs



This web page may be viewed at the following URL: http://finance.ewu.edu/finc335/lectures/Ross Westerfield Jordan/Yield to Maturity.html

The first input error is that the N input should reflect the total number of semiannual periods, not the total number of years. The second input error is that the FV input and the PMT input have opposite signs. This common error leads to incorrect results. Students receive immediate feedback so that they can correct their inputs. Once the inputs are correct, the page calculates the semiannual YTM and instructs the student to multiply it by 2 to find the annual YTM. (See Exhibit 15b)

Exhibit 15b: Online YTM Tutorial – Correct Inputs

An Alcoa, Incorporated 5.01% bond currently sells for $979 per bond and makes semiannual interest payments. If the bond matures in 5 years, what is its yield to maturity (YTM)?

New Problem

In the form below, input the values you would use on a financial calculator to find the yield to maturity for this problem. Assume P/Y and C/Y are both set to 1.

	Calculator Inputs	
N =	10	Yes, there are 10 semiannual payment periods.
PV =	-979	That's correct, $979 is the current price of the bond.
PMT =	25.05	That's correct, the semiannual bond payment is $25.05.
FV =	1000	That's correct, the face value if the bond is $1,000.
P/Y =	1	
C/Y =	1	

Submit

Solving for I/Y (or I% depending on your calculator) produces a semiannual yield to maturity of 2.74803%.
Multiply this by 2 to get the annual YTM of 5.50%.

If students want another practice problem, they can click the "New Problem" button, which generates a new problem and clears the student's previous inputs. The web page also includes video tutorials below the YTM exercise that explain how to calculate YTM with a TIBAIIPlus, a TI84, and an Excel Spreadsheet.

Before implementing the online learning tools described above, 68% of the students in the winter 2013 course were able to calculate the YTM for a bond that makes semiannual interest payments on an exam. After the learning tools were introduced, 73⅓% of the students in the spring 2013 course were able to successfully calculate the YTM. Although the target for improvement was 75%, this information was used as evidence for closing a loop in our college's AACSB reporting.

Calculating book value has long been problematic for many students, which means that they cannot properly determine the tax on the sale of equipment when a capital budgeting project terminates. To address this problem, an HTML5 page was developed that helped students calculate book value using the methods employed in *Fundamentals of Corporate Finance* [Ross, Westerfield and Jordan, 2010].

Exhibit 16: Determining Book Value Using MACRS Depreciation

Determining Book Value using MACRS Depreciation

Book value is the depreciable basis (or historical cost) minus accumulated depreciation. The depreciable basis is the amount paid for the asset, including all costs related to acquisition such as installation, transportation, and modification costs.

Another way to think of book value is that it is depreciation that hasn't been used yet. Thus, summing all the unused depreciation allowances and multiplying that sum by the depreciable basis will yield book value:

$$\text{Book Value}_t = \text{Depreciable Basis} \times \sum_{i=t+1}^{n} \text{Depreciation Allowance Percentage}_i$$

(Where: t = the end of year t; n = tax life of asset based on the selected MACRS property class; i = year of first unused depreciation allowance)

Inputs:

Depreciable Basis: $ 6500000

Depreciation Method: 5-Year MACRS Class

Find the book value at the end of year: 4 Submit

MACRS Depreciation Allowances

Year	Property Class 3-Year	5-Year	7-Year
1	33.33%	20.00%	14.29%
2	44.45	32.00	24.49
3	14.81	19.20	17.49
4	7.41	11.52	12.49
5		11.52	8.93
6		5.76	8.92
7			8.93
8			4.46

The allowances in blue font are the unused depreciation allowances at the end of year 4.

Book Value at the end of year 4 = $6,500,000 × (0.1152 + 0.0576) = $1,123,200

Example: An asset used in a four-year project falls in the five-year MACRS class for tax purposes. The asset has an acquisition cost of $6,500,000 and will be sold for $1,600,000 at the end of the project. What is the book value of the asset at the end of year 4?

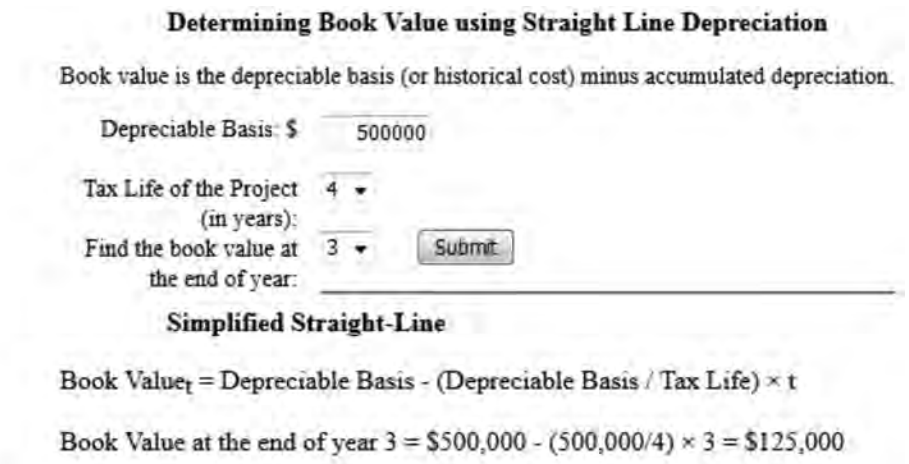
Input the appropriate values into the input section above to see how to arrive at the correct answer for this example. (Book Value = $1,123,200)

Experiment finding other book values. For instance, an asset was acquired for $20,000,000 and depreciated under the 7-year MACRS class. What is its book value at the end of year 3?

This web page may be viewed at the following URL: http://finance.ewu.edu/finc335/lectures/Ross Westerfield Jordan/DeterminingBookValueMACRS.html

Exhibit 16 is a screenshot of the web page that helps students learn how to calculate book value using MACRS Depreciation. Exhibit 17 shows the special case of simplified straight-line, which is used in Ross, Westerfield, and Jordan [2010] and cannot be ignored if an instructor chooses to use Connect for homework assignments.

Exhibit 17: Determining Book Value Using Simplified Straight-Line Depreciation



This web page may be viewed at the following URL: http://finance.ewu.edu/finc335/lectures/Ross Westerfield Jordan/DeterminingBookValueStraightLine.html

To determine whether the Determining Book Value Using MACRS Depreciation web page helped students learn the concept, two sections of an introductory finance course taught in the spring term of 2016 and two sections taught in the summer term of 2016 took an online quiz on book value calculations. Half of each class was randomly selected, given access to the Determining Book Value Using MACRS Depreciation web page, and asked to spend some time using it before taking the quiz. The other half of each class, which did not have access to the web page, was only instructed to take the quiz. Quiz scores for each section are available in Exhibit 18.

Exhibit 18: Book Value Quiz Scores

Section	Average Quiz Score with help	Average Quiz Score with no help	p-value using t-test for means
Spring 2016 -Traditional	9.57/10 (n = 23)	8.52/10 (n = 19)	0.04998*
Spring 2016 - Online	9.65/10 (n = 17)	7/10 (n = 18)	0.00533**
Summer 2016 – Traditional	8.31/10 (n = 13)	8.83/10 (n = 12)	0.33413
Summer 2016 – Online	8.83/10 (n = 12)	9.33/10 (n = 12)	0.26151
Total	9.2/10 (n = 65)	8.29/10 (n = 61)	0.023177*

*Significant at the 0.05 level, ** Significant at the 0.01 level.

Overall, the students who had access to the web page scored significantly higher on the quiz than the students who did not have access to the web page. This was the case even though there was not a significant difference in scores from the summer courses.

To test whether using the web page had any effect on learning retention, a short-answer test question on finding book value was included on each sections' final exam (See Exhibit 19). However, the only statistically significant finding was for the summer online section, where those who had access to the web page actually did worse than those who did not have access to the web page.

Exhibit 19: Book Value Test Question Scores

Section	Average Question Score with help	Average Question Score with no help	p-value using t-test for means
Spring 2016 -Traditional	1.20/2 (n = 23)	1.42/2 (n = 19)	0.11172
Spring 2016 - Online	1.62/2 (n = 17)	1.36/2 (n = 18)	0.13946
Summer 2016 – Traditional	1.58/2 (n = 13)	1.46/2 (n = 12)	0.33724
Summer 2016 – Online	0.67/2 (n = 12)	1.71/2 (n = 12)	0.00029**
Total	1.28/2 (n = 65)	1.47/2 (n = 61)	0.06991

*Significant at the 0.05 level, ** Significant at the 0.01 level.

Summer is an unusual term that packs courses into a much shorter time frame. This often leads to information overload especially when students are taking more than one class during summer. These curious results will be used to refine and revise the web page and lead to continuous improvement in the online learning tools under development. The web page will also be made accessible from many different web locations so students can easily access it when studying for the final exam. During the testing phase, students who had access to the Determining Book Value web page could only access it through the online quiz. Although the results are disappointing, the Determining Book Value web page is a useful tool to bring to the attention of students who are struggling with the concept. It is also a useful in-class tool for demonstrating how to calculate book value.

FUTURE USES

The future of web-based teaching and learning tools is only limited by our imaginations. Many of these technologies are free and easy to learn. There is a

great deal of help available on the web for anyone who is willing to take the time to learn how to use these tools.

Possible future applications include the development of smartphone learning apps for students. Other applications will revolve around new advances in web page development. More interactive tools directly embedded into the online lecture notes hold great promise, as does HTML5's Canvas tool. Its ability to create interactive applications makes it useful for developing online features that engage students as they learn.

RESOURCES

This section provides a short list of resources for anyone who is interested in developing their own web applications

1. If you are interested in setting up a test server:
    1. "AMP" stands for Apache Server, MySQL Database, PHP
    2. WAMP – Windows: http://www.wampserver.com/en/
    3. MAMP – Macintosh: https://www.mamp.info/en/
    4. LAMP - Linux
        - > sudo aptitude install apache2 php5 mysql-server php5-mysql libapache2-mod-php5

2. Books
    1. *JavaScript & jQuery: The Missing Manual* by Luke Welling and Laura Thomson [2009]
    2. *PHP Solutions: Dynamic Web Design Made Easy* by David Powers [2010]
    3. *HTML5 for Masterminds: How to Take Advantage of HTML5 to Create Amazing Websites and Revolutionary Applications* by J.D Gauchat [2012]
    4. *HTML5 and CSS3: Develop with Tomorrow's Standards Today* by Brian P. Hogan [2010]
    5. *Developing Web Pages with jQuery* by Don Gosselin [2013]
    6. *JavaScript & jQuery: The Missing Manual* by David Sawyer McFarland [2012]

3. Websites
    1. php.net - http://php.net/
    2. Dive Into HTML5 - http://diveintohtml5.info/
    3. HTML5 Introduction - http://www.w3schools.com/html/html5_intro.asp

4. lynda.com – This site has great video tutorials on everything from PHP to JavaScript.

CONCLUSION

This is an exciting time to be teaching because so many web-based technology tools are freely available to instructors. It is important for educators to use these tools wisely and to develop teaching and learning tools that are efficient and effective. The technology adoption process suggested earlier should start instructional developers on the right track. Educators must shape the way these tools are used for teaching and learning because they and they alone are in the best position to identify issues that can best be solved using technology. This is something that is simply too important to leave to textbook publishers or LMS providers.

It is also important that teachers share their technological developments with each other and encourage each other to use these tools to enhance the education of our students. Hopefully, this paper is a step in that direction.

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Capital Budgeting and Uncertainty: The NPV Formula and Scenario Analysis as a Heuristic

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Capital Budgeting opportunities are commonly evaluated by calculating the net present value of a proposed investment project. The necessary data are the expected initial investment cost, the expected life of the project, the expected periodic cash flows over that life, and the estimated cost of capital. The point estimates and probability distributions of these variables are inherently uncertain. This paper explores the use of subjective probabilities to assist in the capital budgeting process when precise probability distributions cannot be known. The NPV formula becomes a heuristic, a rule-of-thumb for guiding capital decisions under uncertainty.
Keywords: *NPV, uncertainty, scenario analysis, heuristic, behavioral economics*

INTRODUCTION

A capital budgeting opportunity is commonly evaluated by calculating the expected net present value of a proposed investment project. The data required for this calculation are the following: the estimated cost of the initial investment ($I), its expected income producing life (t=1,…N), the expected cash flow over that life (CF_t), and the firm's calculated cost of financial capital in percent per period (k). Estimates of cash flows depend in turn on estimates of production costs and product prices. All these expected data are single point values or point estimates. The algebraic formula discounts the expected cash flows (I, CF_t) to the present (t=0) at the firm's discount rate k, the DCF:

$$NPV = -I + \sum_{t=1}^{N} \frac{CF_t}{(1+k_t)^t} \quad . \qquad (1)$$

Present value calculations date back at least to the 13th Century in Fibonacci's (Leonardo of Pisa) *Liber Abaci* (1202). He calculated the "alternative use of money" (Goetzmann, 2016, 243-244). For discounted cash flow (DCF) in "historical perspective", see Parker (1968).

Considerable uncertainty is inherent in these values at the time a capital budgeting decision is contemplated. The point estimates are likely mean (or median or modal) values of some subjective probability distributions which, if known (not

subjective), would merit the term "risk". "Subjective probability means the odds you assign to any given situation when you are more or less guessing" (Lewis, 2017, p.183). But such calculations are inherently uncertain. (Al-Najjer et al., 2010; Hampton et al., 1973)

Corporate finance texts approach "riskiness" in capital budgeting decisions by an adjustment to the cost of capital used as the discount rate in equation (1). This hurdle rate is higher for perceived more risky projects, lower for perceived less risky projects. The risk adjusted discount rate (often the weighted average cost of capital) is usually calculated using CAPM for the cost of equity capital; other variables in equation (1) (I, N, CF_t) appear as point estimates in the calculations of NPV. A particular investment project should be evaluated using its own discount rate rather than the firm's overall cost of capital. (Krüger, Landier, & Thesmar,). Capital budgeting techniques—NPV—primarily are taught as if all the variables are algebraic "knowns".

Knight (1921) drew an important distinction: "…[A] *measurable* uncertainty, or 'risk' proper…is so far different from an *unmeasurable* one that it is not in effect uncertainty at all. We shall accordingly restrict the term 'uncertainty' to cases of the non-quantitative type." (p. 20) And later: "Business decisions….deal with situations which are far too unique, generally speaking, for any sort of statistical tabulation to have any value for guidance." (p. 231). Paine (1964) analyzed "Uncertainty and Capital Budgeting" but required knowledge (presumably estimates) of means, variances, and covariances, "probability beliefs."

Keynes (1936) emphasized uncertainty in the *General Theory*, Chapter 12, "The State of Long Term Expectations":

> "Business men play a mixed game of skill and chance." (p. 150)
> "…human decisions affecting the future…cannot depend on strict mathematical expectation, since the basis for making such calculations does not exist; and that it is our innate urge to activity which makes the wheels go round, our rational selves choosing between the alternatives as best as we are able, calculating where we can, but often falling back for our motive on whim or sentiment or chance." (p. 162-163)

Minsky (1975, pp. 62-65) summarized Keynes's emphasis on uncertainty "there is no scientific basis on which to form any calculable probability whatever. We simply do not know." Minsky quoting Keynes (1936, p. 64). But decisions on capital projects—accept or reject—must be made. The U.S. Army Cost Benefit Analysis Guide (2013, pp. 53-64) employs present value techniques and breakeven calculations in evaluating various types of risk.

In their Chapter XV "Treatment of Risk and Uncertainty" Friedrich and Vera Lutz (1951) described the net present value calculations involving the investor's "opinion" of the probability distributions of each of the various values (I, CF_t, N,

k) entering the NPV formula. The estimates (opinions) are based on "correction factors" in turn based on those subjective dispersions (standard deviations, skewness) and on the investor's attitude towards risk. (p. 181) The Lutzes combined the resulting (corrected) subjective probability distributions for each of the variables, adjusting them for any correlations, into a probability distribution of "total net [and discounted] profits" (p. 188). When several opportunities face the investor, his attitude towards risk determines which (if any) of the opportunities are accepted. For each investment opportunity this procedure involves many—hundreds or more—subjective probability distributions: "very lengthy computations on the basis of a large number of probability distributions." (p. 192)

The Lutzes then ignored their theoretical argument: "much cruder methods" are thus used. "[R]evenues may turn out to be lower and costs higher" so in practice the values for I, CF_t, k, and N are adjusted or "corrected" (Lutzes, p. 192):

1. Estimates of revenues and of costs may be subject to a "correction factor" (using lower revenue and higher cost estimates);
2. Adding a risk premium to the discount rate(s):
3. Using a shorter life expectancy; and
4. Reducing the "final figure for estimated net profits."

Thus the investment decision process (NPV) is based on subjective and unknown values adjusted for risk (the Lutzes did not use Knight's distinction between risk and uncertainty). Fama (1977) explored the use of a discount rate adjusted for the uncertainties of cash flows. In common use are risk-adjusted (reduced) expected cash flows and risk-adjusted (increased) discount rates. A practical example of the latter: use one discount rate to calculate NPV of an investment intended to replace assets at the end of their useful lives: use another (and higher) discount rate to evaluate an investment intended to add capacity for existing product lines and markets; and use another (and still higher) discount rate to evaluate investments which anticipate entry into new product lines or geographic markets. Also, the decision to invest (accept a project) can be delayed. This possibility is ignored, but is explored by Meier, et al. (2001) " Jack Welch's phrase 'straight from the gut' crudely sums it up: decisions that matter for investment are intuitive rather than analytical." (Akerlof & Shiller, 2009, p. 144)

This paper explores the implications of uncertainty in investment decisions by suggesting how the NPV formula can be employed in this "mixed game of skill and chance" using subjective probability estimates for the uncertainties of the data (algebraic variables), first each variable considered successively in isolation (sensitivity, one-at-a-time), then the variables considered concurrently (scenario). Then some observations relate the concurrent scenario with some aspects of behavioral economics. The NPV formula remains as an organizing or "framing" concept, describable by the term "heuristic".

Algebraic Interpretations

The decision criterion using the NPV formula: if NPV>0, then accept the opportunity (the investment is expected to increase the owners' wealth); if NPV<0, then reject (expected to reduce the owners' wealth); and if NPV=0, indifferent (the investment would be expected to cover precisely its financial cost, i.e. the normal profit would be exactly covered, with no economic profit or loss) (Graham, Harvey, & Rajgopal, 2008; Lutz and Lutz, 1951). A second criterion also involves equation (1). Setting NPV=0, what is the expected return (in percent per year) of the project: the interest rate or discount rate as a solution (the algebraic root) to equation (1) with k as the unknown, i.e., the internal rate of return. If the IRR>k , accept; if IRR<k, reject; if IRR=k, indifferent (Graham & Harvey, 2001).

Under some conditions (different sizes of investment) the NPV and IRR criteria give different rankings to two (or more) mutually exclusive projects. Osborne (2010) has proposed a resolution to this inconsistency. Also, calculating the IRR may produce multiple roots to the NPV equation. Thus NPV is usually preferred to IRR.

To simplify equation (1): let t be annual (years); CF_t and k_t, all equal ($CF_{t-1}=CF_t$; $k_{t-1}=k_t$) where CF is the difference between annual revenue and annual direct or variable costs; I, undertaken at t=0; receipts (CF_t) at the end of the years; and the disposal cost and salvage value both zero. The k (the discount rate or the cost of capital) is percent per year. With these simplifications equation (1) becomes

$$NPV = -I + \sum_{t=1}^{N} \frac{CF_t}{(1+k)^t} = -I + CF\left[\frac{1-(1+k)^{-N}}{k}\right] \text{ which is the annuity formula.} \quad (2)$$

Viewed algebraically, equation (2) contains five variables: NPV, I, CF, N, and k. If four of them are "known" (i.e. their values can be estimated), then the fifth can be calculated (the equation can be "solved" for the unknown value). Each of these solution values has an economic interpretation. NPV and k (as "unknowns") are emphasized in capital budgeting decisions.

a. If NPV is the unknown, then equation (2) provides the NPV criterion.
b. If k is the unknown (setting NPV=0), then equation (2) provides the internal rate of return to be compared with the cost of capital.
c. If I is the unknown (again setting NPV=0), then equation (2) produces a value for I which can be compared with the expected acquisition cost of the projected investment. If this calculation exceeds the expected acquisition (or investment) cost, then accept; if less, then reject. This value answers the question, "how bad can the point estimate of the acquisition cost (I) be for the project to be not financially viable?"
d. If N is the unknown (NPV=0), then equation (2) produces an estimated value for the income earning life which is the minimum necessary for the project to break even financially, i.e., an estimate of the economic payback period. It suggests an answer to the question "how long must the project last to be financially viable?"

e. If CF is the unknown (NPV=0), then the equation (2) produces an estimated value for the annual economic normal profit, the necessary financial return in dollars per year which will precisely cover the economic (financial) cost of the investment. It suggests an answer to the question, "How great must the annual cash inflow be for the project to be viable?"

Each calculation suggests how "bad" the value would have to be for the NPV to fall to zero.

The idea of economic normal profit as an opportunity cost dates back centuries. Turgot (1770): "…profits [cash flow] at least equal to those which they would derive from their money [financial capital] if they employed it in an entirely different way…" (p. 57; also pp. 53, 61, 64-5, 68-9, 70, 94). Ricardo was the first economist to calculate a numerical value for normal profit, given values for I, N, and k. In Chapter 1 "On Value" in edition 1 (1817) of his Principles and excised by edition 3, he calculated normal profit as the cash flow (equal annually) which would precisely cover the financial cost of an investment. He employed the annuity formula (equation (2) with CF as the algebraic unknown), widely known in financial circles by the early 19th Century. For example "… the present value of an annuity of 16.27£ for ten years, when money is at 10 per cent. is 100£." (p. 55). $\sum_{t=1}^{10} \frac{16.27£}{1.10^t} = 100£$. This is one of over a dozen examples (Chapter 1, first edition) of the calculation of normal profit. Parliamentary debates at this time frequently refer to annuities, suggesting that members seemed generally familiar with the annuity concept if not the calculation. Interest on some loans under Roman law were legitimate: "Interest was recognized as compensation for an alternative use of capital. 'Lucrum Cessans' referred to the return that could have been earned on money that would otherwise been invested in a different asset." (Goetzmann, 2016 p. 235. Also ch. 13).

3.1 First Example (NPV>0)

As the first example of these five ways to view equation (2), consider the following:

I=$-100,000 (at t=0) (expected acquisition cost of the asset or investment)

k= 10% per year (the annual cost of financial capital or the discount rate to compute the DCF)

N= 8 years (expected life of the cash inflow from the investment)

CF= $20,000 per year (expected cash inflow at the end of each year, t=1,…,8).

Inserted as algebraic knowns, in turn, into equation (2) and solved:

a. $NPV = \$ -100{,}000 + \$20{,}000 \left[\frac{1-(1.10)^{-8}}{.10}\right] = \$6{,}698.52 > \$0.$

b. $0 = \$ - 100{,}000 + \$20{,}000 \left[\frac{1-(1+k)^{-8}}{k}\right] \Rightarrow k = IRR = 11.81\% > 10\%.$

c. $0 = -I + \$20{,}000\left[\frac{1-(1.10)^{-8}}{.10}\right] \Rightarrow \text{I} = \$106{,}968.52 > \$100{,}000.$

d. $0 = \$ - 100{,}000 + \$20{,}000\left[\frac{1-(1.10)^{-N}}{.10}\right] \Rightarrow 7.27\ years < 8\ years.$

((d) violates the assumption that the I and CF occur at the end of the relevant years. If the returns and the discount rate are monthly {\$20,000 ÷12 = \$1,666.67 per month and 10%/12 = 0.833%} rather than annual, then the solution to (d) is 84 months or 7 years of monthly returns.)

e. $0 = \$ - 100{,}000 + CF\left[\frac{1-(1.10)^{-8}}{.10}\right] \Rightarrow CF = \$18{,}744.40\ per\ year < \$20{,}000.$

This is the annual economic normal profit which will precisely cover the acquisition cost (\$100,000) at the cost of capital (10%) for an 8 year life. The excess (\$20,000−\$18,744.40=\$1,255.60) is the annual expected supra normal profit.

Each of these calculations in 3.1 suggests acceptance of the project. And each in turn assumes that the other values are "known".

3.2 Second Example (NPV<0)

As a second example consider the same investment opportunity altering only the anticipated annual CF: \$15,000 per year (vice \$20,000) for 8 years, at 10% annual cost of a \$100,000 investment (expected values). Again, considering each of the variables in turn as the algebraic unknown:

a. $NPV = \$ -100{,}000 + \$15{,}000\left[\frac{1-(1.10)^{-8}}{.10}\right] = \$ - 19{,}976.11$ (CF is inadequate to cover all economic costs.)

b. $0 = \$ - 100{,}000 + \$15{,}000\left[\frac{1-(1+k)^{-8}}{k}\right] \Rightarrow k = IRR = 4.24\% < 10\%.$ (IRR is inadequate to cover the cost of capital.)

c. $0 = -I + \$15{,}000\left[\frac{1-(1.10)^{-8}}{.10}\right] \Rightarrow I = \$80{,}023.89 < \$100{,}000.$ (CF is inadequate to cover the acquisition cost of the project.)

d. $0 = -\$100{,}000 + \$15{,}000\left[\frac{1-(1.10)^{-N}}{.10}\right] \Rightarrow N = 11.53 > 8.$ (The CF lasts for too short a period.) (Done monthly as before, exceeds 132 months.)

e. $0 = -100{,}000 + CF\left[\frac{1-(1.10)^{-8}}{.10}\right] \Rightarrow CF = \$18{,}744.40 > \$15{,}000.$ (Again, CF is inadequate.)

As expected, each of these calculations in 3.2 suggests rejection of the project, and suggests how much better the value needs to be for NPV to reach zero (or indifference).

The calculations in 3.1 involve estimated values for I, CF, N, and k which produce a positive NPV. The calculations in 3.2 involve estimated values for I, CF, N, and k which produce a negative NPV. These calculations, however, consider the variables (NPV, k, I, N, CF) one at a time, holding the others constant. All the variables are subject to uncertainty at the time an investment opportunity is considered. An infinite number of combinations of I, CF, N, and k can be calculated to produce precisely where NPV=0, where the firm is indifferent between acceptance and rejection of the project.

4.1 Uncertainty

(I): Subjective probabilities, one variable at a time (*Sensitivity* Analysis)

The calculations in the five examples (3.1), where NPV>0 and the CF_t exceed annual normal profit, provide a useful initial approach to (subjective) uncertainty analysis of a particular investment opportunity. For example, the decision makers could explore an answer to the question " what is the (subjective) probability that the investment cost will exceed $106,698.52?" Similarly: "what is the probability that the investment will not last 7.27 years (or by monthly calculations 84 months)?" And "what is the probability that a rival will enter the industry with a substitute product and drive the CF below $18,744.40?" These probability values are subjective, based on the executives' past experience, knowledge, judgment, and intuition, or Keynes's "whim or sentiment or chance." This is precisely what senior executives are required and expected to do in making investment decisions. These probabilities are not based on precise calculations like the flip of a (fair) coin or the roll of dice or drawing to an inside straight in poker or winning the power ball lottery. Engineering considerations or marketing principles could affect project life or acquisition cost and may impose bounds or restrictions on possible estimates of some values. More important events whose probabilities cannot be calculated but have potentially enormous economic effects include aircraft flying into the World Trade Center or crashing in the Ukraine (and war in the Middle East), the U.S. government's reaction to the mortgage crisis, the Fed's monetary policy, and tax policies introduced by a new president. Economic forecasting is difficult because it is "trying to predict complex systems that can be tipped from one state to another by very small changes" (Mackintosh, 2017).

In contrast, reasonable and useful data-based calculations of probabilities are made by financial firms selling insurance based on mortality tables (as long as individuals' life expectancies are independent), also fire insurance and automobile insurance.

4.2 Uncertainty

(II): Subjective probabilities, more than one variable at a time (*Scenario* Analysis)

Evaluation of uncertainty, using the NPV formula, would be more useful if two (or more) of the algebraic "unknowns" were considered simultaneously, rather than one-at-a-time, as suggested in section 4.1. Setting NPV=0, I= $-100,000, and k=.10, suppose N and CF were considered together, thus both become algebraic unknowns. Equation (2) becomes

$$NPV = 0 = \$ -100{,}000 + CF\left[\frac{1-(1.10)^{-N}}{.10}\right]. \quad (3)$$

Equation (3) then describes the locus (combinations) of CF and N for which normal profit is expected to be precisely covered, i.e., providing neither positive nor negative economic profit (NPV=0). This is the CF-N tradeoff: along this locus, as the expected life N of the project increases, the expected necessary cash flow CF declines. At the extremes, for N=1 and I =$-100,000, the full 10% must be earned in one year, so the necessary CF at t=1 is $110,000. For N=(CF is received in perpetuity) and I =$-100,000, only the 10% or $10,000 must be earned each year, but forever. A geometric plot of CF v. N (I=$-100,000 and k=.10) for NPV=0 is a curve (or locus) in two dimensions; above the curve (to the northeast, away from the origin) the space describes the CF and N combinations for which NPV>0 (given I and k), while below the curve (to the southwest, toward the origin) that space describes the CF and N combinations for which NPV<0. The curve which divides these two spaces is the locus of CF and N for which NPV=0.

Table (1) describes the CF_t values for the various lives of the project (N) (combinations of CF and N) for two of many values of I: $-100,000 and $-125,000.

Table 1. Combinations of Life (N) and normal profit (CF), two values for I, for which NPV=0

N (Life of the project)	For I=-$100,000 Necessary CF_t to cover k=10%	For I=-$125,000 Necessary CF_t to cover k=10%
1	$110,000.00	$137,500.00
2	57,619.05	72,023.81
3	40,211.48	50,264.35
4	31,547.08	39,433.85
5	26,379.75	32,974.69
6	22,960.74	28,700.92
7	20,540.55	25,675.69
8	18,744.40	23, 430.50

Table 1. (*Contiuued*)

N (Life of the project)	For I=-$100,000 Necessary CF_t to cover k=10%	For I=-$125,000 Necessary CF_t to cover k=10%
9	17,364.05	21,705.07
10	16,274.54	20,343.17
11	15,396.31	19,245.39
12	14,676.33	18,345.41
13	14,077.85	17,597.32
14	13,574.62	16,968.28
15	13,147.38	16,434.22
……..	…….....	…….....
20	11,745.96	14,682.45
25	11,016.81	13,771.01
50	10,085.92	12,607.40
∞	10,000.00	12,500.00

Calculations from equation (3) for I=$−100,000 (and then I=$−125,000), at k=10% and NPV=0.

The locus for I=$−125,000 produces values for CF which are 1.25 times the values of CF for I=$−100,000. The necessary (expected and necessary) return (normal profit) declines with a greater expected life of the project for a given investment; and greater expected investment cost requires greater expected CF for any particular life.

If I, N, and CF are considered continuous, then equation (3) becomes

$$NPV = 0 = \$ - I + CF\left[\frac{1-(1.10)^{-N}}{.10}\right]. \quad 4)$$

an equation which has three unknowns (I, CF, N) and defines a surface in these three dimensions. Since NPV=0 this three dimensional surface provides the division of combinations of these three values into accept-reject spaces: accept if the values are above the surface (away from the origin), reject if the values are below the surface (toward the origin).

Further complicating the accept-reject decision is the possibility that the relevant cost of financial capital is uncertain. If k is additionally subject to uncertainty, then the relevant equation becomes

$$NPV = 0 = \$ - I + CF\left[\frac{1-(1+k)^{-N}}{k}\right]. \quad (5)$$

This equation defines the border of NPV=0, a hypersurface in four dimensions (I, CF, N, k) which (by NPV=0) divides the four dimension hyperspace into two regions: accept or reject.

Another complication can be accommodated by a further modification of equation (2). The constancy of the cash flow throughout the life of the project (CF are all equal) ignores the twin mutually exclusive possibilities: that entry by rival firms will occur into the incumbent product's substitutional neighborhood, and that the firm may experience increasing customer acceptance of its product. The first suggests a temporal decline in CF; the second, a temporal growth in CF. Both can be accommodated in the NPV equation by assuming a constant percentage over time, negative for entry of rivals, positive for the firm's growth, applied to an expected CF_t (restoring the t subscript to reflect inequalities in cash flows):

$CF_t = (1 + g)\ CF_{t-1}$ where $g>0$ for growth of the incumbent and $g<0$ for entry of rivals. g and hence CF_t depend on the firm's choice of pricing strategy. Penetration pricing discourages rival suppliers; cream skimming pricing attracts rival suppliers. Simplifying, equation (2) becomes

$$NPV = -I + \sum_{t=1}^{N} \frac{CF_t(1+g)^{t-1}}{(1+k)^t}. \quad (6)$$

Setting NPV=0 provides the hypersurface dividing combinations of I, CF_t, k, N, and now g (hyperspace) into accept-reject regions. If subjective probability distributions are considered for the hundreds or more of variables in the NPV equation (as the Lutzes suggested), then the hyperspace is defined by hundreds (or more) of dimensions. A hypersurface (also in hundreds of dimensions) divides this hyperspace into two regions: accept and reject.

4.3 Heuristics and Prospect Theory for Capital Budgeting Decisions

A heuristic principle "reduce[s] the complex tasks of assessing probabilities and predicting values to simpler judgment operations." (Tversky & Kahneman, 2011, p.1124). The NPV formula for making capital budgeting decisions fits this definition, precisely because the variables are unknown (and unknowable), yet the approach embodies the specific goal of maximizing the value of the firm; the NPV is a rational and logical rule-of-thumb, a short cut, to use when the future values (including the distributions of those values) are unknown. It is a "judgment heuristic".

Tversky's and Kahneman's (2011) early heuristics were three: representativeness, availability, and adjustment from an anchor, which lead to cognitive biases in judgment, "systemic and predictable errors." Jack Welch's "from the gut" likely reflects another: the affect heuristic. Many additional heuristics have been put forward. "The heuristic question is a simpler question [is NPV $\gtreqless$0 ?] that you answer instead." (Kahneman, 2011, p. 97).

The general term covering most of the literature in behavioral economics is "Prospect Theory: An Analysis of Decision Under Risk" (Kahneman & Tversky, 2011; Barberis, 2013). Much of the empirical information is based on decisions

of groups of individuals (samples) answering questions, usually involving preferences and choices in hypothetical settings in laboratory (experimental) settings (Barberis, p. 173). A major conclusion is that individuals regularly (but not always) violate economists' expected utility theory, which earned Kahneman The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel (2002). Had Tversky lived he would have shared the prize. The experimental procedures sometimes involve questions to which the answers should be logically derived, yet the lab subjects often respond with inconsistent (hence illogical) answers. In other lab procedures the subjects are asked to compare ("which would you prefer?") the utilities of different gains (or losses), or of a gain and a loss (for example Kahneman, 2011, pp. 334-336).

In contrast the answer to a (single) capital budgeting question (accept or reject) cannot be "known" in the same sense at the time the decision is made. The "correct" answer (ex ante NPV$>$0 or NPV$<$0) will become known only after some (often considerable) time has passed and the results are realized. How, then, can the NPV heuristic assist in making (improving) the decision? The heuristic/prospect theory literature suggests that the biases which operate (unconsciously) in decisions involving uncertainty may also be operating in capital budgeting. Awareness of some possible biases on the part of CFOs may blunt or remove the unconscious effects of those biases.

Prospect theory can be used in a prescriptive way (in contrast with a descriptive way) to nudge CFOs toward better capital budgeting decisions by being aware of possible biases in their evaluations. Perhaps the greatest benefit from using the NPV heuristic is that it focuses the attention of the CFO ("framing" the problem) on the firm's objective, maximizing the value of the firm by investing in capital projects whose return exceeds the firm's cost of capital, where all the variables should be considered together, as in scenario analysis.

5. Discussion

The equations in sections 3.1 and 3.2 and discussed in section 4.1 allow executives evaluating a proposed investment to evaluate the uncertainties of individual estimates of the variables considered singly to turn out to be less favorable than the ex ante point estimates. If the NPV is estimated to be negative, then the executives can explore the possibility that one (each, considered in isolation) variable might be more favorable than the ex ante point estimates. If the NPV is estimated to be positive, then the executives can explore the possibility that one (or several) of the variables will turn out to be less favorable. These possibilities require the use of subjective probabilities. These one-variable one-at-a-time assessments involve sensitivity analysis: how sensitive is the accept-reject decision to changes in one variable. And this procedure risks the biases in "narrow framing".

Section 4.2 allows several (more than one) of the variables to be considered simultaneously, i.e. sensitivity analysis applied simultaneously to two or more variables. For example, if a point estimate produces NPV>0, what is the risk of CF_t being smaller and N being shorter (eq. 3)? This involves a trade off between CF_t and N. The executives would consider their subjective and joint probability that both CF_t and N would change so that NPV<0 (or hit the curve where NPV=0). This CF−N trade off is illustrated (twice) in table 1, for two values of I. For eq. 4: CF, N and I can be considered jointly: what is the executives' subjective and joint probability that these variables would turn out together to produce NPV<0. Similarly for eq. 5 (with CF_t, N, I, and k) and eq. 6 (adding g).

If the ex ante point estimates produce NPV<0, the question for the decision makers is what is the joint (and subjective) probability that one or more of the variables can be improved (greater CF_t, N, g; smaller I, k) to produce an ex ante calculation for NPV>0? Can the ex ante value for the incumbent firm's expected growth of CF_t (g>0, or market penetration) be legitimately increased, or the ex ante value for expected entry be decreased (smaller negative number)? And how can these more optimistic assumptions for g be combined with other (also more optimistic) assumptions of ex ante values of other variables? The NPV equation, interpreted as embodying variables with joint subjective probabilities, provides business decision makers with an opportunity to exercise their experience, knowledge, and judgment—their "animal spirits"—in making investment decisions. Scenario analysis (section 4.2) focuses the CFOs attention on all the influences together rather than one at a time, i.e. equation 6 "frames" the question appropriately. Awareness of possible cognitive biases in the process lessens the possibility of making errors in judgment. Heuristics and prospect theory do have a place in capital budgeting.

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Financial Calculator Settings, Framing, and the Time Value of Money—a Test of Annuity Problems

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Professors frequently worry about their students substituting memorization for learning. In the field of finance, this often occurs with the reliance on financial calculators, especially in the case of longer problems for which such calculators are useful. In this study, we compare two ways of teaching annuity problems: the traditional textbook method and an alternative method that may decrease the students' conditioning and thus reduce their reliance on financial calculator settings. We then test two groups of introductory-level finance students using the two methods and find significantly higher success rates for the group that learned the alternative method.

INTRODUCTION

Introductory finance textbooks devote considerable space to the teaching of annuities within the context of the time value of money. In most cases, these textbooks present closed-end formulas derived from geometric series to help students solve present value and future value problems. Additionally, there is in most cases a discussion of how to use financial calculators to apply these formulas and find the present or future value of an annuity. While these teaching methods provide a valuable shortcut for obtaining the correct answer, there is an attendant danger that students will become dependent upon the framing of the question when deciding whether to declare the cash flows as occurring at the beginning of the period or at the end.

In the next section of this study, we begin by outlining the traditional method of teaching students how to use their financial calculators to obtain the present or future value of an annuity. We then share the results of a quiz, administered to 28 students in one section of an introductory finance course, on this subject. These

students were all taught the traditional textbook method (and no other methods) of annuity valuation during that class period prior to the quiz.

In the following section, we outline an alternative method of teaching annuity valuation that we believe makes the students less susceptible to conditioning than does the traditional method. We focus specifically on how students can misconstrue the words "beginning" and "ending" in a problem to signal that they should use the same calculator setting to solve for present or future value when in reality the opposite may be the case. We then share the results of a quiz administered to 28 students from a different section of the same introductory finance course. The same professor who taught the previous section of students also taught this section, but this second set of students learned only the alternative method of annuity valuation. We then compare the results of the two groups of students. The key is the comparison of the two groups; while it is likely that several finance professors already teach the alternative method or something like it, our study formally compares the effectiveness of the two methods.

The final section of this paper concludes with a brief summary of the results of the annuity valuation quiz and offers some advice for instructors who will teach that subject, and subjects related to it, in the future.

The Traditional Method of Teaching Annuity Valuation

Ross, Westerfield and Jordan exemplify what we refer to hereafter as the "Traditional Method" in the 10th edition of their textbook Fundamentals of Corporate Finance. On page 155, in the opening part of section 2 of chapter 6, the authors define an ordinary annuity as "a series of constant or level cash flows that occur at the end of each period for some fixed number of periods". They then show the formula for the present value of an ordinary annuity and include short sections on how to use financial calculators and spreadsheets to solve annuity valuation problems. Many other textbooks, including Principles of Managerial Finance by Gitman and Zutter and Fundamentals of Corporate Finance by Brealey, Myers and Marcus, use similar wording to that found in Ross, Westerfield and Jordan.

While there is some value to teaching annuity problems in this manner, it also runs the risk of conditioning students to look for the words "beginning" and "ending" as signals for which financial calculator setting to choose. The power of wording is an important topic in the decision sciences, psychology, and economics literature. In economics, the focus on "framing" traces back to seminal studies by Tversky and Kahneman (1981), and Kahneman and Tversky (1984). While these researchers and others generally present framing in examples showing how the same question worded in different ways causes people to answer it differently, a more basic definition of framing is word-based conditioning. In annuity problems, students may lock on to the words "beginning" and "ending" and thus become more likely to make mistakes while simultaneously avoiding thinking through

the concept. Gardner (2004) makes a similar observation and cautions professors against being too reliant on the words "beginning" and "ending" as well as strictly defining "n" as the number of payments rather than the number of periods. Our aim is to build on these suggestions as well as to formally test student performance after we teach the traditional approach versus the alternative approach.

With this in mind, we study the degree to which such conditioning affects students. In one section of his "Intro to Finance" course, one of the authors of this study presented the Traditional Method. He began by presenting future value and present value annuity problems to the students and defining an ordinary annuity as a series of end-of-period payments and an annuity due as a series of beginning-of-period payments. The annuity word problems given in the lecture reflected most of the annuity word problems presented in textbooks in that they emphasized the words "beginning" and "ending". The students then used their financial calculators to solve the given problems.

After the author gave his lecture, he had each student take a four-question quiz on annuity valuation. We reproduce the quiz, with the correct calculator inputs in italics and the answer also in bold, in Figure 1:

Figure 1: Annuity Valuation Quiz

1. John makes three deposits of $1,000 at the beginning of each year, beginning today. Determine the amount John will have on deposit at the time of his third deposit (includes the third deposit). John earns interest at a 6% annual rate.

Setting = END, N = 3, I = 6%, PV = 0, PMT = $1,000, ***FV = $3,183***.

2. Carlos makes three deposits of $1,000 at the end of each year. Determine the amount Carlos will have on deposit one year after his third deposit. Carlos earns interest at a 6% annual rate.

Setting = BEGIN, N = 3, I = 6%, PV = 0, PMT = $1,000, ***FV = $3,374***.

3. Henry makes three deposits of $1,000 at the beginning of each year, beginning today. Determine the amount Henry will have on deposit one interest compounding period after his last deposit. Henry earns interest at a 6% annual rate.

Setting = BEGIN, N = 3, I = 6%, PV = 0, PMT = $1,000, ***FV = $3,374***.

4. Jenny makes three deposits of $1,000 at the end of each year. Determine the amount Jenny will have on deposit at the time of her third deposit (includes the third deposit). Jenny earns interest at a 6% annual rate.

Setting = END, N = 3, I = 6%, PV = 0, PMT = $1,000, ***FV = $3,183***.

Below, in Table 1, we show the results for the section of the course that learned the Traditional Method just prior to taking the quiz. This group consists of 28 students. We count each answer as correct or incorrect, with incorrect answers falling into three sub-categories. The first sub-category, "switch", indicates that the student would have gotten the correct answer but for one mistake – he or she switched to the incorrect calculator setting. The second sub-category, "P.V.", indicates that the student would have gotten the correct answer except that he or she calculated the present value of the annuity instead of the future value. The third sub-category, "other", indicates an incorrect answer that falls outside of the first two sub-categories, including those cases in which the student made both of the mistakes outlined above.

Table 1. Traditional Method

Answer	#1	#2	#3	#4
Correct	1	7	4	12
Switch	24	12	0	1
P.V.	0	0	3	4
Other	3	9	21	11
All Incorrect	27	21	24	16
N	28	28	28	28

The first thing that one notices about the results above is the prevalence of switched calculator settings for problems #1 and #2. While only one student out of 28 provided the correct answer to problem #1, fully 86% of them (24 students) switched the setting from "ending" to "beginning". By comparison, problem #4, which has the same correct answer as problem #1 but which does not have "beginning" in its wording, yielded 12 correct quiz answers out of 28 and only 1 incorrect calculator setting. This difference in results strongly suggests that using the word "beginning" in the problem conditions students to use the calculator setting of the same word. This is troubling for finance professors, who want their students to think through problems logically rather than become frame-dependent to the extent that they see a word and immediately use the calculator setting that matches that word.

An Alternative Method of Teaching Annuity Valuation

In the other section of his "Intro to Finance" course, the author presented annuity valuation problems in a manner that differed from the traditional method. He began by separating into two different discussions the concept of the present value of an annuity and that of the future value of an annuity. He explained that when a student uses the "ending" setting to calculate the future value of an annuity,

the calculator accounts for one less compounding period. He emphasized that the important point is not when the payments occur (as in at the "beginning" or "end" of the period), but instead whether the last payment earns interest for an additional period.

He then focused on teaching the present value of an annuity, explaining that the "ending" calculator setting equates the number of payments to the number of compounding periods. He also emphasized that the important point is not when the payments occur but instead whether interest applies for the period immediately preceding the first payment.

As in the other section of the course, the professor finished by working through two annuity valuation word problems with the students. We give one example of such a problem below. Notice that the words "beginning" and "ending" are deliberately not used, which is consistent with the way that annuity problems often pose themselves in real life:

> Example: Determine the amount Jan will accumulate immediately after the third annual deposit of $1,000 assuming interest is compounded annually at an annual rate of 2%.

In Table 2 below, we show the results for the section of the course that learned this alternative method just prior to taking the quiz. As with the group that learned the traditional method, this group consists of 28 students, and as with the quizzes from the other section of the course, we sub-categorize all incorrect answers as "switch", "P.V.", or "other".

Table 2. Alternative Method

Answer	#1	#2	#3	#4
Correct	15	16	15	12
Switch	7	6	2	8
P.V.	0	2	1	2
Other	6	4	10	6
All Incorrect	13	12	13	16
N	28	28	28	28

Here, a comparison of problems #1 and #4 shows little difference in either the number of the correct answers or the frequency with which students used the incorrect calculator setting for the timing of the payments. This is also the case with problems #2 and #3, suggesting that after the students learned the alternative method of annuity valuation, their choice of calculator setting was less dependent upon the framing of the question.

Comparing The Traditional Method To The Alternative Method

In this section, we compare the results of the traditional method to that of the alternative method, where the same professor taught both groups of students. The table below shows the percentage of quiz answers that fall into each category ("correct", "switch", "P.V.", and "other") for each question and then compares the results for the questions that have the same correct answer but differ in their wording. We should also mention here that the grade-point-averages for the sections that learned the alternative and traditional methods were 2.726 and 2.659, respectively. The difference between those course G.P.A.s is statistically insignificant even in a one-sided t-test (p-value = 0.35).

Panel A of Table 3 shows the results for the two problems that require the "ending" calculator setting. In problem #1, which features the word "beginning", we see strong evidence of conditioning on the part of those students who learned the traditional method of annuity valuation. 86% of those students used the "beginning" setting, compared to 25% of the students who learned the alternative method. Not surprisingly, just 4% of the students who learned the traditional method (one student out of 28) got the correct answer whereas 54% of the students who learned the alternative method arrived at the correct answer. Both of these differences are statistically significant at the 1% level. Moving over to problem #4, we can see that, in contrast to problem #1, a higher percentage of students who learned the alternative method used the wrong calculator setting. This is likely to be a result of conditioning as well – the use of the word "end" in the problem seems to have caused students who learned the traditional method to select the "ending" setting more often than those who learned the alternative method. Despite this, the students in the alternative method group answered problem #4 correctly with the same frequency as did the students in the traditional method group. Last, we examine the difference between the answers to problem #1 and problem #4. This gives us an indication of the extent to which conditioning affects the students in the two groups. The difference is highly significant, suggesting that for ending setting problems the traditional method can make students frame-dependent to a troubling degree. One can also see this conditioning in the change in the percentage of correct answers; the students who learned the traditional method were much more likely to get the correct answer when the word "end" features in the problem. The difference between the two groups' changes in the percentage of correct answers from problem #1 to problem #4 is also highly significant.

Panel B shows similar results for the two problems that require the "beginning" calculator setting. A much higher percentage of students in the alternative method group correctly answered problem #1 (which features the word "end"), while more students who learned the traditional method used the "ending" setting. However, we do not find the reverse for problem #3, suggesting that the effect of conditioning

Table 3. Analysis of the Data

Panel A	**Problem #1**				**Problem #4**				**Difference (Pr. #1 – Pr. #4)**			
	Correct	Swi.	P.V.	Oth.	Correct	Swi.	P.V.	Oth.	Correct	Swi.	P.V.	Oth.
Traditional	4%	86%	0%	11%	43%	4%	14%	39%	−39%	82%	−14%	−29%
Alternative	54%	25%	0%	21%	43%	29%	7%	21%	11%	−4%	−7%	0%
Alt. – Trad.	50%	−61%	0%	11%	0%	25%	−7%	−18%	50%	−86%	7%	29%
T-statistic	4.88***	−5.67***	0.00	1.08	0.00	2.66**	−0.85	−1.45	3.12***	−6.07***	0.85	2.12**
Panel B	**Problem #2**				**Problem #3**				**Difference (Pr. #2 – Pr. #3)**			
	Correct	Swi.	P.V.	Oth.	Correct	Swi.	P.V.	Oth.	Correct	Swi.	P.V.	Oth.
Traditional	25%	43%	0%	32%	14%	0%	11%	75%	10%	43%	−11%	−43%
Alternative	57%	21%	7%	14%	54%	7%	4%	36%	4%	14%	4%	−21%
Alt. – Trad.	32%	−21%	7%	−18%	39%	7%	−7%	−39%	−7%	−29%	14%	21%
T-statistic	2.54***	−1.73*	1.44	−1.59	3.35***	1.44	−1.03	−3.16***	−0.44	−2.08**	1.65	1.49

* denotes statistical significance at the 10% level in a two-tailed t-test. ** denotes significance at the 5% level. *** denotes significance at the 1% level.

is less severe for problems requiring the "beginning" setting than for problems requiring the "ending" setting. A note of caution, however - the traditional method group had many more answers fall into the "other" category, which includes those cases in which the students made both present/future value errors and used the incorrect calculator setting. Overall, students who learned the alternative method were, as with problem #2, more likely to get the correct answer for problem #3 than were students who learned the traditional method. The "difference" columns show similar, though less severe, results as in Panel A.

Our final table provides some overall comparisons between the answers to the annuity valuation quizzes for the two groups.

TABLE 4. Overall Comparisons

Answer		ALT	TRAD	DIFF	T-STAT
Correct	(#1 and #2)	1.11	0.29	0.82	4.40***
Correct	(#3 and #4)	0.96	0.57	0.39	1.86*
Correct	(All problems)	2.07	0.86	1.21	3.66***
Switch	(#1 and #2)	0.46	1.29	−0.82	−4.55***
Switch	(#3 and #4)	0.36	0.04	0.32	2.88***
Switch	(All problems)	0.82	1.32	−0.50	−2.28**

* denotes statistical significance at the 10% level in a two-tailed t-test. ** denotes significance at the 5% level. *** denotes significance at the 1% level.

The top row above examines the frequency of correct answers for the two problems that frame-dependent students are likely to miss by choosing the incorrect calculator setting. The second row compares the frequency of correct answers for those problems with wording that is consistent with the correct calculator setting. The third row shows the mean number of total problems that each group answered correctly. Those students who learned the alternative method were not only more likely to arrive at the correct answer for problems #1 and #2, but also for problems #3 and #4 (significant at the 10% level for a two-tailed test and at the 5% level for a directional test), wherein the wording was consistent with the correct calculator setting. Overall, the alternative method group averaged 2.07 correct answers out of four while the traditional method group averaged only 0.86 correct answers.

Rows four through six compare the cases in which members of each group would have gotten the correct answer but for using the wrong calculator setting. Not surprisingly, for the problems featuring the word "end" when the correct calculator setting was "begin" and vice versa, the traditional method group fared much worse than the group that learned the alternative method. In problems #3 and #4, where conditioned students may have been led by their conditioning to choose the correct setting, members of the traditional group were indeed less likely to provide answers that were incorrect only because they used the wrong setting.

However, when we aggregate all four problems, we find that those students who learned the traditional method were overall less likely to provide answers that were incorrect only because they used the wrong setting. When we combine this result with the first three rows of Table 4, which show a higher likelihood of the alternative method getting the correct answer overall as well as in both subsets, we conclude that there are significant advantages to teaching the alternative method of annuity valuation over the traditional method.

Limitations To The Study

Although we attempted to remove certain biases from our study as well as test for the presence of others, some limitations remain. First, the sample size - 28 students in each group—is somewhat small, and although the results that we reported are strong, they would likely vary from course to course and university to university. Second, although we used the same professor for both groups of students and had him spend roughly the same amount of time teaching the alternative and traditional methods of annuity valuation, it is always possible that he taught the alternative method more clearly than the traditional method. We did our best to avoid this, but we cannot rule out such possibilities entirely. Overall, however, we feel confident that the results would translate to other instructors and introductory finance classrooms.

CONCLUSIONS

Annuity valuation is one of the core examples of time value of money that introductory finance textbooks provide. As such, it is often one of the first topics covered and is crucial to gaining a full understanding of present and future value. The traditional method of teaching annuities runs the risk of conditioning students to choose their calculator settings in accordance with the wording of the problem, since many (perhaps most) annuity valuation problems explicitly use the words "ending", "beginning" or variations thereof. In real life, however, many annuity problems do not present themselves by using the words "ending" or "beginning". Even if they did, the wording of such problems can be tricky given that the use of "ending" does not always signify the ending calculator setting nor does "beginning" imply that the beginning calculator setting is correct. For these reasons, structuring an annuity valuation lesson that conditions the students to look for those words will often cause them to not only avoid thinking through the problem but may also lead them to the incorrect answer.

With these issues in mind, we designed a quiz containing two problems using the words "end" or "beginning" that required the opposite calculator setting and two problems using the words "end" or "beginning" that required the same calculator

setting. We designed the quiz so that each question that had wording opposite to the correct calculator setting had the same answer as one of the questions whose wording matched the correct calculator setting. One of the authors of this study then taught one of the sections of his Intro to Finance course the alternative method of annuity valuation and followed the lesson by giving his students the quiz. He taught the other section of his Intro to Finance course the traditional method and followed that lesson with the same quiz. In each section of the course, 28 students attended class that day and took the quiz.

For problems #1 and #2, where frame dependence was most likely to cause the students to use the incorrect calculator setting, the group of students that learned the alternative method outperformed the group that learned the traditional method considerably. For problems #3 and #4, where frame dependence may go so far as to make the traditional method group more likely to choose the correct calculator setting and thus arrive at the correct answer, the traditional method group was less likely to provide answers that were incorrect only because of calculator setting. Of course, this conditioning may not be a good thing, especially if the students uncritically chose the correct setting because of it. Even more interestingly, those students who were taught the alternative method were still more likely to get the correct answer to those two problems overall, suggesting that in addition to avoiding frame dependence, those students also learned a better way to think about annuity valuation than the students who learned the traditional method.

Finally, we examine the answers in aggregate and find that for all four quiz problems combined, students who learned the alternative method were more likely to get the correct answer than those who learned the traditional method, with the same professor teaching both groups. Additionally, they were less likely to get the wrong answer solely because they used the incorrect calculator setting, even though for two of the four problems the framing of the question gave the students conditioned by the traditional method an advantage. We believe this is possibly because the alternative method helps students to consider the problem more in terms of the nature of compound interest than with which setting to enter into their calculator. We feel that all of our results, taken together, provide a compelling argument to use the alternative method, rather than the traditional method, to teach the concept of annuity valuation.

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The article

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Threshold concepts in finance and the role of mathematics

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Abstract

Whilst the financial services sector has been a major employer of mathematics graduates, specialist finance programs that develop finance capabilities are in increasing demand. However, research into the role of mathematics in finance programs is limited. Our study addresses this by using threshold concepts to investigate what finance academics identify as essential to understanding finance, with particular interest in mathematics and statistics concepts. Based on qualitative research with finance academics, we make proposals for specific threshold concepts in finance, a significant proportion of which are statistics concepts. In addition, we explore the extent to which mathematics more generally is perceived as fundamental to understanding finance. We conclude that the role of mathematics in finance can be clarified using threshold concepts. Our research informs curriculum design and pedagogical practices to meet the needs of finance students, avoid overcrowding and associated surface learning and improve student engagement and outcomes. The findings have important implications for mathematics and statistics academics, and point to the need to work with other disciplines to articulate the threshold mathematical knowledge that is essential in that discipline.

Introduction

Mathematicians have proposed that mathematics is the base of all discoveries as in *The Unreasonable Effectiveness of Mathematics in the Natural Sciences* (Wigner 1960). But how do other disciplines view mathematics in their disciplines? Our study investigates the views of finance faculty as to what are the threshold concepts in their discipline, with particular interest in mathematics (and statistics) concepts. Our aim is to provide curriculum developers with information to redesign the finance curriculum with an emphasis on threshold concepts.

Curriculum design has many aspects, well defined in the book by Stark and Lattuca (1997) with their model of the academic plan. Our paper fits into the centre of their model concentrating on the content and sequencing of finance programs from the viewpoint of the academic faculty. The findings have important implications for mathematics and statistics faculty and point to the need to work with other disciplines to tease out the threshold mathematical knowledge.

Finance degrees, like so many other degrees offered in Australia and around the world, have to meet the needs of a diverse student cohort (Wood 2001) and increasingly rigorous and multifaceted accreditation requirements (for example, the Australian Securities and Investments Commission Regulatory Guide 146 and Australian Qualifications Framework; the US-based Chartered Financial Analyst Institute; and the Quality Assurance Agency for Higher Education finance benchmark in the UK). The tendency to add more and more material, in part to address these needs, has resulted in an overcrowded curriculum. The aim of our research is to investigate the threshold concepts (Meyer and Land 2003) that are central to the mastery (Cousin 2006) of finance in order to inform curriculum design and pedagogical practices and to improve student outcomes. The context for this research is a degree program in which international finance is taught to international and domestic students, and so the results are relevant to an international audience.

Our primary research question is "What do finance academics consider to be fundamental to understanding their discipline?" following Cousin (2009), and in addressing this question, we also seek to answer the question "What is the role of mathematics in finance?" The significance of mathematics in finance is evident from the fact that the financial services sector is a major employer of mathematics

graduates (Bourner, Greener and Rospigliosi 2009). And indeed, the proposed threshold concepts in finance arising from our research include a significant proportion of statistical concepts as well as (mathematical) modelling. The role of mathematical modelling in financial services and the value of understanding mathematical modelling in the context of the increasing use of information systems, which tend to hide mathematical models, is discussed in the work of Bakker and Kent and their colleagues (Bakker et al. 2006; Kent et al. 2007). Philippon and Reshef (2012) find that the extent to which roles in finance require mathematics varies and that greater deregulation is associated with more mathematics in finance roles and vice versa. In addition, the emergence of behavioural finance, which acknowledges decision biases and non-rational behaviour, considers finance from a different perspective (Frankfurter 2006; Shiller 2006; Statman 2008), with less emphasis on quantitative methods (Coleman 2013). Thus, our findings explore the role of mathematics in the finance curriculum, under the more general themes of how important is mathematics in finance, how much mathematics to include and how to develop mathematics capabilities in students.

Following Cousin's (2009) identification of the benefits of bringing together discipline specialists to identify threshold concepts, focus groups were selected as an effective way to identify threshold concepts in finance. We report on the outcomes of a focus group with staff from the finance department at our institution. In addition, supplementary interviews were conducted with three key finance staff. The data collected were analysed using linguistics techniques to identify proposals for threshold concepts and gain an understanding of the (semantic) content of the focus group discussion (Halliday and Matthiessen 1999).

Threshold concept theory was first developed by Meyer and Land in relation to economics in 2003. Since then, it has been rapidly taken up and widely used in other disciplines, as evidenced by Flanagan's online bibliography (http://www.ee.ucl.ac.uk/~mflanaga/_thresholds.html), as a way to research and inform curriculum design, pedagogical practices and professional development of teaching staff. Threshold concept research allows academics to discuss and identify what is fundamental to their disciplines, explores the difficulties students have in grasping threshold concepts and identifies curriculum design interventions (Cousin 2009).

Threshold concept theory proposes that there are a number of concepts that are central to the mastery of any discipline (Cousin 2006) and, as originally described by Meyer and Land, these concepts have five key characteristics (2003):

(1) Transformative – occasions a shift in the perception of the subject

(2) Integrative – exposes the previously hidden interrelatedness of something

(3) Irreversible – unlikely to be forgotten, or will only be unlearned by considerable effort

(4) Troublesome – conceptually difficult and/or counter-intuitive

(5) Bounded – serves as boundary-markers for the conceptual spaces that make up the disciplinary terrain.

For example, opportunity cost in economics fundamentally changes the way students think about choices, including their own (Meyer and Land 2003). Opportunity cost can be understood in relation to other concepts such as sunk costs, and in this way can be seen as integrative, connecting other concepts, and so contributing to marking the boundaries of economics (Davies and Mangan 2007). It might be a troublesome concept for students because, despite being introduced to the concept, students do not always apply it fully to solve economics problems (Davies and Mangan 2007). Arguably, these characteristics also mean that, once fully understood by a student, opportunity cost is unlikely to be forgotten/difficult to unlearn. More recently, discursive, reconstitutive and liminal aspects have been identified as characteristics or features of threshold concepts (Barradell 2013; Flanagan 2015; Land et al. 2014). However, this research focuses on the transformative, integrative, irreversible and troublesome characteristics in order to identify threshold concepts in finance.

In the following sections we review previous research on threshold concepts in the fields of finance, mathematics and statistics as well as research on the finance curriculum that does not use the threshold concept framework. After describing our methodology, we make proposals for threshold concepts in finance, explore the role and teaching of mathematics in finance more generally and present our conclusions.

Threshold Concepts Research

Despite the rapid take-up of threshold concept theory, other than our own work (Hoadley et al. 2015; Hoadley et al. in press), research on threshold concepts in finance is limited to the work of Diamond and Smith in relation to quantitative finance (Diamond 2014; Diamond and Smith 2011) and business statistics (Diamond 2011). However, more work has been done in the area of threshold concepts in mathematics and statistics. These studies have the potential to inform our own research in relation to specific threshold concepts and more general conceptualisations of threshold concepts in finance.

Proposals for specific threshold concepts

Previous research in quantitative finance, business statistics, mathematics and statistics contains proposals for specific threshold concepts in these disciplines (Table 1). Generally, this research focuses on particular threshold concepts, rather than proposing a definitive list of all the threshold concepts in a discipline. The exception to this is perhaps the work of Diamond (2011) which provides a summary of the threshold concepts in business statistics.

Table 1 Threshold concepts in quantitative finance, business statistics, mathematics and statistics

Discipline area	Threshold concept	Reference
Quantitative finance	change of measure, incomplete markets, Ito's lemma and risk neutrality	Diamond and Smith 2011
	cointegration analysis	Diamond 2014
Business statistics	**basic** - central tendency and dispersion, mean vs. median, standard deviation, probability **discipline** - probability distribution, continuous vs. discrete, hypothesis testing, significance, correlation, regression, time series data, index **procedural** - essential mathematical notation (summation with indexes), operation with equations, polynomials, operations with percentages, meaning of Greek letters, normal distribution tables, ability to visualise distribution and modelling procedures that enable the construction of discipline-specific models, arguments and ways of practising	Diamond 2011
Statistics	confidence intervals, the concept of significance testing (p values), hypothesis testing, analysis of variance and knowing how and when to apply statistical tests and formulae	Bulmer, O'Brien and Price 2007
	notion of patterns of spread or variation, randomness, sampling, the central limit theorem, linear regression, and introductions to Bayes' theorem - hypothesis testing difficult albeit not threshold	Dunne, Low and Ardington 2003
Statistics - in medicine	uncertainty	MacDougall 2010
	sampling theory, normal distribution, statistical significance and the concept of effect size	Thompson 2008
Statistics - in science	hypothesis testing and statistical significance	Quinnell and Thompson 2010
Mathematics	complex number and limit	Meyer and Land 2003
	function, limit, derivative, integral and complex numbers	Pettersson 2011
	Function	Breen and O'Shea 2012
	linearity, complex numbers and limit	Bloom et al. 2011
	proof	Easdown 2007; Jooganah 2009
	secondary to tertiary - fractions, division and algebraic manipulation **professional/metacognitive** - set theory and number theory	Easdown and Wood 2012
Mathematics - in engineering	functions as a symbolic representation of relationship	Galligan, Wandel and Hartle 2010

In relation to business statistics, the basic, discipline and procedural categories used by Diamond (2011) refer to the type and scale of conceptual change (Davies and Mangan 2007). Basic concepts involve a common sense understanding being transformed through integration with ideas from the discipline, and therefore the conceptual change is profound. Discipline concepts connect with and inform other discipline concepts/ideas, and lead to the acquisition of organising schemas. Procedural concepts, such as modelling in the case of finance, are ways to construct narratives and arguments in a discipline, through which other concepts are defined and understood. The specific proposals for threshold concepts in quantitative finance, business statistics and statistics are considered in section 4 in relation to the specific proposals arising from our research

General conceptualisations of threshold concepts in mathematics

In contrast to the specific finance threshold concepts discussed in the previous section, some research has (also) taken a broader view and identified more general or generic threshold concepts. Some of this research is in the area of mathematics and statistics and is therefore relevant to our research. This research generally falls into two categories; mathematics or mathematics and statistics in other disciplines. Each of these categories are discussed in turn below.

In relation to mathematics, Worsley, Bulmer and O'Brien (2012) research conceptual understandings versus procedural knowledge. Easdown (2009; 2011) discusses syntactic versus semantic reasoning in mathematics, and argues that examining syntactic reasoning errors can reveal the deeper semantic reasoning and underlying threshold concepts. Other researchers emphasise the role and use of mathematical discourse in developing the transformed understanding that is implicit in a threshold concept (Jooganah 2009; Pettersson, Stadler and Tambour 2013). In addition, Pettersson (2012) describes a process of extending contextualisation, linked to concrete everyday life examples, gradually enriched to include a more abstract mathematical understanding through which students develop highly personalised understandings.

In relation to mathematics in other disciplines, developing an understanding of mathematics through contextualisation is important for engineering students (Galligan, Wandel and Hartle 2010; Pettersson 2008; Scheja and Pettersson 2010). Wandel (2010) emphasises the importance of making

connections between the threshold mathematics concepts in foundation subjects and engineering concepts in later subjects. In other work related to mathematics for engineering students, interviews with academics reveal different emphasises on content and/or concepts versus being able to think like a mathematician or critically (Worsley 2011). Thinking mathematically is also identified as a more general threshold concept for engineering students by Galligan, Wandel and Hartle (2010). Masouros and Alpay (2010) specify identifying and developing resources to teach mathematics threshold concepts as a key aspect of developing the mathematics skills of engineering students. Quinnell and Thompson (2010) argue that unpacking numerical concepts to underlying non-numerical concepts and improving students' confidence in their numeracy skills reduces medical and life sciences students' resistance towards, and anxiety about, mathematics.

The threshold concepts identified in the related research discussed above have some overlap with our findings. However, because our research focuses specifically on finance and also covers the finance curriculum more broadly, the threshold concepts we propose go beyond the findings of previous research both in terms of relevance and scope.

Finance curriculum research

Finance curriculum research has a number of characteristics that reinforce the need for this study. Firstly, much of the literature focuses on introductory finance rather than the curriculum of an entire finance program (eg Balachandran et al. 2006; Berry and Farragher 1987; Cooley and Heck 1996; Gup 1994; Krishnan et al. 1999). Secondly, some research focuses on the finance curriculum from the point of view of preparing students for specific professional roles (eg Jackling and Sullivan 2007 – financial planners; Lakshmi 2013 – accountants or chief financial officers; Roth, Envick and Anderson 2002 – entrepreneurs). Thirdly, research into the financial curriculum tends to take a more topic based approach than a concept approach.

Notwithstanding the previous points, the finance topics and concepts identified in the finance curriculum research are a point of comparison with the threshold concepts identified in this study. It should be noted also that there is some slippage (see Lai et al. 2009) and overlap between finance *concepts* and finance *topics* in the literature. For example, Cooley and Heck (1996) make the distinction

between topics and concepts, whilst Gup (1994) exclusively uses the term concept. However, Gup's concepts are more like Cooley and Heck's topics than their concepts. The results of relevant studies regarding topics and concepts are shown in Table 2. The results of the research by Balachandran et al. (2006) and Lai et al. (2009) are not included in the table as they are student rankings of (a sub-set of) the seven most important concepts identified by Cooley and Heck.

Table 2 Finance topics and concepts

Berry & Farragher 1987	**McWilliams & Pantalone 1994**	**Gup 1994**	**Cooley & Heck 1996**	**Krishnan et al. 1999**
Academics	**Executives**	**Academics & Executives**	**Academics**	**Students**
Topics	**Topics/subjects**	**Concepts**	**Topics**	**Topics**
Time value of money Capital budgeting techniques Cost of capital/capital structure Valuation theory Working capital	Working capital management Capital budgeting Financial institutions and markets Investments International finance	Present value Cost of capital/CAPM Cashflow and financial statements Risk-Return Capital markets Capital budgeting Capital structure Valuation Accounting	Time value of money Capital budgeting Risk and return Security valuation Cost of capital Financial statement analysis Capital structure	Time value of money Financial statement analysis Security valuation Financial forecasting Investment banking Capital budgeting
			Concepts	
			Present/future value annuity Present/future value single amount Net present value Internal rate of return Valuing stocks Valuing bonds	

There is considerable similarity and overlap between the findings of different studies as summarised in Table 2, with the same topics and/or concepts being identified in multiple studies. For example, the time value of money (or related concepts) appears in most of the lists. In contrast, there is very little overlap between the topics and concepts in Table 2 and the threshold concepts identified in Table 1, due to the mathematics and statistics focus of the threshold concepts research.

Methodology

Cousin (2009) identifies researching threshold concepts as a methodology for researching learning in higher education, involving collaboration with and/or participation by discipline specialists, educational specialist and learners. The primary question that threshold concept research with academics is designed to explore is "What do academics consider to be fundamental to a grasp of their subject?" Cousin advocates focus groups with academics as the primary research activity to answer this question, supplemented by individual interviews with key academics where necessary.

The research activities reported on here consist of a focus group with nine academics from a finance department of an Australian university. All finance academics in the department were invited to attend the focus group, but attendance was entirely voluntary. In addition, in order to ensure the participation of key finance academics, we invited three senior finance academics with responsibilities for the department's finance programs to participate in individual interviews. One of these three academics also attended the focus group. The participants have a diverse range of experience both in teaching finance and working in the finance industry.

At the start of the focus group, as a result of experience from another project and influenced by Appleby and Barton (2012), a brief introduction to threshold concepts was given. The introduction made reference to the five characteristics identified above and included three clear examples of threshold concepts from three different non-finance disciplines. The group was then asked to propose potential finance threshold concepts. The subsequent discussion, which lasted just under 45 minutes, was recorded, transcribed and all personal identifiers were removed from the transcription.

The interviews were based on a semi-structured approach and covered both the identification of finance threshold concepts and how these concepts are embedded in the curriculum. Following Cousin (2009), a visual prompt was used to introduce and explain threshold concepts using the five characteristics mentioned previously. The visual prompt consisted of references from Cousin (2006) and Meyer and Land (2003) to define and explain threshold concepts in the centre, surrounded by images to represent each of the five characteristics, following Meyer, Land and Baillie (2010). For example, the irreversible characteristic was illustrated with a no u-turn road sign. Two of the interviews were

recorded, transcribed and all personal identifiers were removed from the transcription. The third interviewee did not agree to the interview being recorded, however, they did provide a list of finance concepts which they felt represented threshold concepts in finance based on the discussion in the interview. The third interviewee gave permission for this list to be used in the research.

The focus groups and interview transcripts were initially reviewed by one researcher to identify the nominal groups representing proposals for specific threshold concepts in finance. The nominal groups ranged from very simple structures consisting of a single noun (eg arbitrage) to more complex structures consisting of head nouns with pre- and post- modification (eg no-arbitrage arguments, the idea behind the principle of no-arbitrage) (Halliday and Matthiessen 1999). The proposals for threshold concepts identified were provided to the three other researchers along with the de-identified transcripts for verification. The concepts from the list provided by the third interviewee were added to the list from the transcripts. The researchers then met to review the proposed list and refine it to remove duplication, that is, the same concept expressed in different ways or concepts subsumed by a higher level concept (refer Appendix 1). The refined list of proposed threshold concepts was then organised using the framework developed by Davies and Mangan (2007), and considered in relation to proposals from previous research for threshold concepts in quantitative finance, business statistics and statistics.

In addition, prompted by the extent to which the discussion during the focus group focussed on the role and teaching of mathematics in finance more generally, the transcript was analysed linguistically using a top-down approach in order to gain an understanding of the (semantic) content of the entire discussion (Halliday and Matthiessen 1999). The transcript was divided into 28 sections (elements) based on content (primarily change of topic) and structural indicators – such as change of speaker (turn-taking), questioning (change of speech function), discourse markers (comment adjuncts and cohesive conjunctions) (Halliday and Hasan 1976). Fourteen of the 28 sections identified were principally concerned with the role and teaching of mathematics in finance more generally. These 14 sections were given a descriptive title and the key points of each section were identified. This analysis was provided to the three other researchers along with the de-identified focus group transcript for verification.

Results and analysis

Finance threshold concepts

In answer to our primary research question "What do finance academics consider to be fundamental to understanding their discipline?", 25 concepts are identified and proposed as threshold concepts in finance. Of these 25 concepts, eight are statistical concepts. Our research has not identified any other specific mathematical concepts as potential threshold concepts in finance, although the concept of modelling (as a means to define and understand other finance concepts) clearly has a strong mathematical/statistical component. The proposed threshold concepts are shown in Table 3 using the framework developed by Davies and Mangan (2007).

Table 3 Proposed threshold concepts in finance

Type of conceptual change – transformation and integration (Davies and Mangan 2007)	**Finance**	**Statistics**
Basic - Understanding of everyday experience transformed through integration of personal experience with ideas from discipline.	Information asymmetry Leverage/gearing Market structure(s) Pricing Risk versus return Trade offs	Probability/randomness Expected value Regression to the mean Standard deviation Time series
Discipline - Understanding of other subject discipline ideas integrated and transformed through acquisition of theoretical perspective.	Arbitrage Cashflows Diversification Hedging Market efficiency Opportunity cost Risk Short selling Time value of money Utility/risk preference	Central limit theorem and normal distribution Correlation Statistical significance and hypothesis testing
Procedural - Ability to construct discipline specific narratives and arguments transformed through acquisition of ways of practising.	Modelling – building, critiquing, implementing, discipline specific models e.g. pricing models	

From Table 3 it can be seen that, of the 17 finance threshold concepts proposed, six are categorised as bringing about basic conceptual change, that is, concepts in which common-sense understandings based on everyday experience are transformed through integration with ideas from the discipline (Davies and Mangan 2007). For example, the trade off between risk and return is a concept that is generally

understood in terms of risk aversion (fear of making a loss) and desire for a high return (greed). Many people intuitively understand this in the gambling context where a bet may have a high probability of failure but a high payoff in the event of success (e.g. betting odds). The concept of a trade off between risk and return is given deeper and more specific meaning and context in the study of finance and the trade off is conceived of and modelled in various different ways as is the concept of risk.

Ten concepts are categorised as bringing about discipline conceptual change, that is, interrelated finance concepts that are required for/result in a theoretical perspective. For example, hedging is a way to manage risk and derivative instruments are commonly instrument for hedging, thus the concepts of hedging, risk and derivatives inform and transform one another. In combination, they represent an important theoretical perspective in finance.

Finally, modelling is categorised as bringing about procedural conceptual change, being the primary way in which arguments are made in finance, as per Davies and Mangan (2007) in relation to modelling in economics and Diamond (2011) in relation to modelling in business statistics. Modelling enables a more complete understanding of the discipline concepts (Davies and Mangan 2007) through defining and/or quantifying such concepts. That is, in relation to the previous example, modelling enables the calculation and therefore understanding of the amount at risk, the extent to which the risk will be hedged by a particular derivative and the cost of the derivative. The degree of overlap between the finance concepts and the examples of quantitative finance threshold concepts from Diamond and Smith (2011) and Diamond (2014) is limited due to the more specialised nature of quantitative finance.

Of the eight statistical concepts that are proposed as threshold concepts in finance, five are categorised as basic and three as discipline. All of these concepts are identified by Diamond (2011) as threshold concepts in business statistics and our categorisation is consistent with Diamond's (Table 1), with the exception of regression to the mean and time series which we categorise as basic rather than discipline. The additional threshold concepts in business statistics identified by Diamond (2011) reflect a more specific focus, that is, business statistics as opposed to finance. In comparison with previous proposals for statistics threshold concepts more generally (Table 1), four of the statistical concepts we propose – probability/randomness, regression (to the mean), the central limit theorem and normal distribution, and statistical significance and hypothesis testing – are put forward as statistical threshold

concepts in other research (Bulmer, O'Brien and Price 2007; Dunne, Low and Ardington 2003; Quinnell and Thompson 2010; Thompson 2008).

In relation to finance curriculum research more generally (Table 2), the finance threshold concepts we identify overlap with the topics/concepts identified in previous research, for example time value of money and risk versus return. However, more frequently the finance threshold concepts are the concepts that underpin the topics previously identified, for example leverage/gearing, risk, and opportunity costs are all threshold concepts that underpin capital structure. Similarly, the statistics threshold concepts we identify, which overlap with the statistics threshold concepts identified in other research (Table 1), underpin the finance threshold concepts.

Mathematics in finance

The specific concepts identified in Table 1 provide a partial answer to our second research question "What is the role of mathematics in finance?" with reference to the eight statistics concepts and modelling. . However, half of the sections of the focus group discussion are primarily about the role and teaching of mathematics in finance more generally. This discussion indicates that mathematics, beyond the specific concepts identified in Table 1, has a role in finance. In order to explore this role, the 14 sections were given descriptive titles and their key points summarized as shown in Table 4.

Table 4 Focus group discussion sections relating to mathematics with key points

Section	Key points
3. The role of mathematics in finance and mathematics skills	Finance should be *"more mathematical"* because some arguments are *"inherently better presented mathematically".* There was general agreement "*at department meetings recently*" that it would be better if students were *"more mathematically adept"* than they are.
7. Importance of mathematical background	Students without a good level of secondary mathematics struggle in finance.
8. Role of statistics (and mathematics)	A recent program review has recommended a different statistics subject for the finance degree, which will have the twofold effect of making advanced secondary mathematics a prerequisite and developing higher level statistics/mathematics skills.
10. Proposal to develop mathematics skills 1	Existing or new tailored subjects could be used as bridging courses to develop mathematics skills.
13. Proposal to develop mathematics skills 2	Returns to the idea of a specific mathematics subject to develop mathematics skills.
14. Thinking mathematically and working in finance	Mathematics bridging courses will not teach students *"to think how mathematics teaches you to think"* and you have to *"think mathematically"* to work in finance. Students who say they are *"not really that mathematical"* are doing finance programs because there are jobs in finance.
15. Mathematics and university entry requirements	Some finance programs have a lower entry requirement so students are less likely to have done higher level secondary school mathematics, thus some finance students are more likely to find the mathematics difficult.
17. Thinking mathematically 1	Returns to the theme of *"thinking mathematically"* and argues that people who do not think mathematically *"rote learn"* to compensate.
18. Prospective jobs and mathematics skills	Students may want / will end up in jobs that do not require *"high level mathematics skills".*
19. Skills required for working in finance	Mathematics skills are not the most important skills for working in finance.
20. Only seeing the mathematics	Students with mathematics skills may be able to do the mathematics but not understand the financial concepts.
21. Counter proposal to developing mathematics skills	Instead of trying to get students who are weak in mathematics to do more mathematics, *"teach the key financial concepts"* in a contextualised way through *"model building using Excel™"* or similar software.
22. Thinking mathematically 2	Some students *"do think mathematically"* and get through finance easily, but the program needs to be designed to suit all the students.
23. Counter proposal to developing mathematics skills continued	Returns to the idea of using Excel™ to teach finance concepts.

The discussion in these sections is contradictory, for example section 3 in contrast to section 19 and section 10 in contrast to section 21. This, as well extent of the discussion, indicates that the role of mathematics in a more general sense in finance is problematic. Five key interrelated themes are evident in the discussion:

1. Variation in mathematics skills of finance students (sections 3, 14, 15, 22)
2. Variation in (mathematics) skills required for finance jobs (sections 14, 18, 19)
3. How much mathematics should be in finance (sections 3, 20, 21, 22, 23)
4. The importance of mathematics skills for studying finance (sections 3, 7, 17, 20, 21, 22, 23)
5. Responses to the variation in the mathematics skills of students (sections 8, 10, 13, 21, 22, 23).

The role of mathematics in finance is discussed in relation to these five themes in the discussion section below.

Discussion

The aim of our research is to identify the concepts that are fundamental to understanding finance. As with previous research into threshold concepts, our results are both specific and general in nature. That is, they include proposals for specific threshold concepts in finance and also explore more general conceptualisations of threshold concepts in relation to mathematics in finance. We discuss these specific and general results in turn below.

Specific finance threshold concepts

The identification of specific finance threshold concepts was relatively unproblematic in that the proposals could be easily identified in the transcripts. In addition, in the focus group discussion, specific proposals for threshold concepts were not contested by the different participants, rather when a participant made a suggestion other participants agreed or expanded on the suggestion. This indicates that the proposals for threshold concepts in finance are relatively uncontroversial, at least within the group of academics at the focus group. An example of this agreement and expansion is shown in the extract below.

> Participant 1: *Well I think risk measurement, definitions of risk and statistical concepts of distribution and means variances [skewness] and ways to affect your risk by transferring part of it in insurance, reinsurance. Finance derivatives are really just risk transfer.*

Participant 2: *So I guess the concept of hedging in general.*

Participant 1: *Hedging in general, yes.*

The 25 specific finance threshold concepts proposed are divided into two categories: finance concepts, albeit given the interdisciplinary nature of finance a number of these are recognisably from other disciplines; and statistics concepts. Statistics concepts make up a significant proportion (just under one third) of the proposed threshold concepts in finance, indicating the key role of statistics in finance, and hence the importance of the learning and teaching of statistics in the finance curriculum. Most of the statistics concepts identified are supported by previous research relating to threshold concepts in both business statistics and statistics more generally. Although the five characteristics of threshold concepts discussed in the introduction were explained at the beginning of the focus group and interviews, each concept was not "checked off" against all five characteristics. Further testing of the proposed concepts in relation to the characteristics of threshold concepts is reported on in (reference omitted for blind review).

General conceptualisations of threshold concepts: mathematics in finance

In contrast to the identification of specific threshold concepts, the discussion in the focus group about mathematics in finance was much more problematic, as the extracts below demonstrate.

Participant: *I think it's not fair to say all students can't understand, there are some students who do think mathematically and they [unclear] to it and they're really good at it and they just swim through the course. So there's - you know you don't, you just want to design the course to suit the particular people that you're dealing with, and some of them, the mathematical, they're coping. There are some students who...*

Participant: *At the risk of sounding like a broken record here, we're saying, look, our students aren't mathematically strong, let's throw a maths course at them. Or, how about this for an alternative, if our students aren't mathematically strong, let's teach them the key financial concepts through actually going through and model building, using Excel™ […] learning the key concepts, such as arbitrage, efficient markets, short-selling...*

These extracts are just two examples of disagreement during the focus group discussion about mathematics in finance.

In order to better understand the discussion regarding the role of mathematics in finance Figure 1 shows the five key interrelated themes identified in the focus group discussion in relation to one another. The themes of variation in mathematics skills of finance students and variation in (mathematics) skills required for finance jobs are the entry and exit points of the program respectively. The other three themes are impacted by the variation at entry and exit point, but also affect one another.

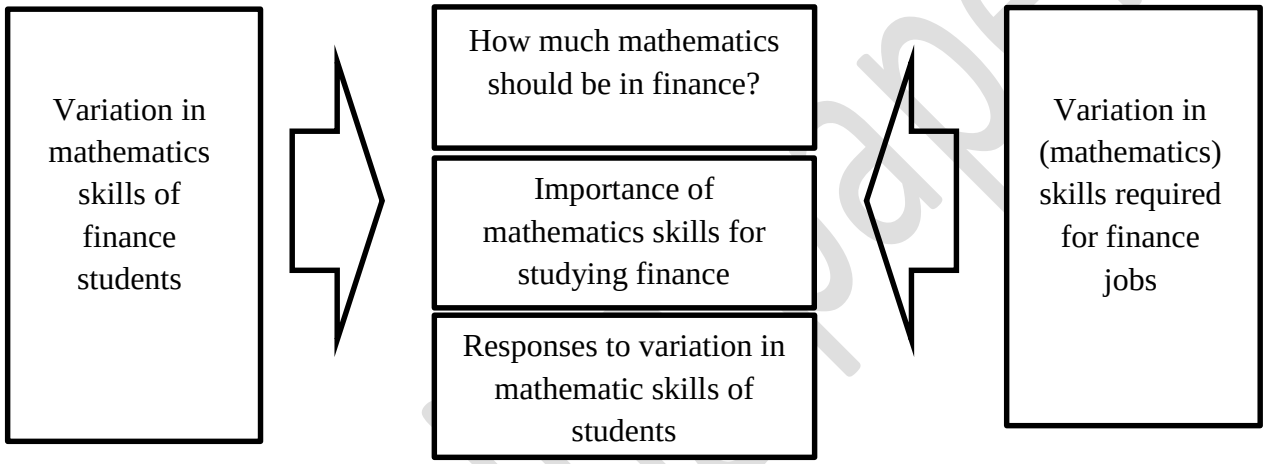

Fig. 1 Key themes of the focus group discussion of mathematics in finance

There is general agreement over the course of the discussion that the mathematics skills of students entering the program vary significantly (sections 3, 14, 15, 22). Although higher level secondary mathematics is assumed knowledge for the finance programs, students can and do enter the programs having done a lower level of mathematics (section 15). According to Quinnell, Thompson and LeBard (2013) based on two Australian studies "students are reducing the level of mathematics they are taking at secondary school".

In contrast, views as to the mathematics skills required to work in finance seem more divergent. At one extreme, it is argued that it is essential to "think" mathematically to work in finance (section 14). However, it is also argued that, whilst some graduates go on to quantitative finance roles which require

high level mathematics skills, other graduates work in more general roles that do not need such high level mathematics skills (section 18). Even for specific finance roles it is argued that mathematical skills are not always the most important skills required (section 19). The idea that the mathematics skills required to work in finance roles varies is supported by other research (Wood, Reid and Petocz 2012).

In line with the lack of agreement about the mathematics skills required to work in finance, the discussion reflects debate about the extent to which finance should be more or less mathematical. At the beginning of the discussion a participant suggests that there should be more mathematics in finance programs (section 3), whereas later in the discussion this is argued against in favour of focusing on teaching finance concepts, using Excel™ for example (sections 21 and 23). The identification of specific threshold concepts in finance offers a way of defining the extent to which finance needs to be mathematical, and indeed the identification of the eight statistics concept as threshold concepts in finance is a first step in doing this. However, the variation in mathematics skills of students entering the program and the mathematics skills required by graduating students creates uncertainty and pressure to go beyond the scope of this definition.

The research of Alcock, Cockcroft and Finn (2008) shows higher and advanced secondary mathematics improves grade outcomes for quantitative business subjects and it is generally acknowledged in the focus group discussion that mathematics skills are beneficial to students studying finance (sections 3, 7 and 22) . However, the variation in the mathematics skills of students coming into the program means that academics cannot rely on students having these skills, or an interest in developing them (Quinnell and Thomspon 2010). Identifying the threshold concepts in finance, as well as identifying essential mathematics concepts, will lead to a more explicit understanding of how these concepts relate to the finance concepts. In this way, the role and relevance of mathematics in/to finance can be made explicit to (potential) students, as emphasised by Wandel (2010) in relation to mathematics for engineering students and Quinnell, Thompson and LeBard (2013) in relation to mathematics for science students.

In relation to possible responses to variation in the mathematic skills of students, one focus of the discussion is reducing the variation in the mathematics skills of the students by assuring/developing mathematics skills through a combination of prerequisites/bridging courses/additional mathematic

subjects (sections 8, 10 and 13). Faced with similar variation in the mathematics skills of engineering students, universities respond by introducing stricter entry criteria to assure the mathematics skills of students coming into the program and/or by providing more remedial mathematics (Masouros and Alpay 2010). However, the former involves changes at an institutional and policy level that are unlikely to occur and the latter involves trying to get students with weaker mathematics skills to do more mathematics (section 21). According to Quinnell and Thompson (2010), students studying other disciplines may be resistant towards mathematics, regardless of their level of mathematics skills. Furthermore, separate mathematics subjects position mathematics as separate to and removed from finance, as a barrier or a hurdle to be overcome before students "get to" finance rather than as integral to finance. In their most recent work, Quinnell, Thompson and LeBard (2013) emphasise the importance of stopping science students from seeing scientific tasks that require mathematics or statistics skills as separate and as mathematics or statistics rather than science.

An alternative response to the variation in mathematics skills of the students proposed in the discussion is to focus on teaching the finance concepts in a contexualised way that is less dependent on (high level) mathematics skills (sections 21 and 23). The importance of contextualisation in developing mathematics threshold concepts is emphasised in other research. Galligan, Wandel and Hartle (2010) identify teaching mathematics in (engineering) contexts as a powerful way to develop students' understanding of mathematics. Further, Pettersson (2008) and Scheja and Pettersson (2010) argue that shifts in students' contextualizations of threshold mathematics concepts represent transformative conceptual development.

Conclusions

Our research supports threshold concepts research as an effective technique to investigate curricula (Cousin 2009). Our focus group was well attended and the discussion lively, and academics were very willing to participate in the research and continue to express an interest in the progress of the project. In addition, our research uses threshold concepts to view the finance curriculum from a "whole of program" perspective, which is different to previous research that tends to be more topic based.

Although the proposals for specific threshold concepts in finance include eight statistics concepts, the extent to which mathematics is central to finance is subject to debate in terms of how much to teach and how to teach it. This debate arises, in part, because the curriculum of finance programs is expected to prepare students with varying skills in mathematics for a wide variety of finance roles, which in turn require varying levels of mathematics skills. Whilst institutional responses could reduce the variation in the mathematics skills of the students coming into finance programs and the variation in the jobs the program aims to prepare students for, such responses need institutional support; take time to implement and have an effect; tend to be mitigated by other factors such as the perceptions of students and the job market; and may be unnecessarily limiting.

Given the previous point, this research indicates the value of using threshold concept theory to gain a better understanding of the finance curriculum and the role of mathematics in finance. Further, threshold concepts offer a way to make the role of mathematics, especially statistics in finance, explicit to students. The findings of this research can inform curriculum design and learning and teaching strategies to meet the needs of students who might reasonably be expected to be primarily interested in finance rather than mathematics.

In other research (Hoadley et al. 2015) we focus on refining and/or validating the potential threshold concepts in finance identified in this paper, using the characteristics of threshold concepts identified by Meyer and Land (2003). In addition, we investigate students' perceptions of threshold concepts in finance, with particular attention to the role of modelling, and consider curriculum models that facilitate the development of threshold conceptual knowledge (Hoadley et al. in press).

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Appendix 1 Proposed threshold concepts in finance from focus group and interviews

Refined list	Original list
Arbitrage	Arbitrage (what it does and does not mean); Arbitrage; Arbitrage pricing No-arbitrage arguments; The idea behind the principle of no-arbitrage
Cashflows	Cashflows
Central limit theorem and normal distribution	Statistical concepts of distribution and means variances; Normal distribution
Correlation	Portfolio theory; Risk/return trade offs; Building a model; Critiquing a model; Benefits of diversification
Diversification	Benefit of diversification; Portfolio theory
Expected value	Risk/return trade offs, Probability, Cashflows, Regression
Hedging	Hedging; Principles of hedging (static, dynamic, what can and can't be hedged.); Ways to affect your risk by transferring part of it in insurance, reinsurance; Risk transfer; Finance derivatives
Information asymmetry	Information asymmetry;
Leverage/gearing	Capital structure theory; Risk and leverage
Market efficiency	Market efficiency; Efficient markets; Efficiency argument; Information efficiency (theoretical ideal, why it's only an ideal)
Market structure(s)	Market microstructure; Operation of markets
Modelling	Model risks and failure (the dangers of taking models too seriously); Black Swans; Non-linearity; Risks not captured by models; Building a model; Critiquing a model; Spread sheet modelling skills; Corporate valuation; Equity value
Opportunity cost	Discounted cashflow
Pricing	Pricing; Factor pricing models (theoretical background and as statistical tools); Relative or risk neutral pricing (links to arbitrage including when its application is questionable); Capital asset pricing model (regression); Valuing flexibility – option pricing; Foreign exchange; Corporate valuation; Equity value
Probability/randomness	Probability
Regression to the norm/mean	Regression to the norm/mean; Capital asset pricing model (regression)
Risk	Risk; Risk preferences; Risk measurement; Measurement of risk; Definitions of risk; Systematic risk; Financial risk; Total risk; Financial distress and bankruptcy –VAR models
Risk versus return	Risk v return; Risk and return; Maximisation of shareholder wealth
Short selling	Short selling
Standard deviation	Standard deviation; Variance
Statistical significance and hypothesis testing	Confidence intervals; Statistical uncertainty (estimation error, sensitivity of quantitative methods to estimation error and approaches to alleviating the problems); Multivariate analysis
Time series	Time series models; Time series properties of variables; Random walk
Time value of money	Time value of money; Discounting; The annuity formula; Bond valuations
Trade offs	Risk/return trade offs
Utility/risk preference	Utility theory, Problems with utility theory

A Newcomer's Guide through the Jungle of Capital Structure: The Case of Amazon.com

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We provide a straightforward and comprehensive framework for understanding how changes in capital structure affect firm value through a host of different benefits and costs. Students and practitioners often struggle with both the theory and practice of estimating how leverage might affect firm value, and tend to get lost in the intricacies of this relationship. Furthermore, while leading textbooks provide present-value tools for estimating the tax benefits of debt, quantitative models for estimating financial distress costs are generally not covered. This paper addresses these important gaps, and provides students and practitioners clear qualitative and quantitative tools for understanding the relationship between leverage and value. We also illustrate an application of the framework to Amazon.com's recent debt issuances.
Keywords: Capital structure, Firm value, Debt issuances, Financial distress costs, Bankruptcy cost, Financial flexibility

INTRODUCTION

The relation between capital structure and firm value is a fundamental concept covered in a typical corporate finance course. The myriad ways leverage affects value can be complex and difficult to understand. Most textbooks will discuss in general terms how leverage might affect firm value. However, while textbooks often demonstrate how to estimate the main benefit of debt—the interest tax shields—the value effects of many other capital structure factors are generally not estimated or demonstrated. In particular, quantitative methods for estimating

financial distress costs are often not covered by leading textbooks.[1] In this paper, we address this important gap, and provide students and practitioners clear qualitative and quantitative tools for understanding the relationship between leverage and firm value. Our purpose is to provide a concise yet comprehensive reference for students and practitioners to estimate how changes in capital structure affect firm value by methodically considering various benefits and costs.

Specifically, we focus on both pedagogical treatment and practical application of concepts and tools to the capital structure decision and its attendant value effects. We refer the reader to Turner (2014) for a clear, extensive, and consistent numerical example that focuses primarily on leverage's effects on tax benefits and financial distress costs, and in turn on the cost of capital and value. Our paper is related to Turner (2014) in that it considers tax benefits and financial distress costs. However, our paper departs from this helpful numerical example by including many other capital structure concepts and by applying these concepts to Amazon.com's recent debt issuance decisions. Furthermore, our approach differs significantly from Turner (2014) by also offering concrete tools for estimating the expected costs of financial distress in the messy context of a real-world firm. Additionally, we provide a framework for distinctly estimating the probability of financial distress by including an application of the Z"-score bankruptcy prediction model (Altman, 1993).

CAPITAL STRUCTURE THEORY

Modigliani and Miller's (1958) seminal work demonstrates that if we assume perfect capital markets (no corporate taxes, no transaction or bankruptcy costs, costless information, etc.), then the capital structure decision does not affect firm value. Subsequently, Modigliani and Miller (1963) allows for the existence of tax savings from interest expense at the corporate level, and suggests that firm value is maximized when the debt to asset ratio approaches 100%. However, observing that firms are generally not extremely leveraged, they surmise that their model misses costs associated with debt financing. Miller (1977) argues that the tax savings from interest expense at the corporate level are at least partially offset by the personal tax disadvantage of debt. Myers (1974) presents an Adjusted Net Present Value (APV) model that expresses project value equal to a base case NPV (NPV for an all-equity financed investment), adjusted for the valuation effects of debt.[2] Firm value increases due to debt's tax savings, but decreases due to its non-

[1] E.g., see Brealey, Myers, and Allen (2011), Ross, Westerfield, and Jaffe (2013), Brigham and Ehrhardt (2013), and Higgins (2012). According to Brotherson, Eades, Harris, and Higgins (2013), these four textbooks are the top-selling MBA level textbooks in corporate finance.

[2] For a numerical illustration of the APV approach to firm valuation, see Datar and Emm (2014).

interest costs (financial distress costs, transaction costs, agency costs, etc.). The model described in Myers (1974) is often presented as the *static trade-off theory*, which posits that the optimal level of debt for a firm occurs where the marginal benefit of issuing debt (interest tax savings) is equal to the marginal cost of issuing debt (the expected financial distress costs).

Nearly every corporate finance textbook presents this theory and its familiar inverted u-shaped graph, showing firm value initially increasing with leverage as the marginal tax benefits from debt dominate the marginal financial distress costs, peaking where the marginal benefits equal the marginal costs, and finally decreasing at an increasing rate as financial distress costs dominate debt's tax benefits (see Ross, Westerfield, & Jaffe 2013).

The following subsections discuss the value effects of these two primary benefits and costs advanced by the capital structure literature, as well as other benefits and costs whose value impacts are less easily estimated.

Tax Benefits

Because interest expense is tax deductible, firms realize tax savings by choosing debt over equity. These savings are relatively straightforward to estimate. Letting T be the firm's marginal tax rate and I its interest expense, the annual tax savings amount to $T \times I$. This amount saved is also known as the interest tax shield, and is widely considered the primary benefit of debt financing. Supposing a firm borrows an amount D at an interest rate of k_D, its annual interest expense will equal $k_D \times D = I$. The values of the future tax savings must be discounted to their values today. For a simplified example, assume that the debt amount D will remain outstanding into perpetuity, implying that the firm has already reached its optimal capital structure. Substituting $k_D \times D$ for I, and using the formula for the present value of a perpetuity, the value today of the future stream of interest tax savings equals

$$\frac{T \times I}{k_D} = \frac{T \times k_D \times D}{k_D} = T \times D.$$

Finally, we can formulate the effect of debt on firm value when considering the tax benefits in isolation. Defining T and D as above and assuming perpetual debt, the value of the firm with debt (the levered firm, V_L) is expressed as:

$$V_L = V_U + T \times D,$$

where V_U is the value of the unlevered firm, i.e. the all-equity firm.

In other words, through the issuance of debt, the value of the firm increases by the present value of the interest tax shields, or $T \times D$.[3]

[3] This result is subject to additional assumptions beyond perpetual debt. For example, the firm's tax rate must be constant; and, to realize the full benefit, the firm's EBIT must reliably exceed interest expense in every future period.

Financial Distress Costs

While debt can generate tax benefits, a firm that borrows money might also encounter difficulty repaying it. The firm will incur various costs when it uses too much debt relative to its future ability to repay. Collectively, these costs are known as the costs of financial distress. The costs include direct bankruptcy costs (legal costs, bank fees, etc.), indirect costs (lost profit opportunities, lost sales to concerned customers, deteriorating supplier relationships, etc.), and conflicts of interest (also known as agency costs, which result from the heightened misalignment of interests among managers, shareholders, and creditors due to financial distress).

Importantly, the nature of the firm's business and assets determines both the likelihood and magnitude of these costs (e.g., see Hortaçsu, Matvos, Syverson, & Venkataraman, 2013; Passov, 2003; Shliefer & Vishny, 1992). A firm that produces and sells durable goods will encounter increased financial distress costs when customers learn of their financial hardship—distressed automobile manufacturers generally encounter lower sales as customers logically steer away from cars that might not have a reliable future market for necessary parts or service. Firms with high proportions of intangible assets (e.g., technology and life sciences companies) will encounter distress at lower levels of debt; to compete in their product markets, these firms must make ongoing expenditures in research and development. Any amount of required debt repayment might delay or eliminate these necessary investments, and jeopardize the firm's competitive position, or even its very existence. As Myers (1977) points out, firms whose value consists primarily of intangible investment opportunities—or "growth options"—will generally avoid debt to limit their greater potential loss in value from underinvestment. By contrast, mature firms with few profitable growth opportunities—firms whose "assets in place" create the cash flows that drive firm value—will have lower expected financial distress costs. As a result, these companies will be better suited to higher leverage ratios. Lastly, the nature of the firm's assets affects the value of collateral, which in turn affects the interest rate paid, and the urgency with which banks might pursue liquidation. Banks are more apt to work with a company towards an amicable resolution when the tangible assets securing the debt give the banks greater comfort. Conversely, banks might take more aggressive foreclosure actions when they perceive the collateral is inadequate, and fear the firm could squander remaining cash, or take gambles by investing in risky projects.

As the above discussion illustrates, financial distress costs are more specific to the firm, and more difficult to quantify than tax shield benefits; but, they are no less important to the capital structure decision. Including an expression for the present value of these costs, our levered firm's value becomes

$$V_L = V_U + T \times D - \text{PV(expected financial distress costs)}.$$

To estimate the present value of expected distress costs, a standard approach long offered by academics is to first assess historical default rates based on the firm's credit rating and outstanding debt maturity to determine the probability of financial distress.[4] This probability is then multiplied by an estimate of firm value lost when distress occurs, and the resulting product (the expected costs of financial distress) is then discounted at a risk-free rate. The decrease in firm value can be estimated using an industry-specific percentage loss in enterprise value (EV) if distress occurs (see Passov, 2003). In summary,

$$\textit{Financial distress costs} = \textit{(Probability of financial distress)} \times \textit{(Percentage value loss)} \times \textit{(EV)},$$

where

$$\textit{EV} = \textit{Market value of common stock} + \textit{Book value of interest-bearing liabilities} - \textit{Cash and investments} - \textit{Marketable securities.}$$

Our analysis thus far suggests that management's choice of debt reflects a fundamental trade-off between tax benefits and financial distress costs (a.k.a. static trade-off theory). Managers should consider the following three firm-specific factors when making financing choices:

1. The ability of the firm to realize the interest tax shields over the life of the borrowing.
2. The increased risk, or probability, of financial distress created by debt.
3. The cost to the firm if financial distress occurs.

Flexibility

The above tax benefits vs. distress costs perspective—the static trade-off theory—treats financing decisions as if they are one-time events. A broader perspective considers the individual decisions within the context of a longer-run financing strategy shaped by a firm's growth potential and its access to capital markets over time. Today's decision to incur debt might well affect the firm's future ability to access capital markets. A prudent amount of debt will also depend on the volatility of a firm's cash flows: the higher the volatility, the lower the debt the firm can support. The upper limit of leverage for a particular firm is often referred to as a firm's 'debt capacity.' Selling bonds now could mean that in coming years a firm may be unable to raise meaningful amounts of additional debt without a proportional increase in equity. Having reached its debt capacity,

[4] In this paper, we provide resources and references for estimating these probabilities in the context of a real-world example.

the firm would find itself dependent on the equity market for any additional external financing. This can be a precarious position because, depending on market conditions and recent firm performance, equity may not be available at a reasonable price—or at any price. Lacking funds, the firm might then be forced to forgo attractive investment opportunities. This could prove costly, as the inability to make competitively mandated investments can result in a permanent loss in market position. Consequently, managers' concern for financing future growth can compel firms to issue equity when market conditions allow, thereby maintaining financial flexibility to meet future contingencies.

Importantly, this concern weighs heavily on real-world capital structure decisions: Chief Financial Officers surveyed in Graham and Harvey (2001) state that financial flexibility is the most important determinant of corporate capital structure.

The value of financial flexibility is arguably higher for smaller firms and many larger ones that are unable or unwilling to sell new equity. For these firms, the financing decision transforms from the more typical choice between debt and equity, to instead a choice between issuing debt to fund growth versus avoiding debt to preserve future options at the expense of current growth. These companies necessarily must place their financing decision in the larger context of managing growth. An optimal approach will preserve flexibility by selecting a prudent capital structure and managing the firm's growth within this constraint.

The value of flexibility in the financing decision is thus particular to the firm's circumstances, and analysts will be hard pressed to generate a defensible numerical estimate of this value. Academics and practitioners can confidently apply the knowledge that the notion of flexibility favors the choice of equity over debt, but beyond that perspective we lack tools for estimating the associated value gained from issuing equity, or value lost from issuing debt.

Signaling

While a concern for future financial flexibility customarily favors equity financing today, a persuasive counterargument is the stock market's typical response. Researchers have explored the stock market's reaction to firm announcements regarding future financing, and show that investors mark down the share prices of firms issuing equity by about 3% on average (e.g., see Asquith & Mullins, 1986; Smith, 1986). The average market reaction to new debt offerings, by contrast, is negative but not significantly different from zero (e.g., see Smith, 1986).

There are several explanations for why these price reactions occur. The explanation most strongly supported by empirical evidence is known as *market signaling*. Signaling theory is premised on the idea that managers have better information than investors. Suppose that a firm's top managers—say, due to a

new manufacturing technology developed in-house—are highly optimistic about the company's ability to generate future cash flows. These optimistic managers will pick debt to finance growth, as the strong expected cash flows will ensure low financial distress costs and more assuredly create income to shield from taxes. Additionally, so long as management expects cash flows (approximated by EBIT) to exceed a breakeven level, the new debt will enable managers not only to avoid equity dilution, but also to ultimately produce higher earnings per share (EPS).[5] Conversely, managers uncertain about future prospects of the firm will choose equity financing. Debt obligates the firm to make a fixed set of cash payments through maturity; if any payments are missed, there are likely serious consequences, including bankruptcy. Equity is more forgiving, because although shareholders ultimately expect cash payouts, managers have more discretion over dividends and can reduce or omit them in times of distress. Adding more debt to the company's capital structure serves as a credible signal of strong and stable expected cash flows (e.g., see Ross, 1977).

Thus, an announcement of an impending equity issuance signals to the market that the managers are concerned about the future and have opted for the safe financing choice. The market signal conveyed by issuing debt is just the opposite; top management is optimistic about future prospects.

More generally, evidence suggests that leverage-increasing transactions are associated with positive stock price reactions, and leverage-reducing transactions are associated with negative reactions. For example, a company's announcement of its intention to repurchase shares is typically greeted by a significant increase in share price; conversely, prices fall significantly in response to the issuance of common shares to retire debt (e.g., see Smith, 1986).

A repurchase also might signal the belief that the current stock price is inexplicably low, so low that a share repurchase constitutes a bargain. Indeed, a modestly different signaling story that arrives at the same conclusion we reached above concerns management's beliefs about whether the firm's stock is under- or over-valued. Managers of firms that have profitable uses for more capital, but believe the shares are undervalued, will generally choose to issue debt rather than equity. Conversely, managers who believe their shares are overvalued will issue equity and/or make acquisitions using the firm's stock. This view sees management as exploiting investors by opportunistically selling shares when they are overpriced and repurchasing them when they are underpriced. Regardless of whether management elects to sell new equity because it is concerned about the firm's future or because it hopes to time markets and sell overvalued shares,

[5] The use of debt increases the sensitivity of EPS for varying levels of EBIT. Compared to a baseline scenario, additional debt amplifies EPS for high levels of EBIT (above the breakeven point), but reduces EPS for lower levels. For more details, see chapter 6 of Higgins (2015).

the signal is the same: new equity announcements are bad news and repurchase announcements are good news.

Management Incentives and the Overinvestment Problem

In most financing decisions, incentive effects are not particularly relevant; however, when relevant, incentive effects can play a dominant role. The origin of incentive effects lies in the conflicts of interest between the managers of a firm and its owners, the stockholders. Generally speaking, the use of debt instead of equity reduces the agency costs of equity—the reduction in firm value that arises from the separation of ownership and management control in large, public companies with diffuse ownership. Managers might be inclined to use their autonomy to pursue their own interests rather than those of the owners, who do not own enough of the company to make costly monitoring worthwhile. This separation between ownership and control enables managers to pursue personal interests such as: retaining profits instead of distributing them to owners; engaging in empire building—that is, making uneconomic acquisitions; pursing market share at the expense of profitability; consuming perquisites; and settling for less than excellent performance. As Jensen (1986) argues, large and mature public companies generate substantial free cash flows, also understood as operating cash flows in excess of profitable investment opportunities. The natural inclination of corporate managers is to use this excess cash in the ways discussed above. Arguably the most damaging inclination is managers' *overinvestment* in their core businesses or pursuing value-destroying acquisitions. Unless management can assure investors that it will resist this tendency, firms that aim to maximize shareholders' value should distribute free cash flows to investors.

In this context, a benefit of aggressive debt financing is that it can reduce the gap between the owners' interests and those of the managers. When a company's principal and interest burden is high, management readily understands that it must generate healthy cash flows or risk losing the business and consequently their jobs. With contractually obligated payments looming large, managers must make value-increasing investments and expend maximum effort. The debt payments ensure that managers lack the excess cash to overinvest, and debt generally serves to squeeze out excess capital. An additional benefit of debt financing for mature businesses is the potential for concentration of equity ownership. By concentrating ownership in the hands of management or small investor groups, incentives are better aligned; the rationale behind many leveraged management buyouts stems from the powerful combination of fiscal discipline and concentrated ownership.

As with flexibility and signaling, the value effects associated with managerial incentives are firm- and situation-specific and thus difficult to estimate. However, in those instances where such incentives matter or overinvestment poses a problem

(as in public companies with diffused ownership and substantial free cash flows), debt is the favored capital choice.

Product Market Competition

As discussed in the Financial Distress subsection, firms producing high-technology durable goods are generally not highly leveraged. These firms have the highest costs associated with financial distress, because their stakeholders will be concerned about their long-term viability.

The product market a firm competes in also affects the capital structure choice in other ways. For example, a firm can sometimes benefit from debt if a higher amount of debt enables increased production output (Brander & Lewis, 1988). The firm might benefit from credibly signaling this increased output to its competition. If its competitors do not respond, the added production will likely reduce prices in the market, and thus reduce profits for both the firm and its competitors. However, if the signal is credible, competitors might reduce their own output instead of engaging in a price war. In this case, increased production will increase the firm's profits. Here, the higher leverage creates a greater incentive for the firm to produce at a high level of output. Competitors observe the firm's high leverage, realize the firm will produce at a high level, and to avoid a price war might accommodate the firm's credible threat of high output by producing at a lower level.

On the other hand, a firm's use of debt can also affect the strategies of its competitors in an entirely different direction. A competitor could have incentives to reduce price if that action would threaten the viability of a more leveraged peer. While the competitor's short-term profits will suffer, in the long run the failure and exit of the levered peer will benefit the predatory competitor. This predatory strategy is especially effective in industries where customers, suppliers, and other stakeholders (e.g., employees) are already concerned about the long-term viability of the business (Hortaçsu et al., 2013). Customers become reluctant to transact with a firm that could encounter financial distress, as this creates greater uncertainty regarding future availability of parts and service for durable or high-tech goods. Thus, highly leveraged firms can be particularly vulnerable to predation by more conservatively financed competitors (e.g., see Bolton & Scharfstein, 1990). To conclude, management must consider how leverage affects the particular competitive dynamics of a firm and its industry, which can either make firms more aggressive competitors, or conversely induce managers to choose more conservative capital structures.

Debt as Takeover Defense

Lastly, debt may also be used as a form of takeover deterrence. Firms with high leverage present less opportunity for a corporate raider to acquire the firm

and use its available debt capacity to finance the purchase. Managers, fearing the prospect of unemployment if their firm is acquired, might respond to increased threat of takeover in their industry by increasing debt levels (e.g., see Harris & Raviv, 1988; Sinha, 1991).

This is not an exhaustive list of the factors management must consider when choosing the firm's capital structure and of its potential effects on firm valuation. There exist myriad other considerations and effects that are minor in most situations, but substantial in others. For example, firms with a highly unionized work force might be inclined to use more leverage to enhance the firm's bargaining position with the union. Students and practitioners are encouraged to think through all the effects the choice of debt versus equity might have on the firm's valuation and its relationships with its many stakeholders.

Summary of Capital Structure Value Effects

Combining these ideas, a full accounting of the value effects driven by the choice of debt in the capital structure becomes:

$$V_L = V_U + T + D - \text{PV(financial distress costs)} - \text{PV(flexibility)} \pm \text{PV)signaling effects)} + \text{PV(managerial incentive effects)} \pm \text{PV)other effects).}$$

With some effort, analysts seeking to estimate the change in firm value from the issuance of debt should find defensible, quantitative estimates for the first three terms on the right-hand side of this equation. However, analysts will be challenged to provide quantitatively supportable estimates for the negative value impact from debt-induced decreases in flexibility, the neutral or perhaps positive signaling effects of debt, the potentially positive incentive effects, as well as the other effects listed above. For these last four factors, managers and analysts will likely identify the value effects in more qualitative terms.

Capital structure and the Cost of Capital

It is important to note that the use of leverage in a firm's capital structure also affects the cost of that firm's capital, i.e., its weighted average cost of capital (WACC). Because investors require a lower return from a firm's bonds than from its stock, the use of debt tends to decrease the cost of capital. The interest paid to creditors is also tax deductible, so the after-tax cost is lower still. However, debt also increases risk to shareholders, which drives up the required return on the firm's equity. And as the level of debt is increased, creditors also face more risk, and thus the cost of debt will also increase. Which of these effects dominate depends on each firm's particular situation. In general, for modest levels of debt relative to the firm's expected cash flows, the low interest cost and tax benefits likely reduce the firm's overall cost of capital. For higher levels of leverage, the resulting increase in

the required returns for both equity and debt could well dominate, thus increasing the firm's overall cost of capital.

Here we remind academics and practitioners to consider not only the direct valuation effects engendered by a change in leverage, but also the concurrent changes in the firm's overall cost of capital. We refer the reader to Turner (2014) for an extensive numerical example showing the effects of leverage on a firm's WACC; in particular, see Table 3, p. 130 of Turner (2014). Also see Berry, Betterton, and Karagiannidis (2014) for an interactive spreadsheet application that helps students appreciate how different capital structures affect the WACC.

ILLUSTRATION: THE CASE OF AMAZON.COM

Next, we illustrate some of the effects of leverage discussed above by analyzing changes in capital structure of the U.S. online retailer Amazon.com (NASDAQ: AMZN).

Company Overview and Major Rounds of Debt Financing

Amazon.com (hereafter, "Amazon") founder and CEO Jeff Bezos launched the company in 1995 in his garage in Bellevue, Washington (Jackson, 2014). Bezos's vision was to tap into the then-unknown Internet to create an e-commerce business platform for selling books. Only two months after its launch, Amazon was generating approximately $80,000 in sales per month and was delivering to over 44 countries and states (Jackson, 2014). "By 1998 he was discounting four hundred thousand bestselling titles and his customer base had grown to 3.1 million people using the site" (Brandt, 2011, p. 11). In May 1997, the company raised $54 million through an initial public offering of stock (Amazon.com, Inc., 1998), and is now one of the largest online retailers in the U.S., selling practically every category of consumer product.

Table 1 summarizes the terms of Amazon's three rounds of debt financing. The first debt issuance took place in 1999. The company offered $1.25 billion of 10-year 4¾ percent convertible subordinated notes, the largest U.S. convertible-bond issue at the time (Zuckerman & Anders, 1999). Investors could convert their notes into Amazon's common stock at a conversion price of $156.05 a share any time before maturity, versus a market price of $122.875 at issue (Zuckerman & Anders, 1999). Because Amazon was still unprofitable, many financial analysts argued that this large debt obligation would steer the firm towards bankruptcy. Moody's Investors Service credit rating agency assigned a Caa3 (noninvestment-grade) rating to these notes. The rating was based on "Moody's expectation that Amazon.com is unlikely to generate positive cash flow for at least another two years, and is expected to invest heavily in fixed assets, intangibles, and working

capital in the near term… and the uncertainty of the company's future growth and operating strategies" (Moody's Investors Service, 1999). The company's stock price experienced significant volatility over the following year. However, by September 2008, Amazon became profitable and was able to pay off its convertible debt before the February 2009 due date.

Table 1. Long-term Debt Obligations of Amazon.com

Issue Date	Long-term Debt Obligations	Maturity Date	Face Value (in millions)
	2.60% Unsecured Senior Notes	December 2019	$1,000
	3.30% Unsecured Senior Notes	December 2021	$1,000
2014	3.80% Unsecured Senior Notes	December 2024	$1,250
	4.80% Unsecured Senior Notes	December 2034	$1,250
	4.95% Unsecured Senior Notes	December 2044	$1,500
	0.65% Unsecured Senior Notes	November 2015	$750
2012	1.20% Unsecured Senior Notes	November 2017	$1,000
	2.50% Unsecured Senior Notes	November 2022	$1,250
1999	4.75% Convertible Subordinated Notes	February 2009	$1,250

The table lists Amazon.com's long-term debt obligations by the issue date. The sources of data are the company's 1999 and 2014 Forms 10-K.

Over a decade after this initial bond offering, in 2012 Amazon issued $3 billion of debt to finance, among other projects, its new corporate headquarters in Seattle (McGee & Bensinger, 2012). Moody's assigned an investment-grade rating of Baa1 to this issue, citing "Amazon.com's very good liquidity and strong balance sheet." However, Moody's also noted that the rating was "constrained by Amazon.com's weak EBIT margin relative to its peers and by the sizable decline in EBIT margin currently being experienced as Amazon.com invests in the infrastructure to support its future growth" (Moody's Investors Service, 2012).

In 2014 Amazon announced its historically largest debt issuance of $6 billion, reportedly for "business purposes" (Chiglinsky, 2014). After Amazon's announcement, Moody's upheld the Baa1 senior unsecured rating stating that "proceeds are to be used for general corporate purposes in support of Amazon's myriad growth initiatives, and it is Moody's expectation that the funds will not be utilized for any form of shareholder returns. … Moody's believes that the company's excellent liquidity provides sufficient cushion to affirm the Baa1 rating." However, the agency changed the outlook for the company from stable to negative expressing concerns that "the new debt will further exacerbate Amazon's already weak interest coverage" (Moody's Investors Service, 2014).

Table 2. Consolidated Statement of Operations of Amazon.com, 2010-2015

AMAZON.COM, INC.

CONSOLIDATED STATEMENTS OF OPERATIONS

(in millions, except per share data)

	Years Ended December 31,					
	2010	**2011**	**2012**	**2013**	**2014**	**2015**
Net sales	$34,204	$48,077	$61,093	$74,452	$88,988	$107,006
Cost of sales	26,561	37,288	45,971	54,181	62,752	71,651
Gross profit	7,643	10,789	15,122	20,271	26,236	35,355
Operating expenses:						
Fulfillment	2,898	4,576	6,419	8,585	10,766	13,410
Marketing	1,029	1,630	2,408	3,133	4,332	5,254
Technology and content	1,734	2,909	4,564	6,565	9,275	12,540
General and administrative	470	658	896	1,129	1,552	1,747
Other operating expense (income), net	106	154	159	114	133	171
Total operating expenses	6,237	9,927	14,446	19,526	26,058	33,122
Income from operations	1,406	862	676	745	178	2,233
Interest income	51	61	40	38	39	50
Interest expense	(39)	(65)	(92)	(141)	(210)	(459)
Other income (expense), net	79	76	(80)	(136)	(118)	(256)
Total non-operating income (expense)	91	72	(132)	(239)	(289)	(665)
Income (loss) before income taxes	1,497	934	544	506	(111)	1,568
Provision for income taxes	(352)	(291)	(428)	(161)	(167)	(950)
Equity-method investment activity, net of tax	7	(12)	(155)	(71)	37	(22)
Net income (loss)	$ 1,152	$ 631	$ (39)	$ 274	$ (241)	$ 596

The exhibit shows Amazon.com's consolidated statement of operations for the period of 2010 to 2015. Some items were combined or separated in order to address changes in the structure of the company's income statement through the years. The sources of data are the company's 2011-2015 Forms 10-K.

Table 3. Consolidated Balance Sheet of Amazon.com, 2010-2015

AMAZON.COM, INC.

CONSOLIDATED BALANCE SHEETS

(in millions, except per share data)

	December 31,					
	2010	**2011**	**2012**	**2013**	**2014**	**2015**
ASSETS						
Current assets:						
Cash and cash equivalents	$3,777	$5,269	$8,084	$8,658	$ 14,557	$ 15,890
Marketable securities	4,985	4,307	3,364	3,789	2,859	3,918
Inventories	3,202	4,992	6,031	7,411	8,299	10,243
Accounts receivable, net and other	1,783	2,922	3,817	4,767	5,612	6,423
Total current assets	13,747	17,490	21,296	24,625	31,327	36,474
Property and equipment, net	2,414	4,417	7,060	10,949	16,967	21,838
Goodwill	1,349	1,955	2,552	2,655	3,319	3,759
Other assets	1,287	1,416	1,647	1,930	2,892	3,373
Total assets	$18,797	$25,278	$32,555	$40,159	$ 54,505	$ 65,444
LIABILITIES AND STOCKHOLDERS' EQUITY						
Current liabilities:						
Accounts payable	$ 8,051	$11,145	$13,318	$15,133	$ 16,459	$ 20,397
Accrued expenses and other current liabilities	2,321	3,751	5,684	7,847	11,630	13,502
Total current liabilities	10,372	14,896	19,002	22,980	28,089	33,899
Long-term debt	184	255	3,084	3,191	8,265	8,235
Other long-term liabilities	1,377	2,370	2,277	4,242	7,410	9,926
Stockholders' equity (deficit):						
Common stock, $0.01 par value	5	5	5	5	5	5
Treasury stock, at cost	(600)	(877)	(1,837)	(1,837)	(1,837)	(1,837)
Additional paid-in capital	6,325	6,990	8,347	9,573	11,135	13,394
Accumulated other comprehensive loss	(190)	(316)	(239)	(185)	(511)	(723)
Retained earnings (accumulated deficit)	1,324	1,955	1,916	2,190	1,949	2,545
Total stockholders' equity	6,864	7,757	8,192	9,746	10,741	13,384
Total liabilities and stockholders' equity	$18,797	$25,278	$32,555	$40,159	$ 54,505	$ 65,444

The exhibit shows Amazon.com's consolidated balance sheets for the period of 2010 to 2015. Some items were combined or separated in order to address changes in the structure of the company's income statement through the years. The sources of data are the company's 2011-2015 Forms 10-K.

Tables 2 and 3 present Amazon's consolidated income statements and balance sheets, respectively, for fiscal years 2010-2015. During that period, the company's total long-term liabilities, i.e., the sum of long-term debt and other long-term liabilities, soared from $1.6 billion in 2010 to $18.2 billion in 2015.[6] Figure 1 shows this remarkable increase in the total long-term debt over the six-year period. Interestingly, Amazon tends to carry substantial current liabilities that exceed its total long-term liabilities. When all liabilities are considered, the degree of the company's financial leverage becomes striking. Over time, Amazon's debt-to-equity ratio rose from 1.7 in 2010 to 2.3, 3.0, 3.1, and 4.1 in 2011, 2012, 2013, and 2014, respectively, only slightly decreasing to 3.9 in 2015.

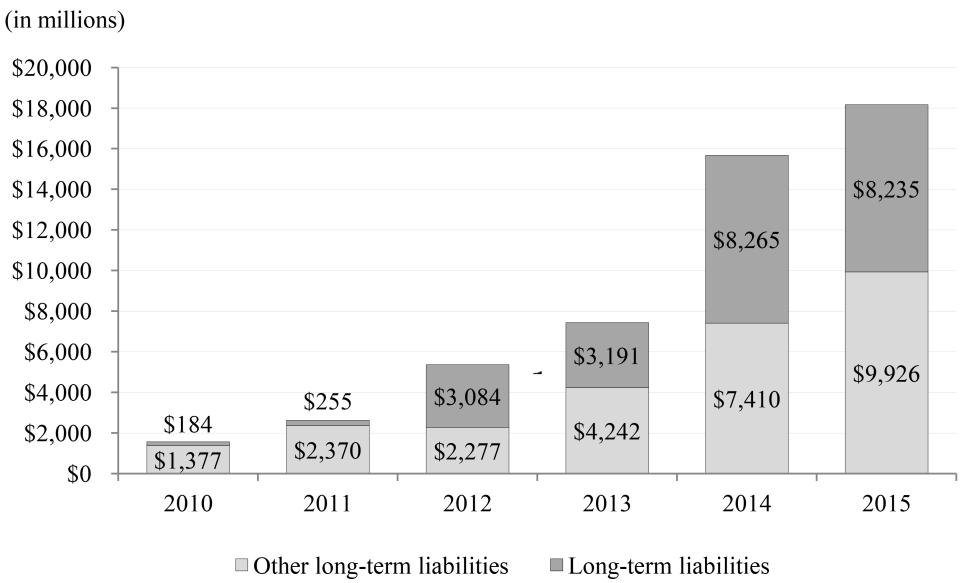

Figure 1. Total long-term liabilities of Amazon.com, 2010-2015. The graph shows book values of long-term liabilities and other long-term liabilities. The sources of data are the company's 2011-2015 Forms 10-K.

Tax Benefits for Amazon

Amazon's debt issuances in 1999, 2012, and 2014 resulted in increasing interest expense on the company's income statements. Table 2 shows that in 2010, Amazon paid $39 million in interest expense. By 2015, the company's annual interest expense ballooned to $459 million. However, as explained earlier, this interest expense generates tax shields, whose present value is T x D (assuming perpetual debt).

[6] The total long-term debt should include any current portion reported as current liabilities. However, we were not able to augment the total long-term debt amounts with the current portions because of data availability.

We can easily estimate the current value of tax benefits for Amazon when it thrice chose to issue long-term debt over equity to finance its growth. For illustrative purposes, we assume that Amazon's income is taxed at a 35% rate, the federal statutory corporate income tax rate in the U.S.[7] With $1.25 billion, $3 billion, and $6 billion in new debt financing in 1999, 2012, and 2014, respectively, the company's incremental present value of tax savings amounts to $437.5 million, $1.05 billion, and $2.1 billion for each issuance. (We assume perpetual debt and calculate the present value of the tax benefit as the tax rate times the debt amount issued.)

Financial Distress Costs for Amazon

As discussed earlier, we can estimate Amazon's financial distress costs as the product of the probability of distress and the value impact of that distress:

$$\textit{Financial distress costs} = \textit{(Probability of financial distress)} \times \textit{(Percentage value loss)} \times \textit{(EV)},$$

where

$$EV = \textit{Market value of common stock} + \textit{Book value of interest-bearing liabilities} - \textit{Cash and investments} - \textit{Marketable securities.}$$

Table 4. Financial Distress Costs of Amazon.com, 2012–2015

(dollar figures in millions, except the stock price)

Firm characteristic	2012	2013	2014	2015
Credit rating (senior unsecured notes)	Baa1	Baa1	Baa1	Baa1
Average maturity (years)	6.6	5.6	12.0	12.0
Default rate (%)	1.760	1.497	3.310	3.310
Stock price	$250.87	$398.79	$310.35	$675.89
Number of shares outstanding	454.55	459.26	464.38	470.84
Market value of equity	$114,033	$183,150	$144,122	$318,237
Book value of total liabilities	$24,363	$30,413	$43,764	$52,060
Cash and cash equivalents	$8,084	$8,658	$14,557	$15,890
Marketable securities	$3,364	$3,789	$2,859	$3,918
Enterprise value (EV)	$126,948	$201,116	$170,470	$350,489

[7] In addition to income taxes in the U.S., Amazon is also subject to taxes in several foreign jurisdictions. Thus, the company's effective tax rate varies significantly from year to year due to variability in its taxable income or losses in different jurisdictions with their own tax rates. This makes calculations of Amazon's effective tax rate challenging.

Table 4. (*Continued*)

(dollar figures in millions, except the stock price)

Firm characteristic	2012	2013	2014	2015
Costs of financial distress (30% x Default rate x EV)	$670	$903	$1,693	$3,480

The table presents Amazon.com's costs of financial distress for the period 2012-2015. Financial distress costs are calculated as follows: *Financial distress costs* = (*Probability of financial distress*) × (*Percentage value loss*) × (*Enterprise Value*), where *Enterprise Value = Market value of common stock + Book value of interest-bearing liabilities − Cash and investments − Marketable securities*. Financial distress costs could not be calculated for years 2010 and 2011 due to Amazon not having bonds outstanding and thus no credit ratings in those years. Stock price is the closing price per share on the last trading day of the year. The sources of data are *Annual Default Study: Corporate Default and Recovery Rates, 1920-2015* (Ou, 2016), finance.yahoo.com, and the company's 2012-2015 Forms 10-K.

Table 4 provides detailed calculations of financial distress costs for Amazon from 2010 to 2015. The probability of default of Amazon in each year is obtained from Moody's 'Annual Default Study: Corporate Default and Recovery Rates, 1920-2015' (Exhibit 35) using Moody's credit rating of Amazon's debt each year and the average maturity of its outstanding long-term debt obligations weighted by face value (Ou et al., 2016). Following Passov (2003), the loss of enterprise value in the event of financial distress in retailing is assumed to be 30%. Table 4 shows that although Amazon's credit rating of Baa1 remained unchanged, the cost of financial distress for the company significantly increased from $670 million in 2012 to $3.48 billion in 2015. This resulted primarily from an increase in the average maturity of its debt obligations due to a large issuance of longer-term notes in 2014 (see Table 1).

Given the non-trivial financial distress costs for Amazon, it is instructive to take a closer look at the likelihood of the company's financial distress. Besides historical default rates, another commonly used method for predicting bankruptcy is the Z-score model developed by Edward I. Altman. Altman (1968) employs multiple discriminate analysis of bankrupt and non-bankrupt U.S. manufacturing firms to determine the factors that best predict financial distress. When the Z-score model was tested on a sample of 86 distressed firms over the subsequent three decades, the model predicted around 80-90 percent of bankruptcies one year prior to the event. However, the model also erroneously predicted bankruptcy in 15-20 percent of firms that did not experience it (Altman, 2000).

To estimate the likelihood of financial distress for Amazon, we use the Z"-score model for non-manufacturing firms, a variation of Altman's original Z-score model. Z"-scores are computed using the following equation (Altman, 1993):

$$Z'' = 3.25 + 6.56X_1 + 3.26X_2 + 6.72X_3 + 1.05X_4,$$

(Table 5 provides variable definitions.)

The Z”-score is a function of four measures of a firm’s performance, specifically, its liquidity (X_1), ability to accumulate and generate earnings using its assets (X_2 and X_3), and financial leverage (X_4). The lower the Z”-score, the higher likelihood of bankruptcy. Firms are classified according to their Z”-scores as follows: Z” < 1.10 = ‘bankruptcy’ classification; 1.10 < Z” < 2.60 = ‘gray’ area; and Z” > 2.60 = ‘non-bankruptcy’ classification. In a study of bankrupt and non-bankrupt retail firms in 2007 and 2008, the Z”-score model correctly predicted financial distress in 90 percent of the sample firms (Hayes, Hodge, & Hughes, 2010).

Table 5. Altman Z”-scores for Amazon.com, 2010-2015

Variable	2010	2011	2012	2013	2014	2015
X_1	0.18	0.10	0.07	0.04	0.06	0.04
X_2	0.07	0.08	0.06	0.05	0.04	0.04
X_3	0.07	0.03	0.02	0.02	0.00	0.03
X_4	0.58	0.44	0.34	0.32	0.25	0.26
***Z”*-Score**	2.5	1.6	1.1	0.9	0.8	0.9

The table shows the Altman *Z”*-score values for Amazon.com for the 2010-2015 period. *Z”*-score values are computed using the following model for non-manufacturing firms (Altman, 2000):

$Z'' = 3.25 + 6.56X_1 + 3.26X_2 + 6.72X_3 + 1.05X_4$,
where

$X_1 = \frac{Current\ assets - Current\ liabilities}{Total\ assets}$,

$X_2 = \frac{Retained\ earnings}{Total\ assets}$,

$X_3 = \frac{Earnings\ before\ interest\ and\ taxes}{Total\ assets}$, and

$X_4 = \frac{Book\ value\ of\ equity}{Total\ liabilities}$.

The lower the *Z”*-score, the higher likelihood of bankruptcy, or financial distress. Specifically, *Z”*-scores in this model are used to classify firms as follows: *Z”* < 1.1 = ‘bankruptcy’ classification; 1.1 < *Z”* < 2.6 = ‘gray’ area; and *Z”* > 2.6 = ‘non-bankruptcy’ classification. The sources of data are the company’s 2011-2015 Forms 10-K.

Table 5 shows Z”-scores for Amazon from years 2010 to 2015. In 2010, Amazon was in the ‘gray’ area with a score of 2.5. This score suggested a low risk of bankruptcy at that time. However, the company’s Z”-score declined over the next four years, slipping into the ‘bankruptcy’ zone beginning with the 2012 debt issuance. As Amazon became more financially leveraged (lower X_4), the company’s liquidity and asset productivity deteriorated (lower X_1, X_2, and X_3). In 2015, Amazon’s Z”-score increased modestly from the prior year, partly due to a slight decrease in its financial leverage (X_4) and its higher asset productivity (X_3). However, the company’s score continued to suggest an elevated risk of bankruptcy.

Summary of Value Effects for Amazon

Using the above framework and analysis, as an example we can approximately estimate the total benefits and costs of Amazon's 2014 debt position.

Tax benefits. As presented above, Amazon's tax benefits from its $6 billion debt issuance in 2014 amounted to $2.1 billion in present-value terms, with the caveat that we have assumed perpetual debt to arrive at this estimate.

Financial distress costs. As shown in Table 4, expected costs of financial distress at Amazon's level of indebtedness amounted to $1.7 billion during 2014.

Static trade-off theory. Based on the above two factors alone, we might conclude that the benefits of Amazon's debt policy in 2014 modestly outweigh the costs, and that debt financing might modestly increase Amazon's value at this degree of financial leverage. On the other hand, managers might be concerned that the firm's 0.8 *Z*"-score implies a high risk of financial distress, despite the firm's relatively comfortable Baa1 credit rating.

However, this is not a full accounting of the potential benefits and costs of Amazon's debt issuance decisions. The other value effects are more difficult to quantify and must be considered in more qualitative terms. Summarizing the potential effects for Amazon, an analyst might arrive at the following conclusions.

Flexibility. As mentioned previously, CFOs regard financial flexibility as the most important determinant of corporate capital structure. Amazon's debt issuances undoubtedly have decreased the firm's financial flexibility; management appears to foresee this fact in the firm's 2008 annual report: "A lack or high cost of credit could limit our ability to obtain additional financing for working capital, capital expenditures, debt service requirements, acquisitions or other purposes in the future, as needed; to plan for, or react to, changes in technology and in our business and competition; and to react in the event of an economic downturn" (Amazon.com, Inc., 2009). The use of debt versus equity could decrease firm value if Amazon's debt capacity becomes an issue that causes the firm to forgo potentially lucrative investment opportunities or important acquisitions.

Signaling. By choosing to issue debt, Amazon avoids the on-average 3% decrease in stock price that might result if the firm were instead to announce an equity issuance (Smith, 1986). Potentially confounding news events and overall market movements make definitive conclusions difficult, but inspecting Amazon's stock price on the debt issuance announcement dates in 2012 and 2014, we observe that it increased 1.6% and 0.10%, respectively. This compares favorably to the on-average negative effect typically expected with equity issuances, as cited above. To appreciate this difference, we can approximately estimate the value effect, using the firm's market capitalization of $144 billion given in Table 4: a 3% averted decrease in Amazon's value as of year-end 2014 translates to (.03)($144 billion) = $4.3 billion in value retained due to the firm's decision to issue debt versus equity.

Managerial incentive effects. Amazon lacks the excess free cash flow that tends to exacerbate agency costs of equity, so these effects are likely not particularly salient here.

Product markets. Amazon's competitors are not well positioned to start a price war or otherwise engage in predatory strategies against the firm. This owes both to Amazon's reputed price leadership in most markets, and the fact that the firm does not principally manufacture and sell big-ticket durable goods. As a result, the value impact through this channel is likely negligible.

The discussion above constitutes a brief example of the quantifiable and qualitative impacts managers might—or should—consider when making capital structure decisions. In Amazon's case, the decision to issue debt in 1999, 2012 and 2014 implies that the firm's managers believed the benefits outweighed the costs.

CONCLUSION

Many students and practitioners struggle to understand the complex relation between leverage and firm value. To help them navigate their way through the dense jungle of capital structure value effects, this paper provides a clear and comprehensive framework for understanding how changes in capital structure affect firm value through myriad benefits and costs. Those considerations include tax benefits, financial distress costs, and other factors such as flexibility, signaling, management incentives, the overinvestment problem, product-market and takeover considerations. Special attention in the paper is given to estimating the costs of financial distress, which is often neglected in textbooks. We also demonstrate the application of the Altman Z"-score as a tool for characterizing the change in risk of bankruptcy. Finally, the effects and complexity of capital structure decisions are illustrated in the case of the U.S. online retailer Amazon.com.

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Financial Market Liquidity: A Review of Contributions to the Literature

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Financial Market Liquidity is an important yet often overlooked topic in undergraduate- and masters-level finance curriculum. This manuscript's intent is to facilitate an understanding of financial market liquidity by advanced-level business students and other interested readers by presenting a review of pertinent contributions to the literature. After considering various definitions of liquidity, we suggest that the most accurate definition of financial market liquidity offered to date is: the costs and time to convert an asset into legal tender. For expository expedience the studies reviewed here are classified into one of three categories; those that attempt to measure liquidity, empirical studies conducted to identify characteristics of liquidity, and institutional factors that impact liquidity. The information presented provides the reader with cutting edge knowledge of important issues associated with financial market liquidity.
Keywords: *bid-ask spread, intraday trading, liquidity, literature review, market depth*

INTODUCTION

The concept of "liquidity" is of interest to both practitioners and academics. Business programs around the globe include coverage of this topic in their Masters and Doctoral level courses. The purpose of this paper is to facilitate the understanding of financial market liquidity by advanced-level students with a review of pertinent contributions to the literature on this topic.

As a prerequisite, consider the definition of liquidity. The word liquid can be a noun or an adjective. When used as the latter, students should appreciate the fact that the meaning of liquid or liquidity, as with other adjectives, can vary based on its' principal subject. According to Merriam-Webster, definitions for

liquid or liquidity include: flowing freely like water, smooth and unconstrained in movement, without friction, or capable of ready conversion into cash. Even when the subject matter is related, such as in financial economics, liquidity can take on several discrete meanings. For example, Wood and Wood (1985) define liquidity as "the inverse of the amount of time that elapses between the decision to sell a security and the receipt of the full market value by the seller." Kluger and Miller (1990) show that this definition is well suited for the real estate market because it quantifies how quickly a fixed asset can be turned into cash. From a corporate finance perspective, however, liquidity usually does not quantify an element of time, but rather it refers to the ability of the firm to meet its debt obligations. Liquidity ratios such as the current ratio or quick ratio quantify a firm's liquidity position. Thus if we say, "firm X is liquid", we mean that they have sufficient cash inflows to satisfy debt obligations.

From a financial markets perspective, liquidity is harder to define. O'Hara (2004) commented on the difficulty of precisely defining liquidity and offered an analogy with pornography, "it is hard to define, but you know it when you see it!" She offered a more precise definition: liquidity relates to the ability to buy and sell assets easily, thus a liquid market (or stock) is one in which buyers and sellers can trade into and out of positions quickly without having large price effects. To illustrate, assume a trader had an equity position of $10 million in each of two assets, stock A and stock B and wanted to immediately exit the positions, thus the trader submits a market sell order for the entire stake in each stock. Suppose each market order is carried out within the day. For stock A, proceeds from the sale totaled $9.85 million, while for stock B proceeds amounted to only $9.20 million. Because liquidation of stock A resulted in the smallest residual loss, according to O'Hara (2004), stock A would be considered to be the more liquid asset.

The simple illustration above highlights one dimension of liquidity, the price impact of trading. Since the price will generally follow the direction of the trade, a higher price impact would imply lower liquidity. Another important dimension of liquidity is how often the asset trades, often called share turnover in equity markets. Stocks with high turnover rates are considered to be liquid stocks, because if there are more shares being bought and sold each day, the easier it is trade. Yet another dimension of liquidity is determined by those that provide it. Some markets designate "liquidity providers" (also known as liquidity suppliers or market makers) such as the NYSE specialists and NASDAQ dealers. The title 'liquidity provider' refers to their responsibility to facilitate trades to both buyers and sellers. Bessembinder et al. (2009):

> "A fundamental issue in trading is the asynchronous arrival of buyers and sellers (Demsetz, 1968). This creates uncertainty as to the amount of time that will be required to locate a counterparty, and regarding the market price that will prevail at the time a trading partner is located. This uncertainty

can be mitigated by the continual presence of 'liquidity suppliers,' who stand ready to serve as counterparties, thereby providing immediacy of trade execution, i.e. 'liquidity'."

Market making is not a charity, thus compensation is required. Market makers need to cover general overhead expenses, potential losses incurred while holding inventory, and losses to informed traders who buy and sell before price changes (Kyle 1985; Glosten et al. 1985). Market makers cover these costs through the bid-ask spread, buying at a lower bid price and selling at a higher ask price. Such transaction costs reduce realized gains and thus are a major component of liquidity. In fact, much of the extant research simply uses quoted or effective spreads as a proxy for liquidity (O'Hara 2004).

The above discussion makes it clear that a simple definition that can fully explain all dimensions of liquidity is difficult to formulate. The O'Hara definition comes close, but lacks consideration of the transaction cost dimension and perhaps other dimensions not considered here. The definition offered by Sarr and Lybek (2002) may be most accurate; they define liquidity as the costs and time to convert an asset into legal tender.

The remainder of the paper is organized into three sections. The next section examines papers that focus on measuring liquidity. The third section contains a review of papers reporting the results of empirical studies conducted to identify various characteristics of liquidity. And in the last section we discuss other institutional factors that can impact liquidity. Of course, new studies are continuously being generated so some very recent works may not be included in our review, but the information presented here provides the interested reader with cutting edge knowledge of issues associated with liquidity.

MEASURING LIQUIDITY

Liquidity is not directly observable and has several dimensions, thus it cannot be captured by a single measure. Sarr and Lybeck (2002) categorize liquidity measures as either transaction cost measures, volume based measures, price-based measures, or market impact measures. In this section we review the most common approaches to measure the several dimensions of liquidity.

TRANSACTION COST MEASURES

The most basic measure of liquidity is the direct cost associated with an equity transaction called the bid-ask spread. The spread is often measured in absolute terms (1), or as a percentage of the bid-ask midpoint (2):

$$Spread = (PA - PB) \tag{1}$$

$$\%Spread = (PA - PB)/((PA + PB)/2) \quad (2)$$

where A is the ask price and B is the bid price. Researchers often use quoted prices as inputs for this metric. For US equities, price quotes are usually obtained through a subset of the Trade and Quotes dataset (TAQ) called the NBBO file (National Best Bid or Offer published by the New York Stock Exchange). As there are thousands of individual dealers and specialists submitting bid and ask quotes, the NBBO reports the lowest ask price and highest bid price available throughout the day.

Because equity transactions do not always occur at the quoted prices, a better measure of the "real" cost of trading is the effective spread described in equation (3):

$$Effective\ Spread_{it} = D_{it}(P_{it} - M_{it})/M_{it} \quad (3)$$

where D_{it} is an indicator variable that equals one for buy orders and negative one for sell orders, and M_{it} is the bid-ask midpoint. Early empirical work using the effective spread show that effective spreads are only half as large as quoted spreads (Petersen and Fialkowski, 1994). Transaction prices are also available in TAQ under the Daily Trades file.

Equity transactions are informative to other traders, just how much information was conveyed can be measured using the price impact measure. While the quoted and effective spread measure transactions costs at time t, the realized spread shows how the price moved after the transaction occurred. Price impact can be estimated as:

$$Price\ Impact_{it} = D_{it}(M_{it+5min} - M_{it})/M_{it} \quad (4)$$

where $M_{it+5min}$ is the bid-ask midpoint five minutes after the trade at time t. While 5 minutes is the industry standard, other time periods, such as 15 or 30 minutes, are also common (Hendershott, Jones, and Menkveld, 2011). Large values for the price impact of trades would indicate a high degree of information asymmetry between traders of that security.

Transaction costs are what market makers charge for their services, and the effective spread is our best estimate of that charge. However, if the market maker is facing information asymmetry with traders (informed traders), then the market makers profits can be dramatically impacted by the price impact of the trade. For example, if the market maker sold inventory to fill an order and the price subsequently improved, then the informed trader benefited on the price improvement at the expense of the market maker. Therefore, to measure the impact on the market maker, we can combine our price impact measure with the effective spread to capture the "realized" spread to the market maker.

$$Realized\ Spread_{it} = Effective\ Spread_{it} - Price\ Impact_{it} \quad (5)$$

VOLUME-BASED MEASURES

A mentioned earlier, the turnover rate is an example of a volume-based measure. The turnover rate measures how many times outstanding shares change hands in a certain period of time. Turnover can be measured as:

$$Turnover = \sum P_i Q_i / S \cdot P \tag{6}$$

where P_i, Q_i are the price and trading volume of the i^{th} trade S, P and are the shares outstanding and the average price of i trades. The higher the turnover rate, the higher is the stock's liquidity. Another form of the turnover ratio is the Hui-Heubel (1984) Liquidity ratio that captures both trading volume and price volatility; but because it is not commonly used we omit its derivation.

Another common volume-based measure is Amihud's (2002) illiquidity metric, which is defined as daily absolute stock return ($|RET_{it}|$) divided by daily dollar volume ($DVol_{it}$) scaled by 10^6:

$$Amihud's\ Illiquidity = \frac{|RET_{it}| * 10_6}{DVol_{it}} \tag{7}$$

This measure is similar in nature to price impact; it measures the dollar trading volume needed to move stock prices. One advantage to using Amihud's measure is that it can be calculated using CRSP data rather than the much larger TAQ dataset.

PRICE-BASED MEASURES

One price-based metric, the Market-Efficiency Coefficient (MEC) developed by Hasbrouck and Schwartz (1988), seeks to measure the ratio between temporary and permanent price movements. MEC is calculated as follows:

$$MEC = \frac{V(R_t)}{T * V(R_t)} \tag{8}$$

where $V(R_t)$ and $V(r_t)$ are the variances of long term returns and short term returns, respectively. And T is the number of short periods in each long period. Porter and Weaver (1997) use one day as the long interval and 30-minutes as the short interval. The closer MEC is to one, the less prices fluctuate when trades occur thus the investor should realize prices closer to the market rate rather than idiosyncratic fluctuations as orders are submitted.

Another price-based metric is the liquidity ratio of Marsh and Rock (1986). It is similar in nature to Amihud (2002), but rather than scaling by volume the Marsh and Rock index scales by the number of transactions in a given time frame. The index is expressed as follows:

$$MR\ Index = \frac{1}{M} \sum_{m=1}^{M} \left| \frac{P_m - P_{m-1}}{P_m} \right| * 100 \tag{9}$$

where M is the total number of transactions for an asset over a given period (e.g., daily), and the summation term represents the total absolute price change between

transactions. If prices move very little between transactions, then that asset is considered to be liquid. A low *MR* index indicates higher liquidity.

MARKET-IMPACT MEASURES

Market-impact measures are designed to describe the impact that trading volume has on stock returns. Hui and Heubel (1984) use the market-model to obtain the portion of a stocks return not explained by systematic risk. The squared residual is then regressed on that day's change in volume:

$$R_{it} = \alpha_{it} + \beta(R_{mt}) + u_{it} \tag{10}$$

$$u_{it}^2 = \gamma_1 + \gamma_2 V_{it} + e \tag{11}$$

where V_{it} is the daily percentage change in dollar volume traded. The more liquid a stock is, the smaller the parameter y_2 is; that is, the smaller the impact volume changes have on stock returns.

As the preceding presentation makes clear there are several ways in which to quantify liquidity. Anecdotally, we have observed that the most frequently used liquidity measures throughout top level journals are the effective spread and share turnover, perhaps two of the easiest measures to compute. In, the next section we focus on the characteristics of liquidity found in empirical research.

CHARACTERISTICS OF LIQUIDITY

To begin our review of liquidity characteristics, we review several papers that document regularities in liquidity. Draper and Paudyal (1997) found that seasonalities exist in several liquidity measures including returns, trading volume, number of trades, average order size, and spreads on the London Stock Exchange. However, their sample was limited to 345 stocks. Ding (1999) found that price volatility is positively related to liquidity as measured by trading costs. Chordia, Roll and Subramanyam (2001) find strong day-of-the-week effects in trading activity and trading costs, that is, trade volume and liquidity are lower on Fridays, while Tuesdays show improvements in both metrics. Lo and Wang (2000) examine weekly turnover data for NYSE and AMEX securities and find a clear time trend from 1962 to 1996. They explain that cross-sectional average turnover is related to expected return, market capitalization, and trading costs. Interestingly, Lo and Wang use one dimension of liquidity to explain another. Therefore, no inference can be made regarding aggregate liquidity. Foster and Viswanathan (1993) explore intraday patterns in trade volume, trading costs, and return volatility using ISSM (Institute for the Study of Security Markets) data from the year 1988. They find that for actively traded firms, adverse selection costs and return volatility are higher in the first half-hour of the day. This evidence is double-edged, while volume

increases (increasing liquidity), costs to market makers also increases (decreasing liquidity). As a result, aggregate liquidity effects are not obvious.

Prior to 2001 the extant literature was limited in breadth and scope, primarily due to limited computing power and data requirements; thus very little research had attempted to explain aggregate liquidity. Chordia et al. (2001) claimed that virtually nothing was known about how aggregate market liquidity behaved over time or what factors cause daily movements in liquidity. Therefore, to quantify market liquidity, they used a variety of liquidity measures as the dependent variable in time series regressions with market return variables, day-of-the-week and seasonal dummies, and macroeconomic variables as explanatory variables. The liquidity measures investigated included trading cost measures such as quoted spread, effective spread, dollar-depth (which refers to the quoted price times the number of shares available), and composite liquidity; as well as trading activity measures such as volume, dollar-volume, and number of trades.

They offered the following five conclusions regarding the determinants of aggregate market liquidity – (1) Trading costs are consistent day-to-day, varying around 2%; while trading volume is much more volatile, ranging from 10-15% each day. (2) An increase in Treasury bond yields relative to the short rate is accompanied by a significant decrease in trading activity, decreased depth, and increased spreads. (3) There are regularities in liquidity. Each measure of liquidity is negatively auto-correlated up to four days, and on day five they are all almost uniformly become positively correlated. (4) Liquidity declines and trading activity slows on Fridays. This is also true on days before major holidays. And (5) both depth and volume increase before important economic news announcements.

The above findings of systematic patterns in liquidity are not the first to be reported. Many of the seminal microstructure studies describe predictable patterns in liquidity. Studies as early as Wood, McInish, and Ord (1985) found that several dimensions of liquidity display a U-Shaped pattern throughout the trading day. McInish and Wood (1992) document that the intraday width of bid-ask spreads for NYSE stocks follow a U-Shaped pattern, where spreads are widest immediately after the open and immediately preceding the close. They used a regression procedure with 12 indicator variables representing half-hour segments of the day (segment 10 omitted). Chan, Christie, and Schultz (1995) use a similar procedure for NASDAQ securities and reported similar results, although spreads did narrow at the close. Liquidity components such as return volatility, trading frequency, number of shares per trade, and the time between trades were all found to behave in a similar fashion. In fact, most of the volatility associated with each component was due to the first 30 minutes of trading. Market liquidity is often measured as the daily value-weighted percentage spread (Chordia, Roll, and Subrahmanyam 2002). McInish and Wood (1992) document a familiar U-shaped (reverse-J) pattern in percentage spreads. After controlling for trade time, they found that the number of trades, shares per trade, spread volatility, and price explain the cross section of

spreads reasonably well. These determinants have become the industry standard when explaining trading costs.

Lee, Mucklow, and Ready (1993) point out that in addition to the bid-ask spread, market depth (which refers to the number of shares available at the quoted price) is also a part of a stock's liquidity. They show that specialists actively manage adverse selection costs by adjusting both spread and depth during times of high information asymmetry. Specifically, they report that spreads widen and depth decreases in response to abnormally high trading volume induced by earnings announcements. Thus, in the presence of asymmetric information specialists strategically decrease liquidity to protect themselves from losses to informed traders. This finding is consistent with Easley and O'Hara's (1992) model, in which specialists use trading volume to infer the presence of informed traders. However, it is inconsistent with the alternative hypothesis, suggested by Harris and Raviv (1993), that increased volume primarily reflects increased liquidity trading and, therefore, higher overall market liquidity. Other papers also find this reaction to increased levels of asymmetric information (e.g., Corwin et al. 2000). Furthermore, spreads remain higher for up to one day while depth returns to normal levels after only three hours. Sadka and Scherbina (2007) also demonstrate that stocks with higher information asymmetry, as measured by analyst disagreement, are on average less liquid.

OTHER FACTORS THAT IMPACT LIQUIDITY

Effects to liquidity are not limited to market price, time, and volatility metrics. Other factors such as the trading exchange, consolidation of trades, transparency, or collusion can have strong impacts on liquidity, specifically on trading costs. Trading costs have been shown to differ across different exchanges. Specialist markets, such as the NYSE, have been shown to have higher volume and lower transaction costs than dealer markets, such as the NASDAQ. Huang and Stoll (1996) were among the first researchers to find that quoted and effective spreads are nearly twice as large for NASDAQ stocks as for NYSE listed stocks. While the exchanges themselves have advanced technologically since the early 1990's, the extant literature maintains that execution costs are still higher in dealer markets (Bessembinder 1999; Chung, Van Ness, Van Ness 2001; Boehmer 2005). To explain the difference in liquidity based on market structure, several studies show that order-fragmentation, common in NASDAQ dealer markets, is the main driver of the higher execution costs (Bennett and Wei, 2006; Nielsson, 2009), although O'Hara and Ye (2011) find evidence to the contrary.

Another research stream seeks to answer the questions, does market transparency improve liquidity? If a market is transparent, then market participants will have the ability to observe information about the trading process, such as the active demand

present in the limit order book. Empirical work in this area is limited, as it is rare that equity markets publish the data required to conduct such analysis. One study conducted by Hendershott and Jones (2005), examines the effects when Island ECN stopped displaying its limit order book in its three most active exchange traded funds (ETFs). They found that Island executed a lower percentage of the ETFs orders, and that trading volume, price discovery metrics, effective, and realized spreads all worsened after the decrease in transparency. However, Madhavan, Porter, and Weaver (2005) concluded that increased transparency reduces market quality as trading costs increased and equity prices decreased following the rule change. Still, other work provides evidence that transparency generates positive liquidity movements (Bessembinder et al. 2006), while others show the change to a more anonymous market improves liquidity (Comerton-Forde et at., 2009). Christie and Schultz (1994) and Christie et al. (1994) made infamous NASDAQ dealers who established a systematic avoidance of odd-eighth quotes, thereby artificially doubling transaction costs and decreasing market liquidity. Chung, Van Ness, and Van Ness (2001) show that this collusion was responsible for 49 percent of the difference between NYSE and NASDAQ's trading costs.

As the preceding presentation makes clear, liquidity has many dimensions, each unique as to their determinants and impact on trading environments. Still today there is much debate on the benefits of liquidity, and whether new phenomenon such as algorithmic trading adds to or detracts from market liquidity.

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